CHAPTER 1

VECTORS

watch video

Vectors and Vector Addition

Let us discuss the differences between scalar and vector quantities through examples.

NOTE: The magnitude of the vector $\vec{A}$ can be denoted by $|\vec{A}|$ or A.

EXAMPLE 1: Which of the following equations involving vectors can be valid?

I. $\vec{A} = 2$ m/s

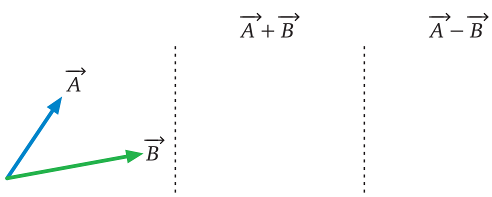
II. $\vec{A} + \vec{B} = \vec{C}$

III. $K = \vec{L} + \vec{M}$

Addition and Subtraction of Vectors

Let us discuss why addition and subtraction operations are needed for vectors.

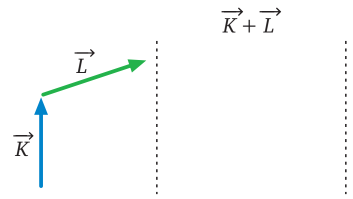
Let us carry out the addition and subtraction of the vectors $\vec{A}$ and $\vec{B}$ shown in the figure.



NOTE: By adding one or more vectors, a **resultant vector** is obtained.

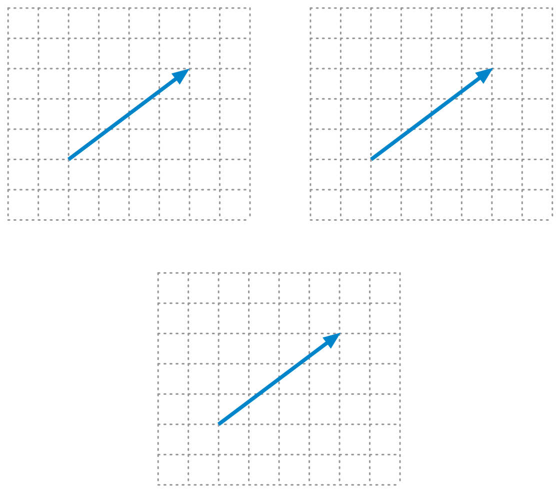
Your turn

1. For the vectors $\vec{K}$ and $\vec{L}$ shown in the figure, find the requested vectors.

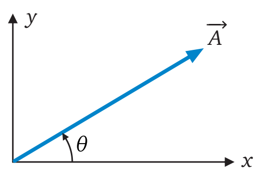


Resolving Vectors into Components

Let us represent the vectors given below on equal unit squares in different coordinate systems. Let us find the components of the vectors and express the vectors in terms of their components.



Let us find the components of a vector shown in the Cartesian coordinate system and express the vector in terms of its components. Let us also express the magnitude of the vector in terms of its components.



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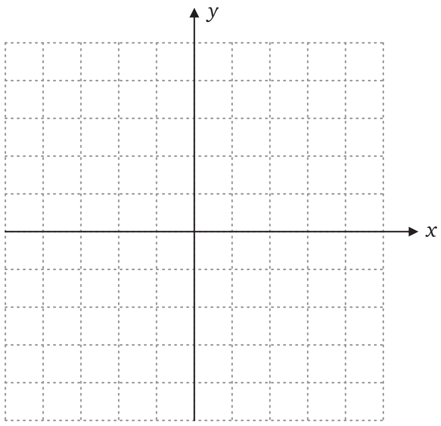
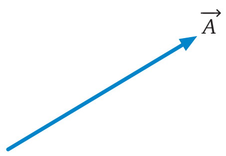
EXAMPLE 2: Draw the vectors $\vec{A} = (3,2)$, $\vec{B} = (-2,2)$, and $\vec{C} = (-1,-4)$ on the coordinate system shown in the figure.



Operations with Vector Components

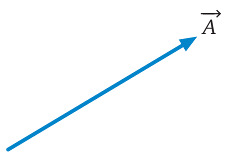
1. Finding the magnitude and direction of a vector

Let us examine how to find the magnitude and direction of a vector.



2. Multiplication of a vector by a scalar

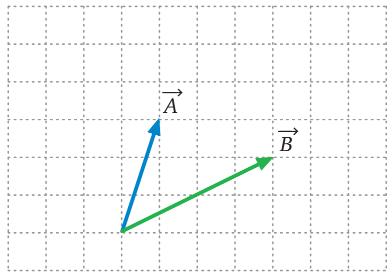
Let us examine the multiplication of a vector by a scalar.



NOTE: Multiplying a vector by a nonzero scalar does not change the direction of that vector.

3. Addition of Vectors

Let us examine how vector addition is carried out.

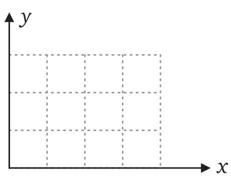


Your turn

2. Given $\vec{A} = (1,-2)$ and $\vec{B} = (3,1)$, find the quantities $|\vec{A}|$, $2\vec{B}$, $\vec{A} + \vec{B}$, and $2\vec{B} - \vec{A}$.

Unit Vectors

Let us examine the unit vectors of the Cartesian coordinate system in two dimensions. Let us express these vectors in terms of their components.



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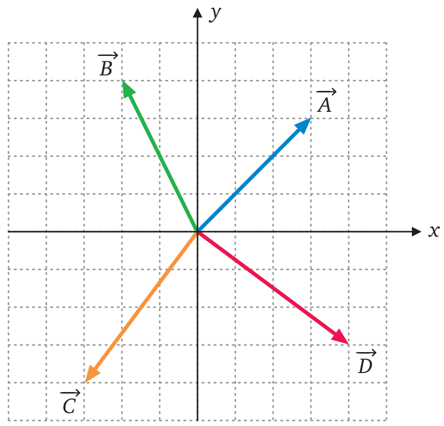
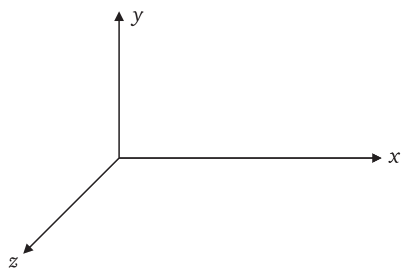
Let us express the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ shown in the Cartesian coordinate system consisting of equal unit squares in terms of unit vectors.

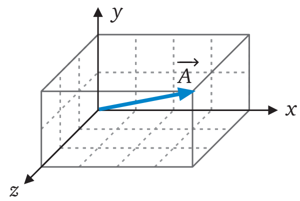


Let us examine the unit vectors of the Cartesian coordinate system in three dimensions. Let us express these vectors in terms of their components.



NOTE: In the Cartesian coordinate system, the unit vectors in the directions of the x, y, and z axes are denoted in many sources by $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, respectively. In some sources, these vectors may also be denoted by $\hat{x}$, $\hat{y}$, and $\hat{z}$.

EXAMPLE 3: Express the vector $\vec{A}$ shown in the Cartesian coordinate system consisting of equal unit squares in terms of unit vectors.



NOTE: Any vector $\vec{A}$ can be expressed as $\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$.

EXAMPLE 4: Given $\vec{v_1} = (\hat{\imath} + \hat{\jmath} - 2\hat{k})$ m/s and $\vec{v_2} = (-2\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$ m/s, obtain the vector $2\vec{v_1} + \vec{v_2}$.

NOTE: In physics, vector quantities are generally expressed with units. For example, the acceleration due to gravity can be written as $\vec{g} = -9,81\hat{\jmath}$ m/s^2.

Your turn

3. Given $\vec{M} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$, find the vectors $-\vec{M}$ and $2\vec{M}$.

4. Given $\vec{F_1} = (\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ N and $\vec{F_2} = (-\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$ N, find the vector $\vec{F_T} = \vec{F_1} - \vec{F_2}$.

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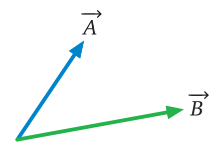
5. Given $\vec{A} = 3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$, find the value of $|\vec{A}|$.

Operations on Vectors

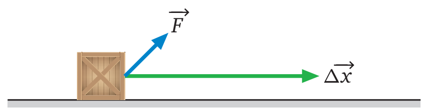
Scalar Product

Let us discuss the meaning of the scalar product for the vectors $\vec{A}$ and $\vec{B}$ shown in the figure and define the scalar product.

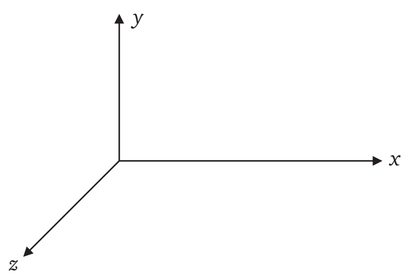


> **NOTE:** Another name for the scalar product is the dot product. The operation $\vec{A} \cdot \vec{B}$ cannot be written as $\vec{A}\vec{B}$. For any two vectors $\vec{A}$ and $\vec{B}$, $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$.

Let us examine the physical meaning of the scalar product through the example of "work done".



Let us examine the scalar products of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ with one another.



For any two vectors $\vec{A}$ and $\vec{B}$, let us write the result of the operation $\vec{A} \cdot \vec{B}$ by using the scalar products of the unit vectors.

EXAMPLE 5: Given $\vec{A} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{B} = -\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, find the scalar product $\vec{A} \cdot \vec{B}$.

EXAMPLE 6: Express the following scalar product operations in different forms.

I. $\vec{A} \cdot \vec{A} =$

II. $\vec{A} \cdot (k\vec{B}) =$

III. $\vec{A} \cdot (\vec{B} + \vec{C}) =$

> **NOTE:** By using the scalar product of two vectors, we can find the angle between them.

EXAMPLE 7: Given $\vec{A} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{B} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$, find the angle between the vectors $\vec{A}$ and $\vec{B}$.

> **NOTE:** For any two nonzero vectors $\vec{A}$ and $\vec{B}$, if $\vec{A} \cdot \vec{B} = 0$, then the vectors are perpendicular to each other.

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Your turn

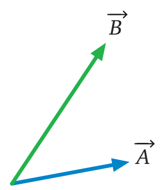
6. Given $\vec{A} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{B} = -\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$, find the scalar product $\vec{A} \cdot \vec{B}$.

7. Given $\vec{M} = (\hat{\imath} - 2\hat{k})$ N and $\vec{N} = (-2\hat{\jmath} - \hat{k})$ m, find the scalar product $\vec{M} \cdot (2\vec{N})$.

8. Find the angle between the vectors $\vec{A} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{B} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$.

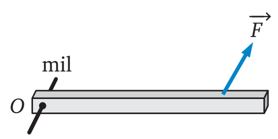
Vector Product

Let us discuss the meaning of the vector product for the vectors $\vec{A}$ and $\vec{B}$ shown in the figure and give the definition of the vector product.

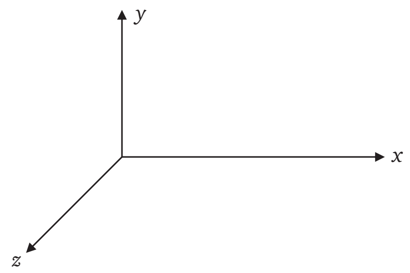


NOTE: For any two vectors $\vec{A}$ and $\vec{B}$, $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$.

Let us examine the physical meaning of the vector product through the example of "torque".



Let us examine the vector products of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ with one another.



NOTE: The coordinate system given above is consistent with the right-hand rule. When drawing the x, y, and z axes of this coordinate system, the relation $\hat{\imath} \times \hat{\jmath} = \hat{k}$ is used.

For any two vectors $\vec{A}$ and $\vec{B}$, let us write the result of the operation $\vec{A} \times \vec{B}$ by using the vector products of the unit vectors.

Let us also express the vector product of $\vec{A}$ and $\vec{B}$ by making use of the determinant operation.

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EXAMPLE 8: Express the following vector product operations in different forms.

I. $\vec{A} \times \vec{A} =$

II. $\vec{A} \times (k\vec{B}) =$

III. $\vec{A} \times (\vec{B} + \vec{C}) =$

EXAMPLE 9: Given $\vec{A} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{B} = -\hat{\imath} - \hat{\jmath} + 3\hat{k}$, find the vector $\vec{C} = \vec{A} \times \vec{B}$.

EXAMPLE 10: Given $\vec{M} = 2\hat{\imath} + \hat{\jmath} + \hat{k}$ and $\vec{N} = 4\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$, find the vector $\vec{L} = \vec{M} \times \vec{N}$.

NOTE: For any two nonzero vectors $\vec{A}$ and $\vec{B}$, if $\vec{A} \times \vec{B} = 0$, then the vectors are parallel.

Your turn

9. Given $\vec{A} = \hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{B} = -\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$, find the vector $\vec{C} = \vec{A} \times \vec{B}$.

10. $\vec{M} = \hat{\imath} - 2\hat{k}$ and $\vec{N} = -2\hat{\jmath} - \hat{k}$, given that;

I. find the vector $\vec{P} = \vec{M} \times \vec{N}$.

II. show that the vector $\vec{P}$ is perpendicular to the vectors $\vec{M}$ and $\vec{N}$. (Hint: The scalar product of perpendicular vectors is zero.)

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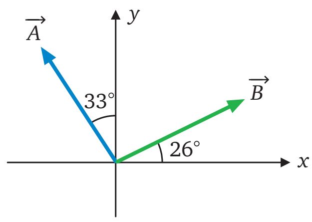
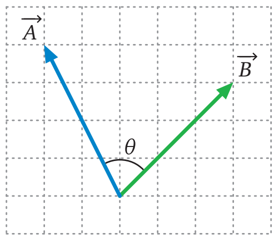
Numerical Problems with Vectors

NOTE: In problems encountered at the university level, we sometimes need to use a calculator.

EXAMPLE 11: $A = 4$ N and $B = 5$ N, let us write the vector $\vec{C} = \vec{A} + \vec{B}$ in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

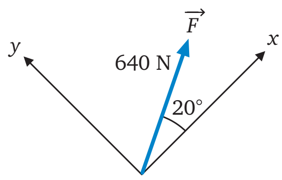


EXAMPLE 12: Find the angle between the vectors $\vec{A}$ and $\vec{B}$ shown in the figure.

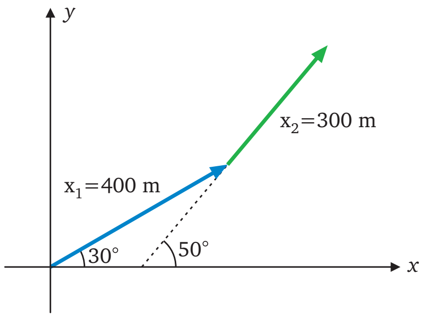


Your turn

11. Write the vector $\vec{F}$ shown in the figure in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.



12. The displacement of a vehicle in two stages is shown in the coordinate system in the figure. Accordingly, what is the total displacement $\Delta\vec{x}$ of the vehicle?



Exercises and Problems

1. $\vec{A} = (1, 0, 1)$ and $\vec{B} = (1, -1, 2)$, find the quantities $|\vec{A}|$, $2\vec{B}$, $\vec{A} - \vec{B}$, and $\vec{B} - 2\vec{A}$.

2. $\vec{A} = -\hat{\imath} + 4\hat{\jmath} + 3\hat{k}$ and $\vec{B} = 2\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$, given that;

I. $|\vec{A}|$

II. $|\vec{A} + \vec{B}|$

III. $-2\vec{A}$

IV. $\vec{A} - 2\vec{B}$

find these quantities.

3. $\vec{A} = (2, 3, 1)$ and $\vec{B} = (1, -1, a)$ are perpendicular to each other. What is a?

4. $\vec{K} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{L} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ form an angle θ. What is $\cos\theta$?

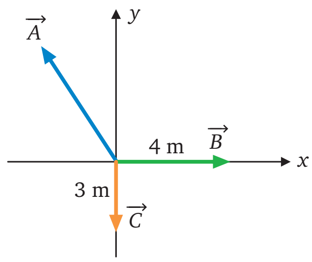
5. $\vec{A} = -\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $\vec{B} = \hat{\imath} - 2\hat{\jmath}$, given that;

I. $\vec{C} = \vec{A} \times \vec{B}$

II. $\vec{D} = \vec{B} \times \vec{A}$

find the vectors.

6. For the vectors shown in the figure, given that $\vec{A} + \vec{B} + \vec{C} = (\hat{\imath} + \hat{\jmath})$ m, find the vector $\vec{A}$.



7. A particle moves with velocity $\vec{v_1} = (2\hat{\imath} - \hat{\jmath} - 2\hat{k})$ m/s. After some time, its velocity becomes $\vec{v_2} = (-\hat{\imath} + \hat{\jmath} + \hat{k})$ m/s. What is the velocity change vector $\Delta\vec{v}$?

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8. $\vec{A}$, $\vec{B}$, and $\vec{C}$ are vectors. For the expressions defined with these vectors

I. $(\vec{A}+\vec{B})\cdot\vec{C}$

II. $(\vec{A}-\vec{B})\times\vec{C}$

III. $(\vec{A}\cdot\vec{B})\times\vec{C}$

IV. $(\vec{A}\times\vec{B})\times\vec{C}$

which of these operations are mathematically defined?

9. Given $\vec{A}=\hat{\imath}-\hat{\jmath}+2\hat{k}$ and $\vec{B}=3\hat{\imath}-\hat{\jmath}-2\hat{k}$, show using the vector product that $\vec{A}$ and $\vec{B}$ are perpendicular to each other.

10. What is the cosine of the angle between the vector $\vec{F}=\hat{\imath}+\hat{\jmath}+\hat{k}$ and the x-axis? (Hint: The unit vector in the positive x-direction can be used.)

11. Find the unit vector in the same direction as the vector $\vec{a}=\hat{\imath}-\hat{\jmath}+2\hat{k}$.

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12. For any vector $\vec{A}$;

I. $\vec{A}\cdot\vec{A}=A^2$

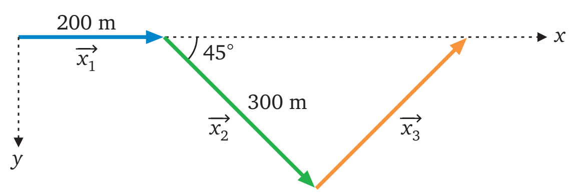
II. $\vec{A}\times\vec{A}=0$

show these equalities.

13. For any two vectors $\vec{A}$ and $\vec{B}$, show that $\vec{A}\cdot(\vec{A}\times\vec{B})=0$.

14. An object moves in the xy plane with velocity $\vec{v}$. Given that $v=50$ m/s and $v_x=-30$ m/s, what values can v_y take?

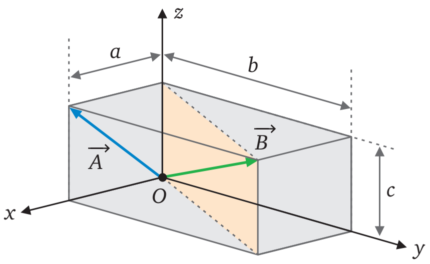
15. The displacements of a moving object in three stages are shown in the figure. Given that the object's resultant displacement is 700 m, find the vector $\vec{x_3}$.



16. In the rectangular coordinate system shown in the figure, the vectors $\vec{A}$ and $\vec{B}$ are given. For these vectors, find the following products.

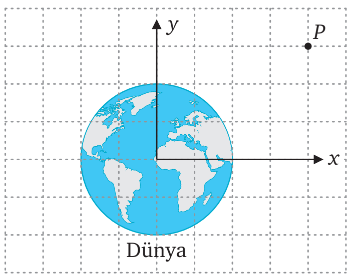
I. $\vec{A} \cdot \vec{B}$

II. $\vec{A} \times \vec{B}$

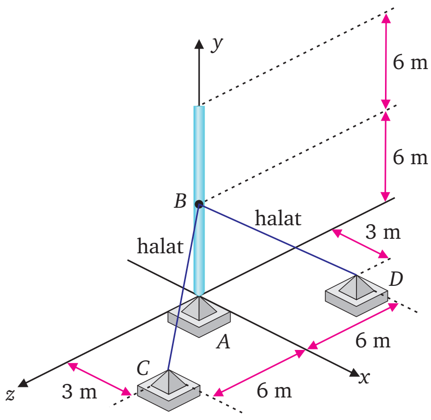


17. At a certain instant, a particle has position $\vec{r} = 2\hat{\imath}$ m and momentum $\vec{p} = 15\hat{\jmath}$ N · s. Since the angular momentum of the object about the origin is $\vec{L} = \vec{r} \times \vec{p}$, find the vector $\vec{L}$.

18. Find the unit vector directed from point P toward the center of the Earth on the equal unit squares shown.



19. In the figure, two ropes attached to point B are used to prevent a pole from tipping over. What is the measure of the acute angle between these two ropes? (Hint: Vectors can be defined in the same directions as the line segments [BD] and [BC], and then the angle between these two vectors can be found.)



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Answers

Example questions

1	Only II
2	Go to the video.
3	$\vec{A} = 4.0\hat{\imath} + 2.0\hat{\jmath} + 3.0\hat{k}$
4	$2\vec{v}_1 + \vec{v}_2 = -1.0\hat{k}$ m/s
5	$\vec{A} \cdot \vec{B} = 6.0$
6	I. A^2 II. $k(\vec{A} \cdot \vec{B})$ III. $\vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$
7	$\theta = 60°$
8	I. $\vec{0}$ II. $k(\vec{A} \times \vec{B})$ III. $(\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$

9	$\vec{C} = 2.0\hat{\imath} - 5.0\hat{\jmath} - 1.0\hat{k}$
10	$\vec{L} = 0$
11	$\vec{C} = (2.32\hat{\imath} + 5.55\hat{\jmath})$ N
12	$\theta \approx 71.6°$

Your turn questions

1	Go to the video.
2	$\lvert\vec{A}\rvert = \sqrt{5}$ $2\vec{B} = (6, 2)$ $\vec{A} + \vec{B} = (4, -1)$ $2\vec{B} - \vec{A} = (5, 4)$
3	$-\vec{M} = -2\hat{\imath} + \hat{\jmath} - 4\hat{k}$ $2\vec{M} = 4\hat{\imath} - 2\hat{\jmath} + 8\hat{k}$
4	$\vec{F}_T = (2\hat{\imath} + 4\hat{k})$ N
5	$\lvert\vec{A}\rvert = 5\sqrt{2}$
6	$\vec{A} \cdot \vec{B} = -7$
7	$\vec{M} \cdot (2\vec{N}) = 4$ Nm
8	$\theta = 60°$
9	$\vec{C} = 2\hat{\imath} - \hat{k}$
10	$\vec{P} = -4\hat{\imath} + \hat{\jmath} - 2\hat{k}$ Go to the video.
11	$\vec{F} = (601.4\hat{\imath} + 218.9\hat{\jmath})$ N
12	$\Delta\vec{x} = (539.2\hat{\imath} + 429.8\hat{\jmath})$ m

Exercises and problems

1	$\lvert\vec{A}\rvert = \sqrt{2}$ $2\vec{B} = (2, -2, 4)$ $\vec{A} - \vec{B} = (0, 1, -1)$ $\vec{B} - 2\vec{A} = (-1, -1, 0)$
2	$\lvert\vec{A}\rvert = \sqrt{26}$ $\lvert\vec{A} + \vec{B}\rvert = \sqrt{5}$ $-2\vec{A} = 2\hat{\imath} - 8\hat{\jmath} - 6\hat{k}$ $\vec{A} - 2\vec{B} = -5\hat{\imath} + 8\hat{\jmath} + 9\hat{k}$
3	$a = 1$
4	$\cos\theta = \dfrac{1}{\sqrt{21}}$
5	$\vec{C} = 4\hat{\imath} + 2\hat{\jmath}$ $\vec{D} = -4\hat{\imath} - 2\hat{\jmath}$
6	$\vec{A} = -3\hat{\imath} + 4\hat{\jmath}$
7	$\Delta\vec{v} = (-3\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$ m/s
8	I, II, and IV
9	Go to the video.
10	$\cos\theta = \dfrac{\sqrt{3}}{3}$
11	$\hat{a} = \dfrac{1}{\sqrt{6}}(\hat{\imath} - \hat{\jmath} + 2\hat{k})$
12	Go to the video. Go to the video.
13	Go to the video.
14	$v_y = \pm 40$ m/s
15	$\vec{x}_3 = (287.87\hat{\imath} - 212.13\hat{\jmath})$ m
16	$\vec{A} \cdot \vec{B} = a^2 + c^2$ $\vec{A} \times \vec{B} = -bc\hat{\imath} + ab\hat{k}$
17	$\vec{L} = 30\hat{k}$ N·m·s
18	$\hat{u} = -\dfrac{4}{5}\hat{\imath} - \dfrac{3}{5}\hat{\jmath}$
19	$\theta = 83.62°$

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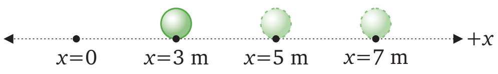
CHAPTER 2

MOTION IN ONE DIMENSION

watch video

Position, Displacement, and Average Velocity

An object starts from the point $x = 3$ m on the axis shown in the figure, is first moved to the point $x = 7$ m, and then to the point $x = 5$ m. Using the motion of the object, let us discuss the concepts of position, displacement, and distance traveled.

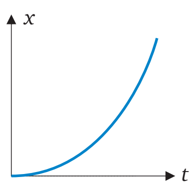


Let us define the concepts of position, displacement, and distance traveled for motion in one dimension.

Let us define the concepts of average velocity and average speed for motion in one dimension.

NOTE: In one-dimensional motion, the distance traveled and the average speed may not always be equal to the magnitudes of the displacement and average velocity vectors.

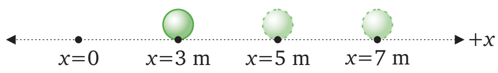
Let us obtain the average velocity over a given time interval from the position–time graph of an object that starts moving from rest.



EXAMPLE 1: In the figure, the object is at position $x = 3$ m at $t = 0$, at position $x = 7$ m at $t = 2$ s, and at position $x = 5$ m at $t = 4$ s. For the time intervals $0 - 2$ s, $2 - 4$ s, and $0 - 4$ s;

a. find the displacements of the object,
b. the distances traveled by the object,
c. the average velocities of the object,
d. the average speeds of the object

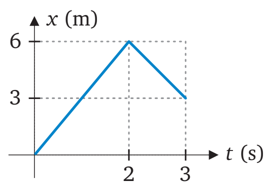
let us determine them.



EXAMPLE 2: In the figure, the position–time graph of an object moving in one dimension is given. For the object, over the time interval $0 - 3$ s,

a. find the amount of displacement,
b. the distance traveled,
c. the magnitude of the average velocity,
d. the magnitude of the average speed

let us determine them.



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Your turn

1. Selin starts walking on a straight track in a park that extends in the north–south direction. From the moment she starts walking, she walks 300 meters northward for 5 minutes. Then she turns back and walks 600 meters southward for 10 minutes. Accordingly, for Selin's 15-minute walk;

a. find the distance traveled,

b. her displacement,

c. her average speed,

d. her average velocity

determine them.

2. A vehicle moves in one dimension on a road extending in the east-west direction. Starting from point A, the vehicle moves to point B with a constant speed of 72 km/h. It then starts moving westward from point B with a constant speed of 54 km/h and arrives at point C after 10 seconds. Accordingly, for the vehicle along the path A-B-C, determine the;

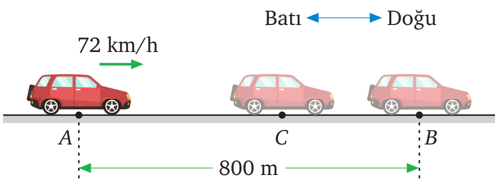
a. displacement,

b. distance traveled,

c. average speed,

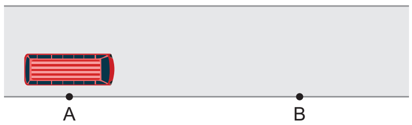
d. average velocity

find.

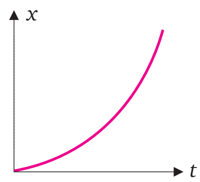


Instantaneous Velocity and Instantaneous Speed

Let us discuss what instantaneous velocity is by making use of the motion of a bus traveling in one dimension from stop A to stop B over a certain time interval.



Let us examine how instantaneous velocity is obtained by using the position-time graph in the figure.



Let us define the x-component of instantaneous velocity using the derivative.

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EXAMPLE 3: The position equation of a toy vehicle as a function of time is given as $x(t) = 15\text{ cm} + \left(10\text{ cm/s}^2\right)t^2$.

a. Let us find the positions of the vehicle at $t = 0$ and $t = 2$ s.

b. Let us find the displacement of the vehicle over the interval from $t = 0$ to $t = 2$ s.

c. Let us find the average velocity of the vehicle over the interval from $t = 0$ to $t = 2$ s.

d. Let us obtain the velocity of the vehicle as a function of time.

e. Let us find the velocities of the vehicle at $t = 0$ and $t = 2$ s.

f. Let us draw the graph of the vehicle's velocity as a function of time.

Your turn

3. A particle moves according to the position function $y(t) = 30\text{ m} - (6.0\text{ m/s})t - \left(1.0\text{ m/s}^2\right)t^2$. Accordingly, for the particle, determine the;

a. positions at $t = 0$ and $t = 9$ s,

b. average velocity over the interval $0 - 9$ s,

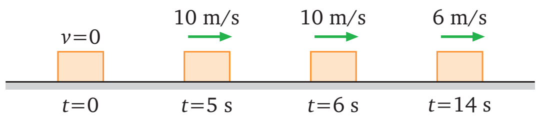
c. average speed over the interval $0 - 9$ s,

d. velocity at $t = 4$ s

find.

Acceleration

Let us discuss the physical meaning of acceleration by making use of the motion of the vehicle in the figure.



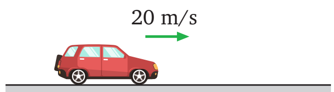
Average Acceleration

Let us define the x-component of the average acceleration.

For the motion of the object given above, let us calculate the average acceleration for the time intervals $0 - 5$ s, $5 - 6$ s, $6 - 14$ s, and $0 - 14$ s.

Instantaneous Acceleration

The vehicle in the figure is moving with a speed of 20 m/s. The driver first takes their foot off the accelerator and, after a while, applies the brakes. Using the motion of this vehicle, let us discuss what instantaneous acceleration is.



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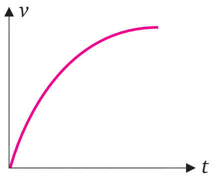
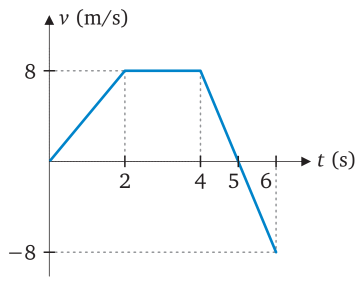
Let us examine how instantaneous acceleration can be obtained from the velocity-time graph in the figure.



Let us define the x-component of instantaneous acceleration using the derivative. Let us examine how instantaneous acceleration is obtained from position.

EXAMPLE 4: The velocity-time graph of an object is given in the figure. For the object, let us find the average acceleration;

a. in the interval 0 — 2 s,
b. in the interval 2 — 4 s,
c. in the interval 4 — 5 s,
d. in the interval 5 — 6 s.



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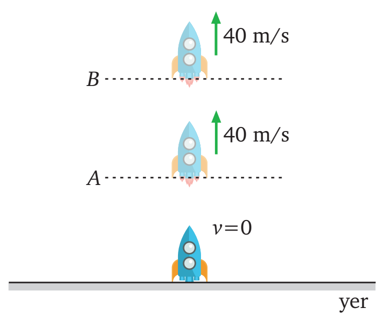
EXAMPLE 5: The time-dependent velocity equation of a particle moving in one dimension is given as $v(t) = -3\ \text{m/s} + \left(2\ \text{m/s}^3\right) t^2$.

a. Let us find the particle's velocities at $t = 1$ s and $t = 3$ s.
b. Let us find the change in the particle's velocity over the interval from $t = 1$ s to $t = 3$ s.
c. Let us find the particle's average acceleration over the interval from $t = 1$ s to $t = 3$ s.
d. Let us obtain the particle's acceleration as a function of time.
e. Let us find the particle's accelerations at $t = 1$ s and $t = 3$ s.
f. Let us draw the graph of the particle's acceleration as a function of time.

Your turn

4. The rocket prototype shown in the figure, prepared by a group of students in their project course, starts moving from rest at ground level and continues in vertical motion. The rocket reaches level A in 5 seconds with constant acceleration, and its speed at that level becomes 40 m/s. After level A, it continues moving for 5 more seconds without changing its speed of 40 m/s and reaches level B. Accordingly, for the rocket, find;

a. the average acceleration during the first 5 seconds,
b. the average acceleration during its 10-second motion.



5. The time-dependent velocity equation of a vehicle moving in one dimension is given as $v(t) = 3.5\ \text{m/s} + \left(0.70\ \text{m/s}^3\right) t^2$. It is known that this equation is valid over a certain time interval. For this time interval;

a. find the time-dependent equation of the vehicle's acceleration,

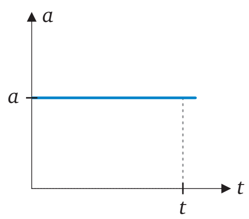
b. the speed and acceleration of the vehicle at $t = 3.0$ s,

c. and the acceleration of the vehicle when its speed is 21 m/s

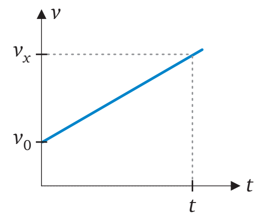
find.

Motion with Constant Acceleration in One Dimension

Using the definition of acceleration, let us find the change in velocity over a time interval t for the object whose acceleration-time graph is given. Then let us obtain the time-dependent equation of the object's velocity.



Using the velocity-time graph of the same object, let us obtain its average velocity in terms of v_0, a, and t. Then, using this result, let us find the displacement of the object over the time interval t and then obtain its time-dependent position equation.



Using the time-dependent equations of position and velocity, let us obtain the kinematic equation that does not contain time.

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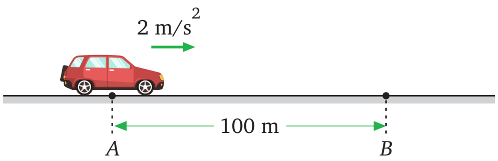
EXAMPLE 6: The vehicle shown in the figure moves along a straight track in one direction. Starting from rest, the vehicle accelerates with a constant acceleration of 2 m/s^2 and after some time reaches point B, which is 100 meters ahead. If the vehicle is treated as a particle,

a. how many seconds does it take for the vehicle to reach point B?

b. What is the speed of the vehicle when it reaches point B?

c. How far ahead of point A will the vehicle be after 5 s?

d. What will be the speed of the vehicle after 5 seconds?



EXAMPLE 7: The motorcycle shown in the figure is moving along a straight road and passes point A with a speed of 54 km/h. From that point on, it accelerates with a constant acceleration of 2 m/s^2 for 5 s and reaches point B. Then, starting from point B, it continues at a constant speed for 10 s and reaches point C. At point C, the rider notices the traffic light, applies the brakes, and begins to slow down with a constant acceleration.

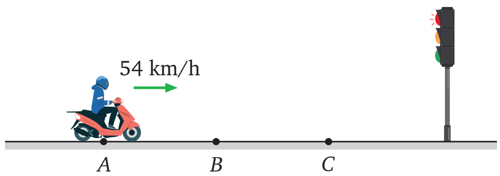
a. What is the distance between points A and B?

b. What is the speed of the motorcycle at point B?

c. What is the distance between points B and C?

d. Given that the distance between point C and the traffic light is 125 m, what is the minimum acceleration required for the motorcycle to stop before reaching the traffic light?

e. For this minimum-acceleration case, how much time does it take for the motorcycle to go from point C to the traffic light?



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EXAMPLE 8: Of the objects shown in the figure, X moves toward object Y with a constant speed. When the distance between the objects is 48 m, object Y starts moving from rest toward object X with a constant acceleration of 4 m/s^2.

a. After how many seconds do the objects meet?

b. What is the speed of object Y when they meet?

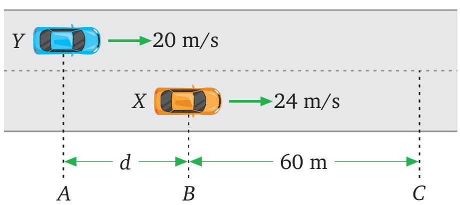
Your turn

6. The objects X and Y shown in the figure move in one dimension, parallel to each other, in two different lanes of a road. While vehicle X moves with a constant speed of 24 m/s, vehicle Y starts accelerating with a constant acceleration of 6 m/s^2 from the line through point A, where it is moving at 20 m/s, and catches X at the line through point C. Assuming the sizes of the vehicles are negligible,

a. after how many seconds does vehicle Y catch vehicle X?

b. What is the distance d?

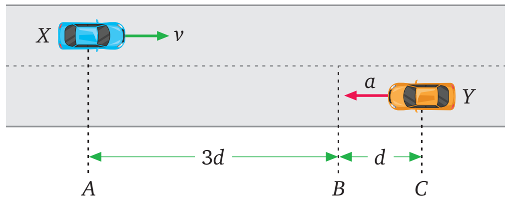
c. What is the speed of vehicle Y when it catches vehicle X?



7. As shown in the figure, the vehicles X and Y move in one dimension and parallel to each other in two different lanes of a road. At the instant vehicle X starts moving from level A toward Y with a constant speed of magnitude v, vehicle Y also starts moving from rest from level C toward X with an acceleration of magnitude a. The vehicles meet at level B after a time interval of t.

a. Find a in terms of v and d.

b. When the vehicles meet, what is the speed of vehicle Y in terms of v?



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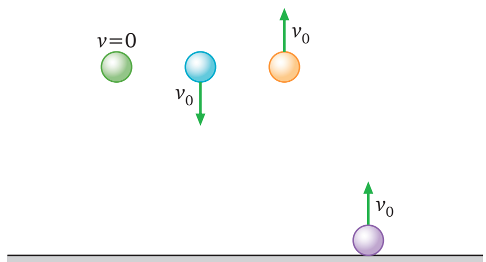
Free-Fall Motion

Let us examine the motion of the point objects shown in the figure.



Free fall Let us examine the conditions required for this motion to occur and define this motion.

Let us write the kinematic equations for free-fall motion.

EXAMPLE 9: A fruit on the tree shown in the figure is at a height of 30 m above the ground on a branch. The fruit breaks off the branch and starts falling toward the ground. Neglecting friction, let us answer the following questions.

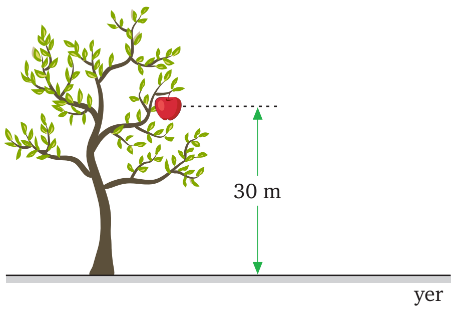
a. How many seconds does it take for the fruit to hit the ground?

b. What is the height of the fruit above the ground after 1.5 s?

c. What is the speed of the fruit after 1.5 s?

d. What is the speed of the fruit when its height above the ground is 20 m?

e. How many meters does the fruit fall during the time interval 1 s — 2 s?



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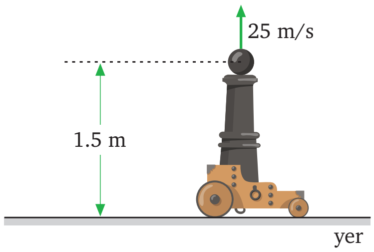
EXAMPLE 10: The cannon shown in the figure fires a projectile vertically upward with a speed of 25 m/s. Assuming air resistance is negligible, let us answer the following questions.

a. What is the maximum height the projectile can reach?

b. How many seconds does it take for the projectile to reach its maximum height?

c. What is the height of the projectile above the ground at the end of 3.5 s?

d. What is the speed of the projectile when it hits the ground?



Your turn

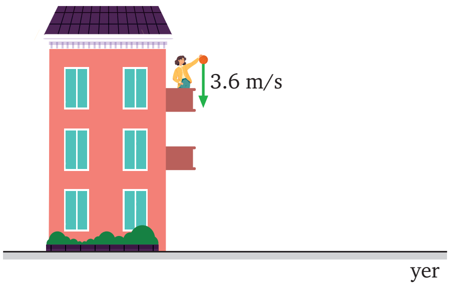
8. The child on the balcony of the building shown in the figure throws the negligibly small ball in his hand vertically downward with a speed of 3.6 m/s. In an environment where air resistance is negligible, the ball hits the ground after 2.8 s. Accordingly,

a. What is the height above the ground from which the ball is thrown?

b. What is the speed of the ball when it hits the ground?

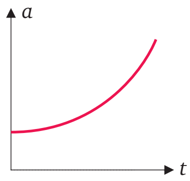
c. What is the speed of the ball after it has fallen 15 m from the instant it is thrown?

d. What is the time elapsed until the ball has fallen 15 m from the instant it is thrown?

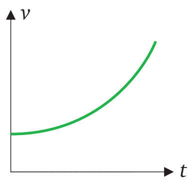


Obtaining Velocity and Position by Integration

Let us discuss how to obtain the velocity of an object at a given instant by using its acceleration-time graph.



Let us discuss how to obtain the position of an object at a given instant by using its velocity-time graph.



Let us derive the kinematic equations of an object moving with initial position x_0, initial velocity v_0, and constant acceleration a_0.

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EXAMPLE 11: A rocket, initially at rest, starts moving vertically upward from the ground. During the first 8.0 s, the vertical acceleration of the rocket is $a_y(t) = \left(3.0\ \mathrm{m/s^3}\right) t$. The $+y$ direction is upward.

a. Find the height of the rocket above the ground at $t = 8.0$ s.

b. Find the velocity of the rocket when it is 108 m above the ground.

Your turn

9. The acceleration of a car in the x direction is given by $a_x(t) = At - Bt^2$. Here, $A = 3.0\ \mathrm{m/s^3}$ and $B = 0.25\ \mathrm{m/s^4}$. The car is at the origin and at rest at $t = 0$.

a. Find the position function $x(t)$ and the velocity function $v_x(t)$ for the car.

b. Find the magnitude of the maximum speed reached by the car and the time at which it reaches this speed.

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Exercises and Problems

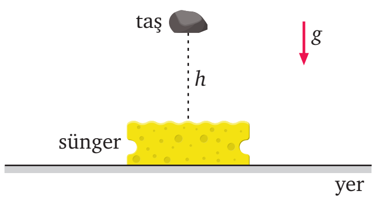
1. In the figure, the stone is released from rest from a height of h above the top of the sponge in an environment where friction is neglected. After being released, the stone strikes the sponge and comes to rest after compressing it by $h/3$. Let t_1 be the time from the release of the stone until it strikes the sponge, and let t_2 be the time from when the stone strikes the sponge until it comes to rest. Assuming the stone has constant acceleration while in contact with the sponge,

a. the speed of the stone when it strikes the sponge,

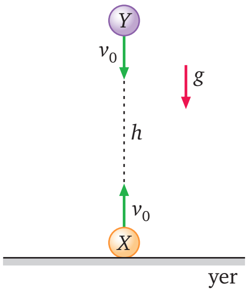
b. the magnitude of the acceleration of the stone while it is being brought to rest by the sponge,

c. t_1/t_2 ratio

find these quantities in terms of the given variables and the acceleration due to gravity.



2. The distance between the point particles X and Y shown in the figure is h. The particles are launched vertically toward each other with speed v_0 in an environment where air resistance is neglected. Find the collision time of the particles in terms of the given quantities.



3. The position function of a particle as a function of time is given by $y(t) = (6.0\ \mathrm{m/s^2})\, t^2 - (1.0\ \mathrm{m/s^3})\, t^3$. Accordingly;

a. the instants when the particle is at rest,

b. the maximum value that y can take,

c. the instant when the particle's acceleration is zero,

d. the greatest value that the particle's velocity can take

find.

4. Two particles start moving at $t = 0$ and move in one dimension. Their position functions are given by $x_1(t) = 5.0\ \mathrm{m} - (1.0\ \mathrm{m/s^2})\, t^2$ and $x_2(t) = (2.1\ \mathrm{m/s})\, t + (1.5\ \mathrm{m/s^2})\, t^2$. Accordingly, determine the following for the particles:

a. At what time are they at the same position?

b. What is the position value when they are at the same position?

c. Can the velocities of these two particles be equal at any instant?

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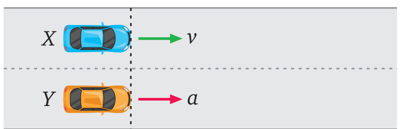
5. Of the vehicles shown in the figure, X moves in one dimension with constant speed v. When X comes alongside Y, vehicle Y starts moving from rest with constant acceleration of magnitude a. The vehicles come side by side again after a time t from the first instant they are side by side. Accordingly,

a. a in terms of v and t,

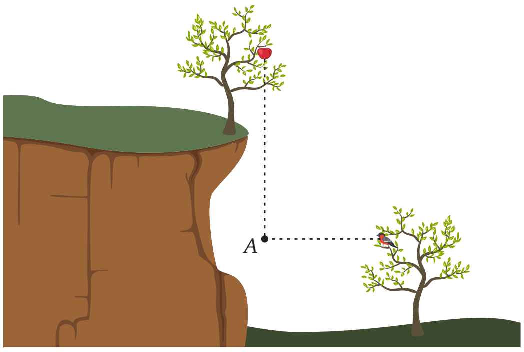
b. Y's average speed in terms of v,

c. the speed of Y at the instant the vehicles come side by side again,

find.



6. The fruit growing on the tree on the rocky cliff shown in the figure breaks off from its branch and starts to fall. At the instant the fruit starts falling, a bird perched on a tree on the ground starts flying horizontally with constant acceleration $a = 2.5\ \text{m/s}^2$ to catch the fruit and catches it at point A. Given that the distance from point A to the initial position of the fruit is 40 m, how many meters is the bird's distance to point A?



7. The position function of a drone initially at rest on the ground up to a certain altitude is given by $y(t) = \left(0.8\ \text{m/s}^2\right) t^2$. The $+y$ axis represents the vertical upward direction, and the drone is at ground level at $t = 0$ s. Accordingly,

a. Find the position of the drone at $t = 1$ s.

b. Find the position of the drone at $t = 2$ s.

c. Find the average speed of the drone in the time interval $0 - 2$ s.

d. Find the speed of the drone at 2 s.

e. At 2 s, if a very small particle is released from the drone while being at rest relative to the drone, what will be the speed of this particle when it hits the ground?

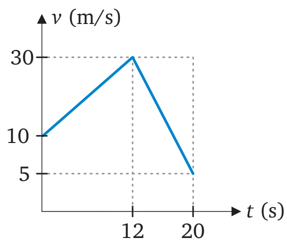
8. The velocity-versus-time graph of a vehicle moving in one dimension is given in the figure. According to the graph,

a. What are the accelerations of the vehicle at $t = 10$ s and $t = 15$ s?

b. Given that the vehicle is at position $x = 40$ m at $t = 0$ s, what is its position at $t = 12$ s?

c. What is the displacement of the vehicle in the interval $0 - 20$ s?

d. What is the average speed of the vehicle in the interval $0 - 20$ s?



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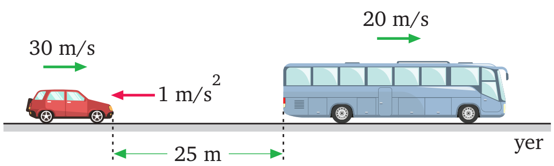
9. The automobile and the bus shown in the figure are moving in one dimension and in the same direction. In a foggy environment, the bus moves with constant speed 20 m/s while the automobile moves with constant speed 30 m/s. When the automobile driver is 25 m behind the bus, he applies the brakes and decelerates with constant acceleration 1.0 m/s^2. Accordingly,

a. How many seconds after the automobile starts slowing down does it hit the bus?

b. How many meters does the automobile travel from the instant it starts slowing down until the instant it hits the bus?

c. What is the speed of the automobile when it hits the bus?

d. If the automobile decelerates with acceleration 1.0 m/s^2, what should be the minimum distance between them so that it does not hit the bus?

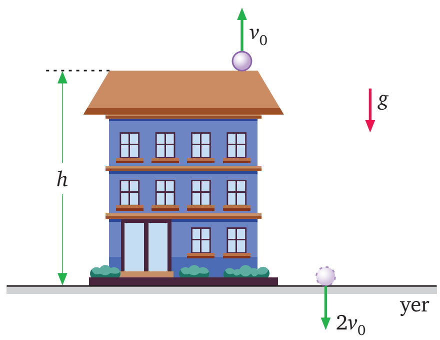


10. A point particle is thrown vertically upward from the top of the building shown in the figure with speed v_0 and hits the ground with speed $2v_0$. Air resistance in the environment is negligible.

a. Find the building height h in terms of v_0 and g.

b. How many h above its launch position does the particle rise?

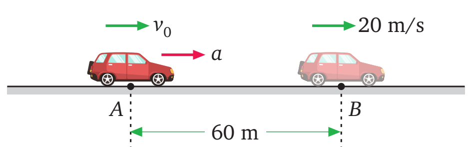
c. If the particle reaches its maximum height in time t from the instant it is launched, what is the total flight time of the particle?



11. The vehicle shown in the figure, moving in one dimension, starts to accelerate with constant acceleration a as it passes point A with speed v_0. It takes 4.0 s for the vehicle to go from point A to point B, and its speed at point B is 20 m/s. Accordingly,

a. Find the value of v_0.

b. Find the value of a.



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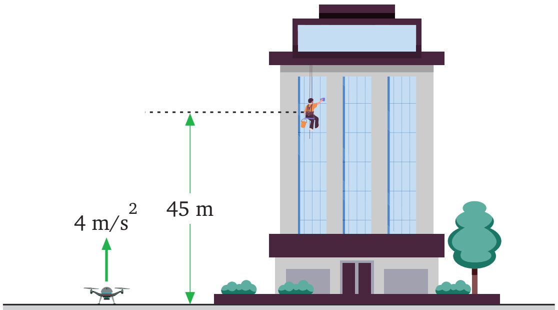
12. A window cleaner is cleaning the exterior of the building shown in the figure using a safety system. While the worker is cleaning, a drone starts accelerating vertically upward from rest on the ground with constant acceleration $4.0\ \mathrm{m/s^2}$ in order to record a video of the worker. At the instant the drone leaves the ground, the worker drops the brush in his hand. The worker, the drone, and the brush are to be treated as point objects, and air resistance in the environment will be neglected.

a. How many seconds does it take for the drone and the brush to be at the same height?

b. What is the height of the drone above the ground when they are at the same height?

c. When the brush hits the ground, what is the height difference between the drone and the worker?



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Answers

Example questions

1	a. $\Delta x_1 = 4.0\ \mathrm{m}$, $\Delta x_2 = -2.0\ \mathrm{m}$, $\Delta x_3 = 2.0\ \mathrm{m}$ b. $d_1 = 4.0\ \mathrm{m}$, $d_2 = 2.0\ \mathrm{m}$, $d_3 = 6.0\ \mathrm{m}$ c. $v_1 = 2.0\ \mathrm{m/s}$; $v_2 = -1.0\ \mathrm{m/s}$, $v_3 = 0.50\ \mathrm{m/s}$ d. $s_1 = 2.0\ \mathrm{m/s}$, $s_2 = 1.0\ \mathrm{m/s}$, $s_3 = 1.5\ \mathrm{m/s}$
2	a. $3.0\ \mathrm{m}$ b. $9.0\ \mathrm{m}$ c. $1.0\ \mathrm{m/s}$ d. $3.0\ \mathrm{m/s}$
3	a. $x(0) = 15\ \mathrm{cm}, x(2.0) = 55\ \mathrm{cm}$ b. $\Delta x = 40\ \mathrm{cm}$ c. $v_{ort} = 20\ \mathrm{cm/s}$ d. $v(t) = (20\ \mathrm{cm/s^2})t$ e. $v(0) = 0\ \mathrm{cm/s}, v(2.0) = 40\ \mathrm{cm/s}$ f. Go to the video.
4	a. $4.0\ \mathrm{m/s^2}$ b. $0\ \mathrm{m/s^2}$ c. $-8.0\ \mathrm{m/s^2}$ d. $-8.0\ \mathrm{m/s^2}$
5	a. $v(1.0) = -1.0\ \mathrm{m/s}, v(3.0) = 15\ \mathrm{m/s}$ b. $\Delta v = 16\ \mathrm{m/s}$ c. $a_{avg} = 8.0\ \mathrm{m/s^2}$ d. $a(t) = (4.0\ \mathrm{m/s^3})t$ e. $a(1.0) = 4.0\ \mathrm{m/s^2}, a(3.0) = 12\ \mathrm{m/s^2}$ f. Go to the video.
6	a. $10\ \mathrm{s}$ b. $20\ \mathrm{m/s}$ c. $25\ \mathrm{m}$ d. $10\ \mathrm{m/s}$

7	a. $x_{AB} = 100$ m b. $v_B = 25$ m/s c. $x_{BC} = 250$ m d. $a = 2.5$ m/s^2 e. $t = 10$ s
8	a. $t = 2.0$ s b. $v = 8.0$ m/s
9	a. $t = \sqrt{6}$ s b. $h = 18.75$ m c. $v = 15$ m/s d. $v = 10\sqrt{2}$ m/s e. $\Delta h = 15$ m
10	a. $h = 32.75$ m b. $t = 2.5$ s c. $h = 27.75$ m d. $v = 25.6$ m/s
11	a. $y = 256$ m b. $\vec{v} = 54$ m/s, upward

Your turn questions

1	a. 900 m b. 300 m, southward c. 60 m/min d. 20 m/min, southward
2	a. 650 m, eastward b. 950 m c. 19 m/s d. 13 m/s, eastward
3	a. 30 m ve −105 m b. −15 m/s c. 15 m/s d. −14 m/s
4	a. 8 m/s^2, upward b. 4 m/s^2, upward
5	a. $a(t) = (1.4 \text{ m/s}^3)t$ b. $v = 9.8$ m/s, $a = 4.2$ m/s^2 c. 7.0 m/s^2
6	a. 2.5 s b. 8.75 m c. 35 m/s
7	a. $a = \dfrac{2v^2}{9d}$ b. $\dfrac{2v}{3}$
8	a. 49.3 m b. 31.6 m/s c. 17.6 m/s d. 1.4 s
9	a. $x(t) = \left(\frac{1}{2} \text{ m/s}^3\right) t^3 - \left(\frac{1}{48} \text{ m/s}^4\right) t^4$ $v_x(t) = \left(\frac{3}{2} \text{ m/s}^3\right) t^2 - \left(\frac{1}{12} \text{ m/s}^4\right) t^3$ b. $v_{max} = 72$ m/s and $t = 12$ s

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Exercises and problems

1	a. $\sqrt{2gh}$ b. $3g$ c. 3
2	$t = \dfrac{h}{2v_0}$
3	a. $t = 0$ s and $t = 4$ s b. 32 m c. $t = 2$ s d. 12 m/s
4	a. 1.1 s b. 3.9 m c. No

5	a. $\frac{2v}{t}$ b. v c. $2v$
6	10 m
7	a. 0.8 m b. 3.2 m c. 1.6 m/s d. 3.2 m/s e. 8.6 m/s
8	a. 1.67 m/s^2 and -3.13 m/s^2 b. 280 m c. 380 m d. 19 m/s
9	a. 2.9 s b. 82.8 m c. 27.1 m/s d. 50.0 m
10	a. $\frac{3v_0^2}{2g}$ b. $h/3$ c. $3t$
11	a. 10 m/s b. 2.5 m/s^2
12	a. 2.5 s b. 12.5 m c. 27 m

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CHAPTER 3

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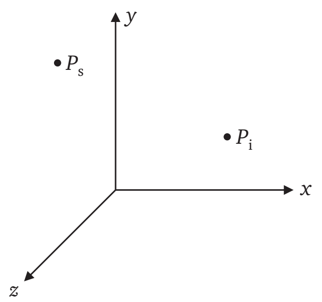
MOTION IN TWO OR THREE DIMENSIONS

Position and Velocity Vectors

By examining a particle moving from point P_i to point P_s;

- let us define the position vector of the particle at points P_i and P_s,
- write the displacement vector between the two points,
- write the average velocity vector for its motion between the two points,
- write the instantaneous velocity vector for any instant during its motion,
- show the components of the instantaneous velocity vector separately.



EXAMPLE 1: The time-dependent x and y coordinates of a point particle moving in two dimensions

$x(t) = 5.0\ \text{m} + \left(1.0\ \text{m/s}^3\right) t^3$

$y(t) = (1.0\ \text{m/s})\, t + \left(2.0\ \text{m/s}^2\right) t^2$

are given. For the particle, determine:

a. its positions at $t = 0$ and $t = 2$ s,

b. the time-dependent position and velocity vectors,

c. its instantaneous velocity at $t = 2$ s,

d. its average velocity over the time interval $0 - 2$ s.

Your turn

1. The position of a point on a digital screen is

$$\vec{r}(t) = \left(\left(2.5\ \text{cm/s}^2\right) t^2\right)\hat{\imath} + \left(2.0\ \text{cm} + (4.0\ \text{cm/s})\ t\right)\hat{\jmath}$$

Find the following for the point:

a. find the average velocity vector between $t = 0$ and $t = 2$ s

b. find the instantaneous velocity vector at $t = 1$ s.

c. draw its trajectory for the time interval $0 - 2$ s.

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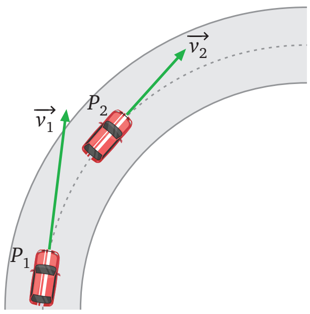
The Acceleration Vector

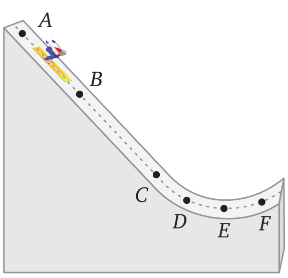
By examining the motion of a car, let us determine:

- the change in its velocity,
- its average acceleration,
- its instantaneous acceleration,
- the components of the instantaneous acceleration vector.



NOTE: We can speak of the components of acceleration that are parallel and perpendicular to the trajectory of the moving object.

Let us discuss the acceleration of an athlete sliding downhill at different points on a surface where friction is neglected.



Let us examine the possible directions of the acceleration of a car passing over the top of a curved road.

EXAMPLE 2: The time-dependent x and y coordinates of a point particle moving in two dimensions
$x(t) = 5.0\,\text{m} + \left(1.0\,\text{m/s}^3\right) t^3$
$y(t) = (1.0\,\text{m/s})\,t + \left(2.0\,\text{m/s}^2\right) t^2$
are given. For the particle, determine:

a. the time-dependent acceleration vector,
b. its instantaneous acceleration at $t = 2$ s,
c. its average acceleration over the time interval $0 - 2$ s.

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Your turn

2. The position of a point on a digital screen is

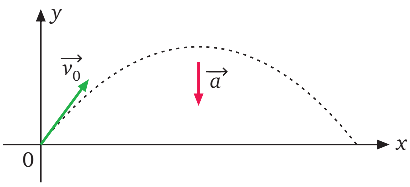
$$\vec{r}(t) = \left(\left(2.5\,\text{cm/s}^2\right)\,t^2\right)\hat{\imath} + (2.0\,\text{cm} + (4.0\,\text{cm/s})\,t)\,\hat{\jmath}$$

Find the following for the point:

a. find the average acceleration vector between $t = 0$ and $t = 2$ s
b. find the instantaneous acceleration vector at $t = 1$ s.

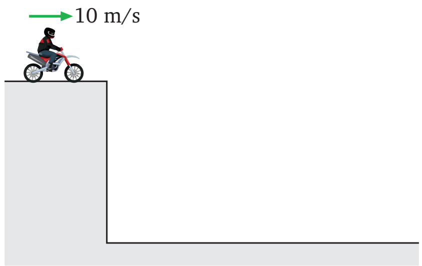
Projectile Motion

Let us examine projectile motion and its properties using the given figure.



Let us write the equations for position, velocity, and acceleration in projectile motion.

Let us examine the horizontal projectile motion of the object shown in the figure, which leaves the surface at $t = 0$ s. Let us determine the position, velocity, and acceleration of the object at $t = 1$ s and $t = 2$ s.



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EXAMPLE 3: A soccer player kicks a ball from the ground, launching it with a speed of $v = 20$ m/s at an angle of $\alpha = 37°$ with the horizontal. Air resistance is neglected.

a. Find the position and velocity of the ball at $t = 2$ s.

b. Find the time required for the ball to reach its maximum height and its height at that instant.

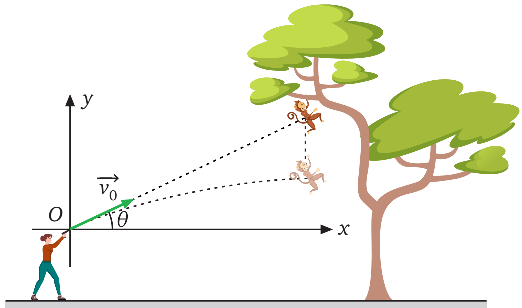
c. Find the range of the ball and its velocity just before it hits the ground.

EXAMPLE 4: A student throws a ball from a window 12 m above the ground with a speed of $v = 15$ m/s at an angle of $\alpha = 30°$ below the horizontal.

a. Find the time for the ball to hit the ground.

b. Find how far the ball travels horizontally before it hits the ground.

Let us discuss how a zoo worker who wants to hit a monkey jumping from a tree with a tranquilizer dart should aim.



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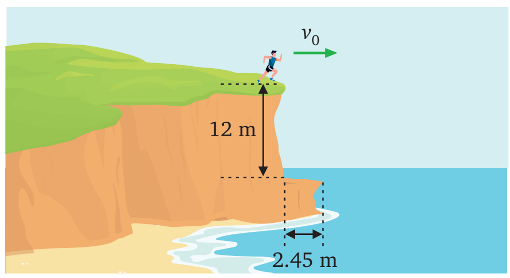
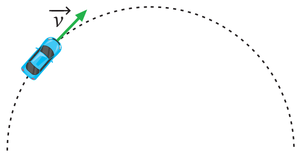
Your turn

3. An athlete runs off the edge of a cliff and jumps horizontally toward the water. At the bottom of the cliff, there is a ledge 12.0 m below the top of the cliff and 2.45 m wide. Find the minimum horizontal speed the swimmer must have at the instant of leaving the top of the cliff in order to avoid hitting the ledge.

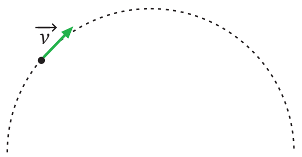


Circular Motion

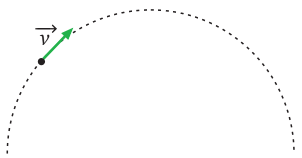
Let us examine circular motion for a car moving along a circular road. Let us define the concept of **uniform circular motion**.



Let us calculate the acceleration of an object moving in uniform circular motion.



Let us discuss the acceleration of an object moving in nonuniform circular motion.



EXAMPLE 5: Passengers on a Ferris wheel move along a circle with constant speed. The radius of the circle is $R = 12$ m, and the period for one complete revolution is $T = 20$ s. Find the acceleration of the passengers.

Your turn

4. The radius of Mercury's orbit around the Sun is 5.79×10^7 km, and Mercury completes this orbit in 88 days.

a. Calculate the magnitude of Mercury's average orbital speed around the Sun.

b. Calculate the magnitude of Mercury's centripetal acceleration.

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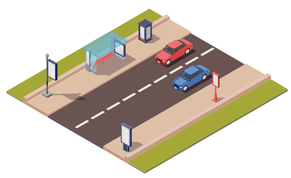
Relative Velocity

Let us use the following figures to examine why motion is a relative concept.



Relative Velocity in One Dimension

Let us examine this through an example:

- A child is moving on a platform with a speed of $+1.0\ \mathrm{m/s}$ relative to the platform.
- The platform is moving with a speed of $+2.0\ \mathrm{m/s}$ relative to the ground.

Let us find the velocity of the child relative to the ground.

Let us derive mathematically the expression for the child's velocity relative to the ground. For this, let us begin by obtaining the child's position relative to the ground. Let us compare this result with what we learned in high school.

Relative Velocity in Two or Three Dimensions

Let us derive the results we obtained in one dimension for two and three dimensions as well.

EXAMPLE 6: The compass of an airplane moving horizontally shows that the airplane is flying eastward. The airplane's airspeed indicator measures its speed relative to the air as $300\ \mathrm{km/h}$. There is a wind blowing from south to north at a speed of $80\ \mathrm{km/h}$. Let us find the airplane's velocity relative to the ground.

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Your turn

5. A ship is sailing due east with speed $v_1 = 20\ \mathrm{km/h}$. A second ship is moving on a course 30° east of north. What should its speed be so that it always remains exactly due north of the first ship?

Exercises and Problems

1. A bomber airplane is moving horizontally in the $+x$ direction with speed $v_U = 60$ m/s at a height of $h = 125$ m above the ground. The airplane releases a bomb just as it passes directly over point O. On the ground, there is a truck moving at constant speed in the $+x$ direction. At the instant the bomb is released, the truck is located 200 m in the $+x$ direction from point O. Neglect air resistance and the height of the truck, and take $g = 10$ m/s^2. It is known that the bomb hits the truck.

a. What must the speed v_K of the truck be in m/s?

b. What is the flight time t of the bomb in seconds?

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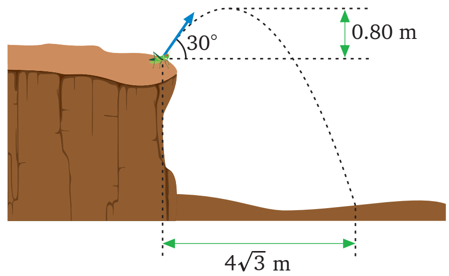
2. A grasshopper jumps from the edge of a vertical cliff at an angle of 30° with the horizontal. The highest point of its path is 0.80 m above the launch point. The grasshopper lands at a horizontal distance of $4\sqrt{3}$ m from the base of the cliff. Accordingly,

a. find the magnitude of the grasshopper's initial speed.

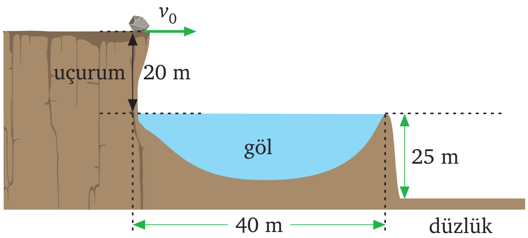
b. find the height of the cliff.



3. A stone is thrown horizontally from the top of a vertical cliff that is 20 m above the surface of a lake. The lake is 40 m wide, and beyond the lake there is a flat ground located 25 m lower. Accordingly,

a. what is the minimum speed required for the stone to clear the lake and reach the flat ground?

b. how many meters beyond does the stone land on the flat ground?



4. You are standing on the loading platform at the top of a straight ramp inclined downward by an angle α from the horizontal. You push a small box horizontally from the platform with speed v_0 so that it lands on the ramp. How much vertical displacement does the box undergo before hitting the ramp? Obtain this distance in terms of the given quantities and the magnitude of the gravitational acceleration g.

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Answers

Example questions

1	a. $\vec{r}_0 = 5.0\hat{\imath}$ m, $\vec{r}_2 = (13\hat{\imath} + 10\hat{\jmath})$ m b. $\vec{r}(t) = (5.0 + 1.0t^3)\hat{\imath} + (1.0t + 2.0t^2)\hat{\jmath}$ m $\vec{v}(t) = (3.0t^2)\hat{\imath} + (1.0 + 4.0t)\hat{\jmath}$ m/s c. $\vec{v}_2 = (12\hat{\imath} + 9.0\hat{\jmath})$ m/s d. $\vec{v}_{avg} = (4.0\hat{\imath} + 5.0\hat{\jmath})$ m/s
2	a. $\vec{a}(t) = (6.0t)\hat{\imath} + (4.0)\hat{\jmath}$ m/s^2 b. $\vec{a} = (12)\hat{\imath} + (4.0)\hat{\jmath}$ m/s^2 c. $\vec{a}_{avg} = (6.0)\hat{\imath} + (4.0)\hat{\jmath}$ m/s^2
3	a. $\vec{r} = (32\hat{\imath} + 4.0\hat{\jmath})$ m, $\vec{v} = (16\hat{\imath} - 8.0\hat{\jmath})$ m/s b. $t = 1.2$ s, $h = 7.2$ m c. $R = 38.4$ m, $\vec{v} = (16\hat{\imath} - 12\hat{\jmath})$ m/s
4	a. $t = 0.97$ s b. $x = 12.6$ m
5	$a = 1.18$ m/s^2
6	$\vec{v} = (300\hat{\imath} + 80\hat{\jmath})$ km/h

Your turn questions

1	a. $\vec{v}_{avg} = (5.0\hat{\imath} + 4.0\hat{\jmath})$ cm/s b. $\vec{v} = (5.0\hat{\imath} + 4.0\hat{\jmath})$ cm/s c. See the video
2	a. $\vec{a}_{avg} = (5.0\hat{\imath})$ cm/s^2 b. $\vec{a} = (5.0\hat{\imath})$ cm/s^2
3	1.58 m/s
4	a. $v = 4.79 \times 10^4$ m/s b. $a = 3.95 \times 10^{-2}$ m/s^2
5	$v_2 = 40$ km/h

Exercises and problems

1	a. $v_K = 20$ m/s b. $t = 5$ s
2	a. $v_0 = 8$ m/s b. $h = 1.0$ m
3	a. $v_0 = 20$ m/s b. $d = 20$ m
4	$\dfrac{2v_0^2}{g}\tan^2\alpha$

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CHAPTER 4

NEWTON'S LAWS OF MOTION

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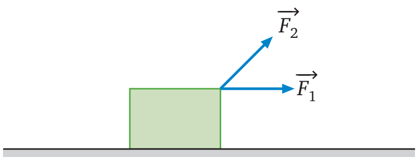
Force and Interactions

Let us define the concept of force. Let us explain the following concepts:

- Normal force and frictional force
- Tension
- Contact force and noncontact force
- Weight

Net Force

Let us examine what happens when more than one force acts on an object. Let us explain the **principle of superposition**.



Let us express the **net force** acting on an object as a sum.

Let us write the x- and y-components of the net force.

Your turn

1. On a horizontal surface, there are two toy cars pulling two ropes attached to the same pole. The angle between the ropes is 32.0°. The first car exerts a horizontal force of magnitude 124 N, and the second car exerts a horizontal force of magnitude 176 N.

a. Find the magnitude of the resultant force.

b. Find the angle that the resultant force makes with the rope of the first car.

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Newton's First Law

1st Law: An object on which no net force acts has a constant velocity (which may be zero) and zero acceleration.

Let us discuss in detail what this law means.

NOTE: This law is also called the *law of inertia*.

Let us state what it means for an object to be in **equilibrium**.

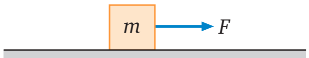
Let us examine what an **inertial frame of reference** is. Let us discuss why it is important.

Newton's Second Law

2nd Law: If a net external force acts on an object, the object accelerates. The direction of the acceleration is the same as the direction of the net force. The product of the object's mass and acceleration vector is equal to the net force vector.

Let us discuss in detail what this law means.

Let us consider the concept of **mass** as a measure of inertia. Let us define the quantity 1 N.



Let us examine the use of Newton's second law in three dimensions.

NOTE: $m\vec{a}$ is not a force.

EXAMPLE 1: A skater is gliding at a speed of 2.0 m/s on rough horizontal ice. Because of friction, the skater comes to a complete stop after 4.0 s. If the mass of the skater is 60 kg, find the magnitude of the friction force.

Your turn

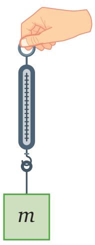
2. A man applies a constant horizontal force of magnitude 60.0 N to an ice block resting on a horizontal frictionless surface. Starting from rest, the ice block travels 12.0 m in 4.0 s.

a. What is the mass of the ice block?

b. If the man stops pushing at the end of 4.0 s, how far does the ice block travel during the next 4.0 s?

Mass and Weight

Let us consider the difference between the concepts of **mass** and **weight**. Let us write the weight of an object on the Earth's surface.



NOTE: In mechanics problems, when the mass of an object is given, information is provided about both its weight and its inertial mass.

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Your turn

3. A handheld camera to be used on the Moon weighs 21.4 N on Earth. When the camera is on the surface of the Moon, it weighs 3.53 N. Take the gravitational acceleration on Earth to be 9.8 m/s^2.

a. What is the gravitational acceleration on the Moon?

b. What is the mass of the camera on the Moon?

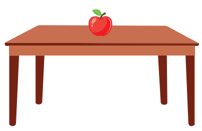
Newton's Third Law

3rd Law: If an object A exerts a force on an object B (action), then object B also exerts a force on object A with equal magnitude but opposite direction (reaction). These two forces have the same magnitude and opposite directions. These forces act on different objects.

Let us discuss in detail what this law means.

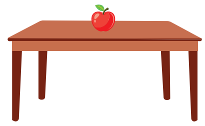
NOTE: Every real force exists as part of an action-reaction pair.

Let us show the forces acting on the objects in the figure and the action-reaction pairs.



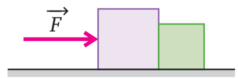
Free-Body Diagrams

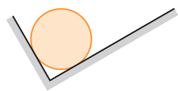
Let us construct the free-body diagrams of the objects through an example.

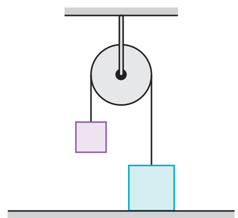


NOTE: When showing a force acting on an object, it is helpful to answer the question of which object exerts that force.

Let us draw the free-body diagrams of the objects in the figures.

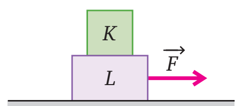






Your turn

4. Draw the free-body diagrams of the objects shown in the figure. All surfaces are rough.



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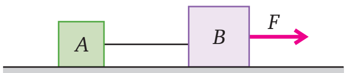
Exercises and Problems

1. There is friction between two stacked blocks, but there is no friction between the lower object and the ground. The mass of the upper object is $m_A = 2$ kg, while the mass of the lower one is $m_B = 8$ kg. When the lower mass is pulled by a horizontal force of magnitude 16 N, its acceleration has magnitude $a_B = 1.8\ \mathrm{m/s^2}$. Accordingly, what is the magnitude of the acceleration of the upper object?

2. On a frictionless horizontal surface, two blocks connected by a massless string are arranged as shown. The mass of block B is 12.0 kg, and the mass of block A is m. A constant horizontal force of $F = 50.0$ N is applied to block B as shown. During the first 4.00 s after the force begins to act, block B undergoes a displacement of 20.0 m.

a. When the blocks move together, what is the tension T in the string connecting the two blocks?

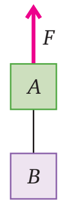
b. What is the mass m of block A?



3. Boxes A and B are connected by a massless string as shown. A constant upward force of magnitude $F = 56.0$ N is applied to box A. The system is released from rest, and box B moves downward by 9.00 m after 3.00 s. Given that the tension in the string is $T = 32.0$ N;

a. What is the mass of box B?

b. What is the mass of box A?



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Answers

Example questions

1	$f = 30$ N

Your turn questions

1	a. 2.89×10^2 N b. 18.9°
2	a. 40.0 kg b. 24.0 m
3	a. $g_{\mathrm{Moon}} = 1.62\ \mathrm{m/s^2}$ b. $m = 2.18$ kg
4	See the video.

Exercises and problems

1	$a_A = 0.80\ \mathrm{m/s^2}$
2	a. $T = 20.0$ N b. $m = 8.00$ kg
3	a. $m_B = 4.00$ kg b. $m_A = 3.00$ kg

CHAPTER 5

APPLICATIONS OF NEWTON'S LAWS OF MOTION

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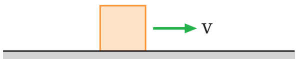
Using Newton's First Law: Particles in Equilibrium

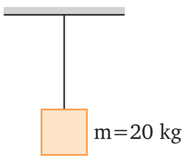
Let us discuss what it means for an object to be in equilibrium. Let us interpret the state of motion of an object in equilibrium.



EXAMPLE 1: 20.0 kg object is suspended from the ceiling by a rope and held in equilibrium.

a. What is the force exerted by the rope on the object when the mass of the rope is neglected?

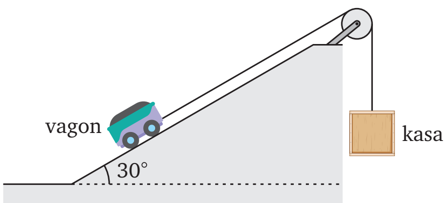
b. If the mass of the rope is 2.0 kg, what are the tensions at the top and bottom ends of the rope?



EXAMPLE 2: A cart on a ramp and a 100 kg crate are in equilibrium as shown in the figure by means of a rope of negligible mass. Friction is negligible.

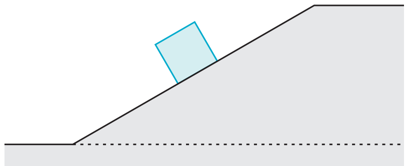
a. What is the tension in the rope?

b. What is the mass of the cart?

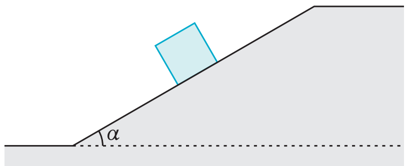


Using Newton's Second Law: Dynamics of Particles

Let us discuss how to treat an object under a net force.



EXAMPLE 3: Show that the magnitude of the acceleration of the object shown in the figure, released on a frictionless inclined plane, is $g \sin \alpha$.



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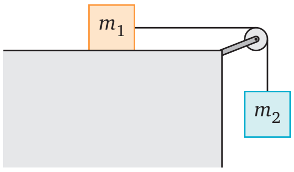
Objects with Equal Magnitudes of Acceleration

Let us obtain the accelerations of the objects in the system shown, where friction is neglected.

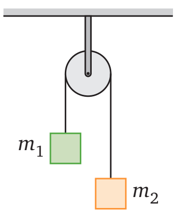


Your turn

1. Objects of mass m_1 and m_2 are released in the system shown, where friction is neglected. Given that $m_1 > m_2$;

a. Find the magnitudes of the acceleration of the objects in terms of g, m_1, and m_2.

b. Find the tension in the rope in terms of g, m_1, and m_2.



Motion in Accelerated Frames of Reference

Let us examine what kind of differences observers in accelerated reference frames will observe compared to those in inertial reference frames.

Let us discuss what we feel when we are inside an accelerating elevator or in a car taking a turn.

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EXAMPLE 4: A person of mass $m = 70$ kg inside an elevator accelerating upward at $2\ \mathrm{m/s^2}$ is being weighed at that moment. What is the value shown by the scale in this case?

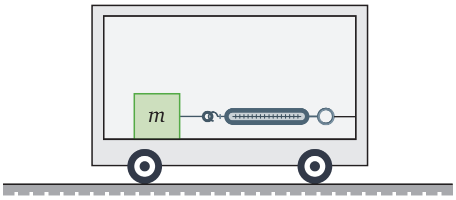
Your turn

2. An object of mass $m = 50.0$ kg is connected to a spring scale of negligible mass inside a frictionless wagon as shown. The wagon moves with a constant acceleration in the horizontal direction. While the object remains stationary relative to the wagon, the spring scale shows a value of $T = 150$ N.

a. Find the magnitude of the acceleration of the wagon.

b. Draw the free-body diagram showing the forces acting on the object while the wagon is moving with constant acceleration.

c. What can be said about the motion of the object if the rope connecting the object to the spring scale suddenly breaks?



Forces of Friction

Kinetic and *static* friction forces. Let us examine the following table.

Surface Pair	Coefficient of Static Friction (μ_s)	Coefficient of Kinetic Friction (μ_k)
Rubber - Concrete	1.0	0.8
Wood - Wood	0.4	0.3
Steel - Ice	0.03	0.02
Aluminum - Steel	0.61	0.47
Glass - Glass	0.9	0.4
Tire - Dry Asphalt	1.2	1.0

Let us examine the parameters on which the magnitude of the kinetic friction force depends and express this mathematically.

Let us draw a graph showing the magnitude of the friction force acting on an object at rest on a rough surface as a function of the magnitude of the applied force.

EXAMPLE 5: A 30 kg crate is pulled with a rope that makes an angle of 37° with the horizontal and moves with constant speed. Given that the coefficient of kinetic friction is $\mu_k = 0.75$, find the magnitude of the pulling force required to move the crate with constant speed.

Let us explain the concept of the **coefficient of rolling friction**.

NOTE: The friction force and the normal force are always perpendicular to each other.

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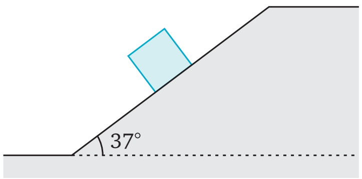
Your turn

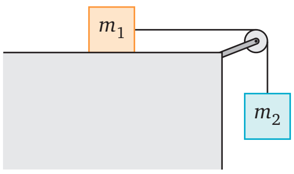
3. An object of mass m at rest on a ramp inclined at 37° is released. The coefficient of kinetic friction between the object and the surface is $\mu_k = 0.20$. Knowing that friction slows down the motion, find the magnitude of the object's acceleration along the inclined plane. Take $g = 10\ \mathrm{m/s^2}$.



4. $m_1 = 2$ kg and $m_2 = 4$ kg objects are released from rest in the system shown, where only the horizontal surface is rough. It is known that the system starts moving and that the coefficient of kinetic friction between the surface and the object is $\mu_k = 0.20$. Take $g = 10\ \mathrm{m/s^2}$.

a. Find the magnitudes of the accelerations of the objects.

b. Find the tension in the string.



Fluid Resistance and Terminal Speed

Let us examine the motion of an object released from rest in air and discuss the concept of **terminal speed**.

Assuming that air resistance is proportional to the square of the object's speed, let us obtain the terminal speed of the object. Let the mass of the object be m, the magnitude of the gravitational acceleration be g, the density of air be ρ, the cross-sectional area of the object be A, and the drag coefficient be D.

Assuming that fluid resistance is proportional to speed, let us obtain the time-dependent equations for the velocity and acceleration of an object released from rest.

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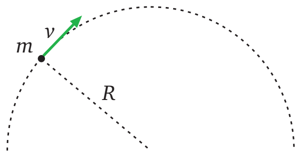
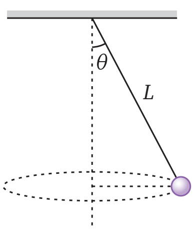
Dynamics of Circular Motion

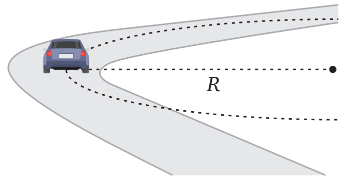
Let us find the net force required to make an object of mass m and speed v undergo uniform circular motion in a path of radius R.



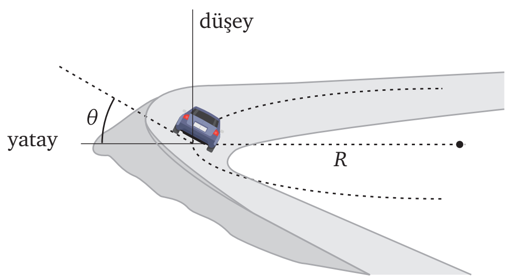
EXAMPLE 6: A conical pendulum is formed by attaching an object to the end of a string fixed to the ceiling. The string of the pendulum makes an angle θ with the vertical, and the length of the string is L. Given that the magnitude of the gravitational acceleration is g, obtain the period for one revolution of the pendulum, in terms of the given quantities, when it is rotated as shown.



EXAMPLE 7: A car is rounding a horizontal curve of radius R as shown in the figure. If the coefficient of static friction between the tires and the road is μ_s, what is the maximum speed v_{max} at which the driver can take the curve without skidding, in terms of the given quantities and the magnitude g of the acceleration due to gravity?



EXAMPLE 8: A car is to round a curve with speed v. The curve is to be banked so that the car can safely make the turn at this speed without the need for friction. If the radius of the curve is R, what should the banking angle θ be, in terms of the given quantities and the magnitude g of the acceleration due to gravity, for the car to turn safely without friction?



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EXAMPLE 9: A passenger of mass m on a Ferris wheel moves with constant speed v along a vertical circle of radius R. The seat remains upright throughout the motion. Find the force exerted by the seat on the passenger at the top and bottom points of the circle.

Your turn

5. A bowling ball whose weight is 70 N is suspended from the ceiling by a string of length 4.0 m. The ball is pulled to the side and released, and it swings like a pendulum. As the string passes through the vertical position, the speed of the ball is measured to be 4.0 m/s. At this instant:

a. Find the magnitude and direction of the acceleration of the bowling ball.

b. Find the tension in the string.

6. A toy car of mass 1.2 kg moves with constant speed on the inside surface of a vertical circular track of radius 4.0 m. If the normal force exerted on the car by the track has a magnitude of 8.0 N at the top point of the track, what is the magnitude of the normal force acting on the car at the bottom point?

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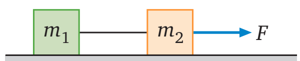
Exercises and Problems

1. Two objects connected by a string are on a horizontal surface. The masses of the objects are $m_1 = 3.0$ kg and $m_2 = 2.0$ kg, respectively. The surface is rough for both objects. The coefficient of static friction is given as $\mu_s = 0.40$, and the coefficient of kinetic friction is given as $\mu_k = 0.30$. The objects are initially at rest, and a force of magnitude F is applied horizontally to the object of mass m_2. Take $g = 10\ \mathrm{m/s^2}$.

a. Find the minimum magnitude of the force, $F_{\min}$, that must be applied for the system to start moving.

b. When the system is set into motion, find the magnitude of the acceleration of the objects and the tension in the string under the effect of this minimum force.

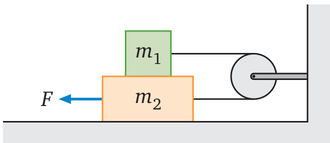


2. In the arrangement shown in the figure, an object of mass $m_1 = 2.0$ kg rests on top of an object of mass $m_2 = 4.0$ kg. The objects are connected to each other by a string passing over a pulley. The string passing over the pulley is massless, and the pulley is frictionless. There is friction on all surfaces of the system. The coefficient of kinetic friction between m_1 and m_2 is $\mu_{k1} = 0.30$, and the coefficient of kinetic friction between m_2 and the horizontal floor is $\mu_{k2} = 0.20$. Take the magnitude of the gravitational acceleration as $g = 10\ \mathrm{m/s^2}$. The lower object is set into motion by a force of magnitude $F = 30.0$ N.

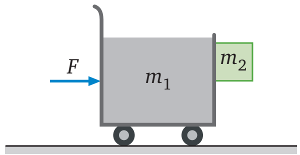
a. Draw the free-body diagrams of the objects.

b. Write Newton's second law for the objects.

c. Find the magnitude of the acceleration of the objects and the tension in the string.



3. A block of mass m_2 is placed against the vertical front face of a cart of mass m_1 that is pushed horizontally by a force of magnitude F. What is the minimum value of F required for the block to not slide down? The magnitude of the gravitational acceleration is g, and the coefficient of static friction between the block and the surface of the cart is μ_s. Friction between the cart and the floor is negligible.



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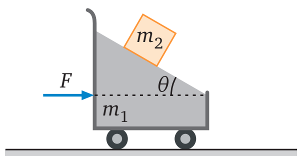
4. A block of mass m_2 is placed on the frictionless inclined upper surface of a cart of mass m_1 that is pushed horizontally by a force of magnitude F. What should F be for the block to remain without falling? The magnitude of the gravitational acceleration is g, and friction between the cart and the floor is negligible.

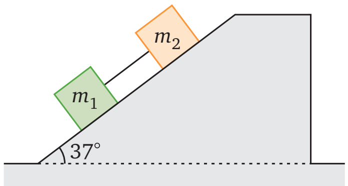


5. Two objects of masses $m_1 = 2.0$ kg and $m_2 = 3.0$ kg, connected to each other by a string, are sliding downward on an incline at 37°. The coefficient of kinetic friction between the 2.0 kg object and the inclined plane is $\mu_{k1} = 0.10$, and the coefficient of kinetic friction between the 3.0 kg object and the inclined plane is $\mu_{k2} = 0.35$. Take $g = 10\ \mathrm{m/s^2}$.

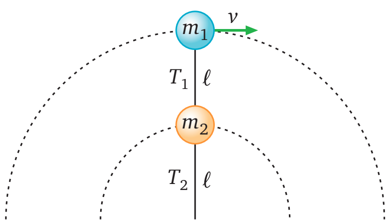
a. Find the magnitudes of the accelerations of both objects.

b. Calculate the tension in the string.

c. If the positions of the objects are interchanged, what changes occur compared with the previous case? Explain.



6. An object of mass $m_1 = 2.0$ kg is connected by a string of length $\ell = 0.50$ m to a second object of mass $m_2 = 2.0$ kg. These two objects are rotated in a vertical circular plane, as shown in the figure, by being attached to the end of a second string. The length of the second string is also $\ell = 0.50$ m. During the motion, the two strings remain collinear. When object m_2 passes through the top point of the circular path, its speed is $v = 8.0$ m/s. What are the tensions T_1 and T_2 at this instant? Take $g = 10\ \mathrm{m/s^2}$.



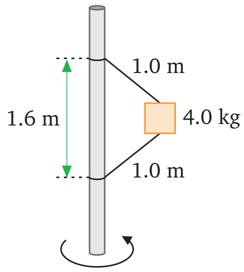
7. An object of mass 4.0 kg is rotated with the help of two strings attached to a vertical rod, as shown in the figure. The strings are taut while the system rotates about the axis of the rod. The tension in the upper string is measured to be $T_{\text{upper}} = 75.0$ N.

a. In this case, how many newtons is the tension force T_{lower} in the lower string?

b. What is the tangential speed of the object?

c. What is the minimum speed at which the lower string can remain taut?

d. What happens if the system rotates more slowly than this speed? Explain.



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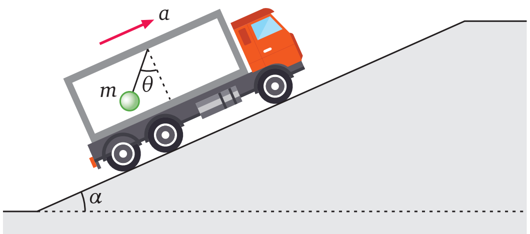
8. A vehicle is accelerating upward with a constant acceleration of magnitude a on an incline that makes an angle α with the horizontal. A small object of mass m is suspended from the ceiling of the vehicle by a light string. As the truck accelerates, the object remains in equilibrium such that the string makes an angle θ with the normal to the truck's ceiling. The magnitude of the gravitational acceleration is g.

a. Draw the free-body diagram of the object.

b. What is the magnitude a of the vehicle's acceleration in terms of the given quantities?



Answers

Example questions

1	a. $T = 2.0 \times 10^2$ N b. $T_{bottom} = 2.0 \times 10^2$ N, $T_{top} = 2.2 \times 10^2$ N
2	a. $T = 9.8 \times 10^2$ N b. $m = 2.0 \times 10^2$ kg
3	Go to the video.
4	$N = 826$ N
5	$F = 1.8 \times 10^2$ N
6	$T = 2\pi\sqrt{\frac{L\cos\theta}{g}}$
7	$v_{maks} = \sqrt{\mu_s gR}$
8	$\theta = \arctan\left(\frac{v^2}{gR}\right)$
9	$N_{top} = m\left(g - \frac{v^2}{R}\right)$, $N_{bottom} = m\left(g + \frac{v^2}{R}\right)$

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Your Turn Questions

1	a. $a = \dfrac{m_1 - m_2}{m_1 + m_2} g$ b. $T = \dfrac{2m_1 m_2}{m_1 + m_2} g$
2	a. $a = 3.0\ \text{m/s}^2$ b. Go to the video. c. Go to the video.
3	$a = 4.4\ \text{m/s}^2$
4	a. $a = 6\ \text{m/s}^2$ b. $T = 16$ N
5	a. $a = 4.0\ \text{m/s}^2$, upward b. $T = 98$ N
6	$N_{bottom} = 32$ N

Exercises and Problems

1	a. $F_{min} = 20$ N b. $a = 1.0\ \text{m/s}^2$, $T = 12$N.
2	a. Go to the video. b. Go to the video. c. $a = 1.0\ \text{m/s}^2$, $T = 8$N.
3	$F_{min} = (m_1 + m_2)\dfrac{g}{\mu_s}$
4	$F_{min} = (m_1 + m_2) g \tan\theta$
5	a. $a = 4.0\ \text{m/s}^2$ b. $T = 2.4$ N c. Go to the video.
6	a. $T_1 = 108$ N, $T_2 = 152$ N.
7	a. $T_{lower} = 25$ N b. $v = 3.0$ m/s c. $v_{min} = 2.12$ m/s d. Go to the video.
8	a. Go to the video. b. $a = g(\tan\theta\cos\alpha - \sin\alpha) = g\dfrac{\sin(\theta - \alpha)}{\cos\theta}$

CHAPTER 6
WORK AND KINETIC ENERGY

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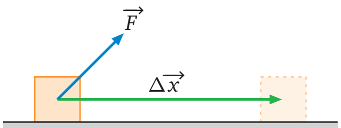
Work

Let us define the concept of work and express work mathematically. Let us obtain its SI unit by performing a dimensional analysis of work.

Let us examine how to determine the work done when the force and the displacement are not in the same direction.



Let us discuss what it means for the work done to be positive, negative, or zero.

Let us examine through an example what it means for the work done to be positive, negative, or zero.

Let us define the concepts of **total work** and **net work**.

Kinetic Energy and the Work-Kinetic Energy Theorem

Let us discuss how the speed of an object changes when positive or negative net work is done on it.

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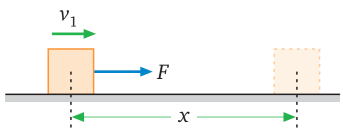
Let us examine the change in the speed of an object of mass m with initial speed v_1 that is pulled through a distance x by a net force of magnitude F. Then, by defining the concept of **kinetic energy**, let us explain the **work-kinetic energy theorem**.



EXAMPLE 1: An object is thrown downward toward the ground with a speed of 10.0 m/s from a height of 15 m above the ground. The mass of the object is $m = 2.0$ kg, and the magnitude of the acceleration due to gravity is $g = 10\ \mathrm{m/s^2}$.

a. What is the work done by the gravitational force until the object strikes the ground?

b. What is the change in the kinetic energy of the object during its motion?

c. What is the speed of the object when it strikes the ground?

Your turn

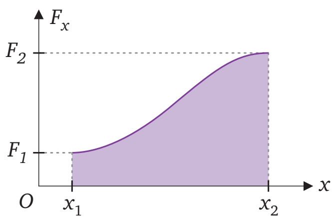
1. An object of mass $m = 3.0$ kg is moving on a frictionless horizontal plane. A constant horizontal force is applied to the object, whose initial speed is $v_i = 4.0$ m/s. When the object has traveled 10.0 m under the influence of the force, its speed becomes $v_f = 6.0$ m/s.

a. Find the change in the kinetic energy of the object.

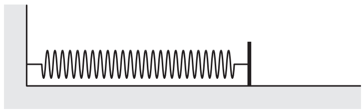
b. Find the magnitude of the applied force.

Work Done by a Varying Force

For one-dimensional motion, let us show the work done by a force that varies along the path.



Let us examine the work done on a spring as it is compressed.



NOTE: The work done on the spring and the work done by the spring are not the same thing.

Let us show that the work–kinetic energy theorem also holds for work done by a varying force.

EXAMPLE 2: The end of a special type of spring is at its natural length at the position $x = 0$. When the spring is compressed by an amount x, the force exerted by the spring on the object is found to be $F(x) = -kx - bx^2$. It is given that $k = 100$ N/m and $b = 1500\ \mathrm{N/m^2}$. The spring is compressed from the position $x = 0$ to $x = 0.10$ m. Find the work done by the external force during the compression.

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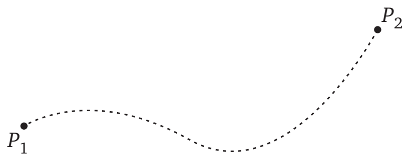
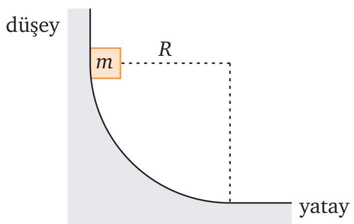
Work Done Along a Curved Path

Let us examine how to obtain the total work done along a curved path.



EXAMPLE 3: An object is released from rest at the highest point of the quarter-circular path of radius R shown in the figure. The mass of the object is m, and the magnitude of the acceleration due to gravity is g. What is the work done by the gravitational force on the object along this portion of the path?



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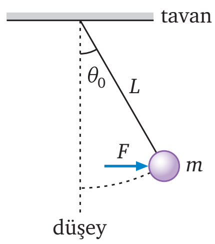
Your turn

2. An object of mass m is suspended from the ceiling by a string of length L. A varying horizontal force is applied to the object so that it moves slowly and remains nearly in equilibrium throughout the motion. By means of this force, the object is brought to the position shown in the figure, where it makes an angle θ_0 with the vertical. What is the work done by the force during this motion?



Power

Let us define the concept of power. Let us express it mathematically and examine its SI unit.

Let us explain the concepts of average power and instantaneous power. Let us examine the relationship between instantaneous power and instantaneous velocity.

EXAMPLE 4: A small motor attached to a toy car produces a constant thrust force that pushes the car forward. The magnitude of the thrust force produced by the motor is $F = 5.0$ N. Because of frictional effects, the toy car moves with a constant speed of $v = 2.0$ m/s. What is the power output of the motor at this instant?

Your turn

3. A mountain climber of mass 60.0 kg climbs to a peak of height 450 m in 30.0 min. Take the magnitude of the acceleration due to gravity as $g = 10\ \text{m/s}^2$.

a. Find the work done by the climber against gravity during the climb.

b. Calculate the climber's average mechanical power.

Exercises and Problems

1. An object of mass $m = 2.0$ kg is released from rest at the top of a rough ramp inclined at 37°. The object moves 6.0 m down along the ramp. The coefficient of kinetic friction is $\mu_k = 0.10$, and the magnitude of the acceleration due to gravity is $g = 10\ \text{m/s}^2$.

a. Find the work done by the friction force along this path.

b. Find the work done by the gravitational force along this path.

c. Find the speed of the object after it has moved 6.0 m.

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2. A block of mass m is released from rest at the top of an inclined plane that makes an angle θ with the horizontal. The coefficient of kinetic friction between the block and the inclined plane is μ_k. The top of the inclined plane is vertically a height h above its base. The magnitude of the acceleration due to gravity is g.

a. Find, in terms of the given quantities, the work done by the friction force as the block moves from the top of the inclined plane to its base.

b. If the angle θ is increased, does the magnitude of the work done by the friction force increase or decrease? Explain.

3. You are asked to design a spring bumper for a parking-lot wall. When a car of mass 1000 kg collides with the bumper at a speed of 0.80 m/s, it must come to rest after compressing the spring by at most 0.10 m. What must the force constant k of the massless spring be?

4. A horizontal force is applied to a crate on a horizontal floor. The force is $\vec{F} = (-6.00\ \text{N})\,\hat{\imath} + (4.00\ \text{N})\,\hat{\jmath}$. At a certain instant, the velocity of the crate is measured to be $\vec{v} = (2.50\ \text{m/s})\,\hat{\imath} + (1.50\ \text{m/s})\,\hat{\jmath}$. Find the instantaneous power delivered to the crate by the force at the specified instant.

5. A box sliding on a horizontal plane enters a rough region starting at $x = 0$. The magnitude of the box's velocity at $x = 0$ is $v_0 = 5.0$ m/s. The coefficient of kinetic friction in this region is not constant; it starts from $\mu_k = 0.20$ at $x = 0$, and the coefficient of friction increases linearly with x. At the position $x = 10.0$ m, it reaches $\mu_k = 0.40$. Take the magnitude of the acceleration due to gravity as $g = 10\ \text{m/s}^2$.

a. Using the work–kinetic energy theorem, find how many meters the box travels before coming to rest.

b. What is the value of the coefficient of friction at the stopping point?

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Answers

Example questions

1	a. $W = 300$ J b. $\Delta K = 300$ J c. $v = 20$ m/s
2	$W = 1.0$ J
3	mgR
4	10 W

Your turn questions

1	a. $\Delta K = 30$ J b. $F = 3$ N
2	$W_F = mgL(1 - \cos\theta_0)$
3	a. $W = 2.7 \times 10^5$ J b. $P_{\text{avg}} = 150$ W

Exercises and problems

1	a. $W_f = -9.6$ J b. $W_g = 72.0$ J c. $v = 7.9$ m/s
2	a. $W_f = -\mu_k mgh\cot\theta$ b. Go to the video.
3	$k = 6.4 \times 10^4$ N/m
4	$P = -9.0$ W
5	a. $x_s = 5.0$ m b. $\mu_k(x_s) = 0.30$

CHAPTER 7

POTENTIAL ENERGY AND CONSERVATION OF ENERGY

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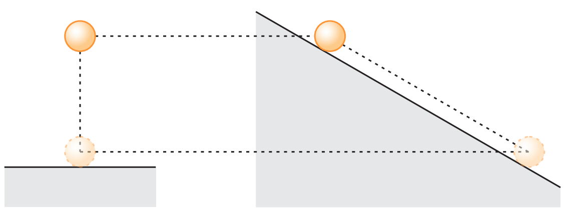
Gravitational Potential Energy

Let us discuss our motivation for defining the concept of potential energy. Let us define the concept of **gravitational potential energy**. Let us establish the relationship between the work done by the gravitational force acting on an object and the change in the object's potential energy.

NOTE: Gravitational potential energy is a property of the object–Earth system, not of the object alone.

Let us define **the mechanical energy** of a system in which only gravitational force does work. Let us show the conservation of mechanical energy for this object.

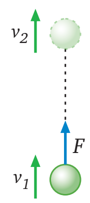
NOTE: When defining potential energy, we can choose any height as the zero level.



EXAMPLE 1: Calculate the maximum height reached by an object projected with speed v_0 at an angle θ with the horizontal by using conservation of mechanical energy.

Let us examine how to treat the case in which a force other than the gravitational force also does work on an object.

NOTE: The work done by nonconservative forces changes the mechanical energy of the system.



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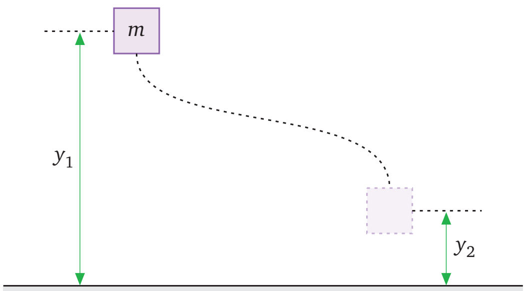
EXAMPLE 2: An object of mass 3.0 kg is moving vertically downward. In addition to its weight, a constant downward force of magnitude 33 N acts on the object. Initially, the object's speed is $v_0 = 2.0\ \mathrm{m/s}$. What is the magnitude of its velocity after the object moves an additional 6.0 m under the influence of this force? Take the magnitude of the acceleration due to gravity as $g = 10\ \mathrm{m/s^2}$.

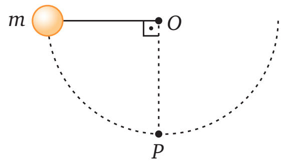
Show that the work done by the gravitational force on an object is independent of the path and depends on the change in the object's height.



EXAMPLE 3: In the vertical plane shown in the figure, an object of mass m is attached to point O by a massless and inextensible string. When the object is released from rest at the same level as point O, it speeds up and passes through point P. The magnitude of the acceleration due to gravity is g.

a. What is the magnitude of the object's centripetal acceleration as it passes through point P?

b. What is the tension in the string as the object passes through point P?



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Your turn

1. A crate of mass $m = 10.0$ kg is projected up from the bottom of a sufficiently long ramp inclined at an angle of 37° with a speed of $v_0 = 6.0\ \mathrm{m/s}$. The crate moves 2.0 m up the ramp, comes to rest, and then slides back down to the bottom of the ramp. Assume that the magnitude of the friction force is constant throughout the motion. Take the magnitude of the acceleration due to gravity to be $g = 10\ \mathrm{m/s^2}$.

a. Find the magnitude of the friction force acting on the crate.

b. Find the magnitude of the crate's velocity when it reaches the bottom of the ramp again.

Elastic Potential Energy

Let us define the concept of **elastic potential energy**. Let us establish the relationship between the work done by a spring and the change in elastic potential energy.

Let us examine situations in which gravitational potential energy and elastic potential energy are both involved.

EXAMPLE 4: 800 kg elevator is moving vertically downward with a speed of 4.0 m/s when it falls onto a spring on the floor. The elevator comes to rest when the spring is compressed by 2.0 m. During the compression of the spring, a constant friction force of magnitude 2.0 kN acts upward on the elevator. Take the free-fall acceleration to be $g = 10\ \mathrm{m/s^2}$. What is the spring constant k in N/m?

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Conservative and Nonconservative Forces

Let us explain the concepts of conservative and nonconservative forces. Let us give examples of these forces.

EXAMPLE 5: A force field defined in two dimensions is of the form $\vec{F}(x, y) = Ay\hat{\imath}$, where $A = 5$ N/m. A particle moves in the xy plane along a rectangular path. The particle passes through the points $(0, 0)$, $(3, 0)$, $(3, 2)$, and $(0, 2)$, respectively, and returns to the point $(0, 0)$.

a. Find the work done on the particle by the given force along each side of the rectangle.

b. Find the total work done on the particle by this force.

c. Discuss whether this force field is conservative.

Let us discuss what the principle of conservation of energy means for a system.

Force and Potential Energy

Let us examine how the force acting on an object can be obtained from potential energy for one-dimensional problems.

Let us obtain the forces acting on an object from gravitational and elastic potential energies.

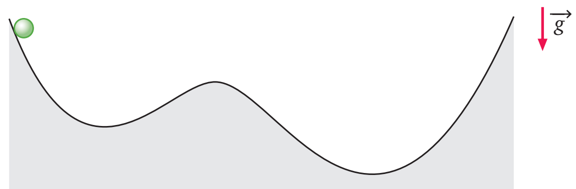
EXAMPLE 6: A particle with electric charge q_1 is fixed at the position $x = 0$. A second charge q_2 is free to move along the x-axis. Given that the electric potential energy of this system is $U(x) = k\frac{q_1 q_2}{x}$, determine the position-dependent function for the force acting on the second particle.

Let us examine how the force acting on an object can be obtained from the potential energy for three-dimensional problems.

EXAMPLE 7: An object moves in the xy-plane. The potential energy of this object attached to the end of a spring and the spring is given as $U(x,y) = \frac{1}{2}k(x^2 + y^2)$. Determine the force exerted by the spring on the object as a function of position.

Energy Diagrams

Let us draw an energy diagram for an object released from the position shown in the figure and moving in a frictionless surface. Let us indicate the points of stable and unstable equilibrium. For the same object, let us also draw a graph giving the force as a function of position.



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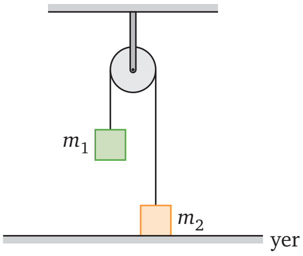
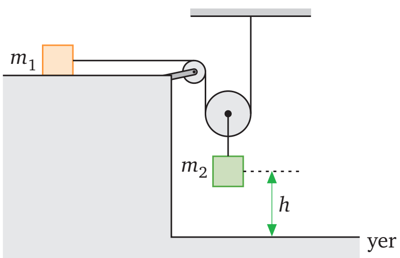
Exercises and Problems

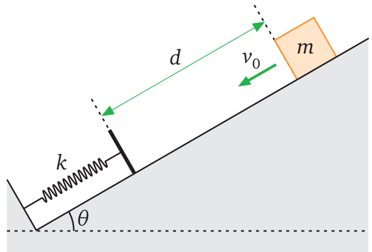
1. In the system shown in the figure, $m_1 = 6.0$ kg and $m_2 = 4.0$ kg. The height of the object of mass m_1 above the ground is 4.0 m. All friction is negligible, and the magnitude of the acceleration due to gravity is $g = 10\ \mathrm{m/s^2}$. When the system is released, what is the maximum height reached by the object of mass m_2?



2. In the system shown in the figure, the object of mass $m_1 = 6.0$ kg is on a horizontal surface. The coefficient of kinetic friction between the object and the surface is $\mu_k = 0.20$. The object of mass $m_2 = 4.0$ kg is suspended from a movable pulley system as shown. When the system is released, the object of mass m_2 moves downward. The object m_2 is released from rest when it is at a height $h = 2.0$ m above the ground. The magnitude of the acceleration due to gravity is $g = 10\ \mathrm{m/s^2}$. Assuming that the weights of the pulleys and friction in the pulleys are negligible, calculate the magnitude of the speed of the object of mass m_2 just before it strikes the ground.



3. The object of mass m shown in the figure is launched on an inclined plane with speed v_0 and moves down the ramp, compressing a spring at the bottom of the ramp by a maximum distance x before coming to rest. Initially, the object is a distance d from the spring. Given the spring constant k, the coefficient of kinetic friction μ_k on the surface, and the magnitude g of the acceleration due to gravity, what is the initial speed v_0 of the object in terms of the given quantities?



4. A small object of mass m slides without friction on the inside surface of a vertical circular track of radius R. The object is required to complete this path without leaving the track. The magnitude of the acceleration due to gravity is g. For this to occur, what must be the minimum speed of the object at the lowest point of the track in terms of the given quantities?

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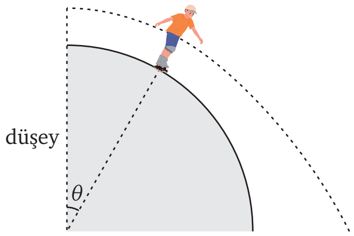
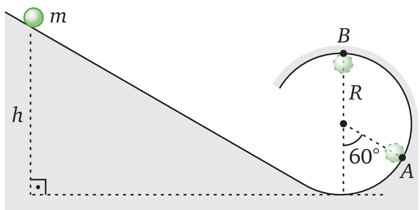
5. A skater slides downward from the top of a very large frictionless spherical surface. What is the value of the cosine of the angle θ that the point where the skater leaves the surface makes with the vertical?



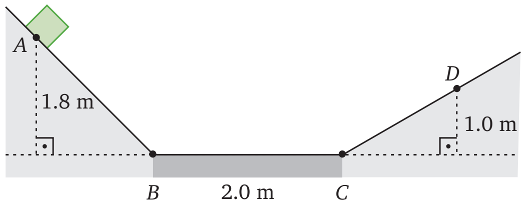
6. A point object of mass m slides down a frictionless track whose vertical cross section is shown in the figure. When the object is at height h, its speed is zero. The object passes through point B of the circular part of the track of radius R with the minimum speed required so that it does not lose contact with the track. The magnitude of the acceleration due to gravity is g. Therefore,

a. what is the height h in terms of R?

b. what is the magnitude of the normal force exerted on the object by the track as the object passes through point A, in terms of the given quantities?



7. An object is released from rest at point A on the track whose vertical cross section is shown in the figure, and it can rise at most to point D. If friction exists only between points B and C on the track, what is the coefficient of kinetic friction μ_k in this region?



8. The position-dependent expression for the force exerted by a spring that does not obey Hooke's law is $F_x(x) = -kx - \lambda x^2$. Here, $k = 50.0$ N/m and $\lambda = 12.0$ N/m^2.

a. Write the potential energy function $U(x)$ for this spring. Define the potential energy so that $U = 0$ at $x = 0$.

b. An object of mass $m = 1.00$ kg is attached to this spring on a frictionless horizontal surface. The spring is pulled 2.00 m from the equilibrium position $x = 0$ and released. Find the speed of the object as it passes through the position $x = 1.00$ m.

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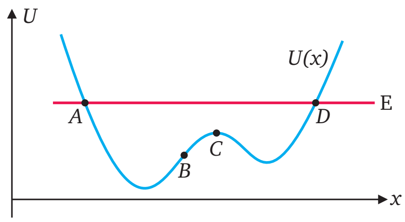
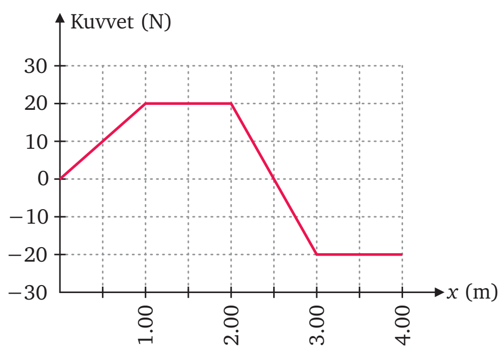
9. A particle moves along the x-axis under the influence of a conservative force parallel to the x-axis. The graph of the potential energy of the particle in this system is shown in the figure. The total energy of the particle is denoted by E. For the particle:

a. Between which points does it move?
b. Where is its kinetic energy greatest?
c. What is the direction of the force acting at point B?
d. What is the force acting at point C?
e. Which points are stable equilibrium points?
f. Which points are unstable equilibrium points?



10. A particle moves along the x-axis under the influence of a force parallel to the x-axis. The graph of the force acting on the particle as a function of position is shown in the figure. For the particle that is initially at $x = 0$ and has no initial velocity:

a. What is the maximum value of its kinetic energy?
b. What is the net work done on it over the interval 0 – 4 m?



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Answers

Example questions

1	$h = \frac{v_0^2 \sin^2\theta}{2g}$
2	$v = 16$ m/s
3	a. $2g$ b. $3mg$
4	$k = 9.2 \times 10^3$ N/m
5	a. $W_{(0,0)\to(3,0)} = 0$ J, $W_{(3,0)\to(3,2)} = 0$ J, $W_{(3,2)\to(0,2)} = -30$ J, $W_{(0,2)\to(0,0)} = 0$ J b. $W_{net} = -30$ J c. Nonconservative
6	$\vec{F}(x) = \frac{kq_1q_2}{x^2}\hat{\imath}$
7	$\vec{F} = -kx\hat{\imath} - ky\hat{\jmath}$

Your turn questions

1	a. $F = 30$ N b. $v = 3.46$ m/s

Exercises and problems

1	$h_{max} = 4.8$ m
2	$v_{impact} = 1.51$ m/s
3	$v_0 = \sqrt{\frac{k}{m}x^2 + 2g(\mu_k\cos\theta - \sin\theta)(d + x)}$
4	$v_0 = \sqrt{5gR}$
5	$\cos\theta = \frac{2}{3}$
6	a. $h = \frac{5}{2}R$ b. $N_A = \frac{9}{2}mg$
7	$\mu_k = 0.40$
8	a. $U(x) = \frac{1}{2}kx^2 + \frac{1}{3}\lambda x^3$ b. $v = 14.4$ m/s
9	Go to the video.
10	a. $K_{max} = 35$ J b. $W = 10$ J

CHAPTER 8

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LINEAR MOMENTUM AND COLLISIONS

Momentum and Impulse

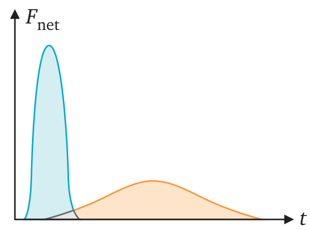
Let us express Newton's second law of motion mathematically in terms of momentum for an object of constant mass. Let us define the concept of **momentum** and examine its SI unit.

> **NOTE:** Because the momentum of an object is a vector quantity, it is represented by its components.

Let us define the concept of **impulse** by using Newton's second law of motion. Let us emphasize that impulse is a vector quantity and examine its unit.

Let us explain the **impulse–momentum theorem**.

Let us examine and interpret the areas under the net force–time graphs given for one-dimensional motion.



Let us discuss the similarities and differences between momentum and kinetic energy. Let us compare the impulse–momentum theorem with the work–kinetic energy theorem.

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EXAMPLE 1: A ball of mass 2.0 kg is initially moving west with a speed of 3.0 m/s. Starting at this instant, a constant force of magnitude 4.0 N is applied to the ball eastward for 2.0 s.

a. Find the initial momentum of the ball.

b. Find the impulse applied to the ball during the 2.0 s interval.

c. Find the new momentum and velocity of the ball at $t = 2.0$ s.

Your turn

1. A ball of mass 0.50 kg is moving horizontally in the $+x$ direction with a speed of 25 m/s when it strikes a rigid wall. After the collision, the ball rebounds in the $-x$ direction with a speed of 15 m/s.

a. Find the change in the momentum of the ball during the collision.

b. If the collision lasts 0.080 s, calculate the magnitude of the average force exerted by the wall on the ball.

2. A spacecraft fires its engine at $t = 0$. The engine exerts a force on the spacecraft in the $+x$ direction that increases with time. This force is given by the relation $F_x(t) = At^2$. The magnitude of the force is 60 N at $t = 2.0$ s.

a. Find the value of the coefficient A.

b. Calculate the total impulse exerted by the engine on the spacecraft during the 2.0 s interval from $t = 1.0$ s to $t = 3.0$ s.

c. Find the change in the velocity of the spacecraft of mass $m = 200$ kg during this time interval.

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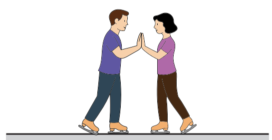
Conservation of Momentum

Let us discuss conservation of momentum for a simple system. Let us show the forces acting on two objects and express conservation of momentum mathematically.



NOTE: If the vector sum of the external forces acting on a system is zero, the total momentum of the system is constant.

NOTE: Conservation of momentum is the conservation of the components of momentum.

EXAMPLE 2: A rifle of mass 4.0 kg fires a bullet of mass 9.0 g horizontally with a speed of 600 m/s.

a. Find the recoil velocity of the rifle immediately after the bullet is fired.

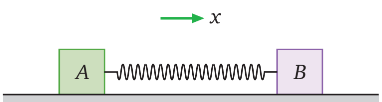
b. Calculate the momenta of the bullet and the rifle separately, and show that the total momentum is conserved.

Your turn

3. Two objects of masses $m_A = 2.0$ kg and $m_B = 4.0$ kg are held next to each other with a compressed spring placed between them. The system is released from rest on a frictionless horizontal surface. The mass of the spring is neglected, and the spring separates from the blocks when it is released. After being released, object B moves in the $+x$ direction with a speed of 2.0 m/s.

a. Find the final velocity of object A.

b. Find the potential energy initially stored in the spring.



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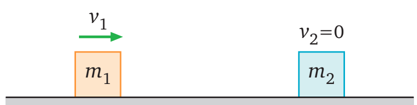
Conservation of Momentum and Collisions

Let us discuss **elastic**, **inelastic** and **perfectly inelastic** collisions. Let us examine these collisions in terms of the conservation of momentum and energy.

Perfectly Inelastic Collisions

Let us examine the change in kinetic energy in a perfectly inelastic collision between two objects, one initially at rest and the other initially moving.



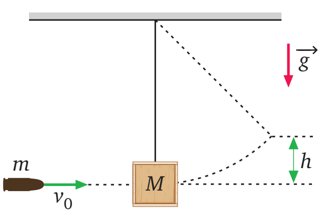
EXAMPLE 3: A car of mass 1500 kg is traveling north at 20 m/s, while a truck of mass 2500 kg is traveling east at 12 m/s. The two vehicles collide at an intersection and stick together as they move.

a. Find the magnitude and direction of the velocity of the wreckage immediately after the collision.

b. Compare the total kinetic energy before and after the collision.

Your turn

4. In a ballistic pendulum setup, a bullet of mass m becomes embedded in a wooden block of mass M suspended as a pendulum. After the collision, the bullet–block system swings upward together and rises to a maximum height h. The magnitude of the free-fall acceleration is g. Express the magnitude v_0 of the initial velocity of the bullet in terms of h, g, m, and M.



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Elastic Collisions

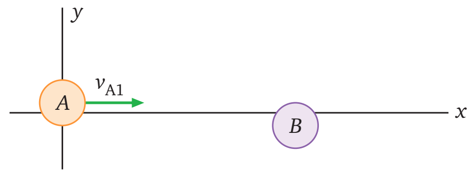
Let us examine an example of an elastic collision in one dimension.

EXAMPLE 4: In an experiment performed in a laboratory, two gliders move without friction on a straight air track. One glider has a mass $m_A = 0.40$ kg, and the other has a mass $m_B = 0.60$ kg. Glider A moves in the $+x$ direction with velocity $v_{A1x} = 4.0$ m/s, while glider B moves in the $-x$ direction with velocity $v_{B1x} = -1.0$ m/s. The gliders collide elastically with each other. Let us find the velocities of the gliders (v_{A2x} and v_{B2x}) after the collision.

EXAMPLE 5: Two pucks can move without friction on an air hockey table. The masses of the pucks are $m_A = 1.00$ kg and $m_B = 0.60$ kg, respectively. Puck A is initially moving in the $+x$ direction with a speed of 4.0 m/s; puck B is initially at rest. The pucks undergo an elastic (perfectly elastic) collision. After the collision, puck A has a speed of 2.0 m/s and moves at an angle α with the x axis. Puck B has speed v_{B2} in a direction that makes an angle β with the x axis.

a. Find the magnitude v_{B2} of the velocity of puck B after the collision.

b. Find the angles α and β after the collision.



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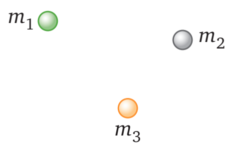
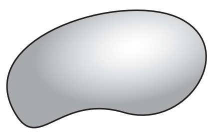
Center of Mass

Let us examine how we determine the center of mass of a system in two dimensions. Let us derive mathematically the vector $\vec{r}_{\text{cm}}$, which gives the position of the center of mass.

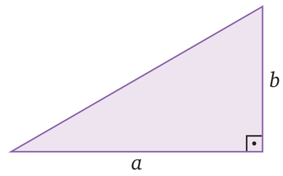


EXAMPLE 6: A uniform cube with mass 0.60 kg and volume 0.0080 m^3 rests on the ground. A uniform sphere with radius 0.30 m and mass 1.20 kg rests at the exact center of the top face of the cube. Find the height of the center of mass of the system consisting of these two objects above the ground.

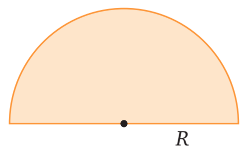
Let us examine how to determine the center of mass of an object with a continuous mass distribution in three dimensions.



Let us determine the location of the center of mass of a thin, homogeneous plate in the shape of a right triangle.



EXAMPLE 7: Determine the location of the center of mass of a thin, homogeneous semicircular plate.



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Motion of the Center of Mass

Let us examine the motion of the center of mass of a system in two dimensions. Let us derive mathematically the vector $\vec{v}_{\rm cm}$, the velocity of the center of mass.

Let us derive mathematically the relationship between the velocity of the center of mass and the total momentum.

EXAMPLE 8: Ali and Banu are standing on a frictionless horizontal surface. Ali's mass is $m_A = 80$ kg and Banu's mass is $m_B = 50$ kg. Initially, Ali is at position $x_{A1} = -8.0$ m, while Banu is at position $x_{B1} = 8.0$ m. Ali and Banu start moving by pulling each other with the help of a massless rope.

a. Find the initial position of the center of mass of the system.
b. Find how far Banu moves when Ali has moved 4.0 m.
c. How much does the center of mass move when Ali has moved 4.0 m?

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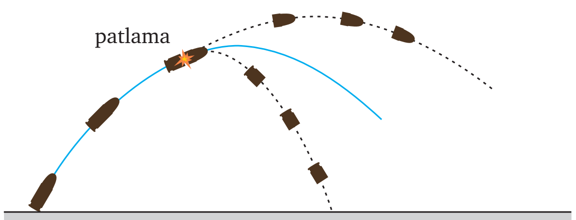
External Forces and the Motion of the Center of Mass

Let us discuss how the center of mass of a system acted on by external forces moves.

NOTE: According to Newton's third law, all internal forces cancel in pairs and $\sum \vec{F}_{\text{int}} = 0$.

Let us examine the trajectories of the fragments and the center of mass of an artillery shell that explodes in midair in the absence of air resistance.



Your turn

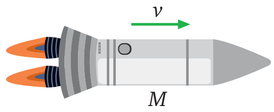
5. An object of mass $m = 4.0$ kg is launched with a speed of 50.0 m/s at an angle of 37° above the horizontal. When the object reaches the highest point of its trajectory, it explodes into two fragments of equal mass. One of the fragments falls only vertically downward with zero initial speed immediately after the explosion. Neglect air resistance and take the magnitude of the acceleration due to gravity to be $g = 10\ \text{m/s}^2$.

a. Assuming the ground is level, how far horizontally from the launch point does the other fragment land?

b. How much energy is released during the explosion?

Rocket Propulsion

Let us derive the force acting on a rocket accelerating in one direction in empty space, the instantaneous acceleration of the rocket, and its final speed.



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Your turn

6. In space, very far from all other objects, a rocket has a speed of 6.0×10^3 m/s relative to Earth. The rocket engines are fired, and fuel is expelled in the direction opposite the rocket's motion with a speed of 4.0×10^3 m/s relative to the rocket.

a. When the mass of the rocket decreases to one-half its initial value, what is its speed relative to Earth?

b. If the rate at which the rocket fuel is burned is 50 kg/s, what is the thrust force acting on the rocket?

Exercises and Problems

1. A net force is applied to an object starting at $t = 0$. The time-dependent equation of the applied force

$$\vec{F}(t) = ((0.20\ \mathrm{N/s})\, t)\,\hat{\imath} + \left((-0.30\ \mathrm{N/s^2})\, t^2\right)\hat{\jmath}$$

is given. If the initial momentum of the object is $\vec{p}_0 = (-3.0\hat{\imath} + 4.0\hat{\jmath})\ \mathrm{kg \cdot m/s}$, what is the momentum of the object at $t = 2.0$ s?

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2. The momentum of an object is given by

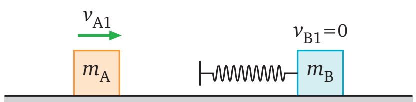
$$\left((0.50\ \mathrm{kg \cdot m/s^3})t^2 - (4.0\ \mathrm{kg \cdot m/s})\right)\hat{\imath} + \left(0.50\ \mathrm{kg \cdot m/s^2}\right) t\,\hat{\jmath}$$

Find the net external force acting on the object.

3. A block of mass $m_A = 2.0$ kg is initially moving in the $+x$ direction with a speed of 6.0 m/s on a frictionless horizontal track. This block collides with a massless spring attached to a second block of mass $m_B = 1.0$ kg. The collision takes place in one dimension, and the second block is initially at rest. The spring constant is $k = 600$ N/m.

a. Find the velocities of the blocks after the collision, when they separate from each other and begin to move independently.

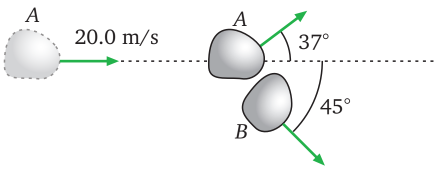
b. Find the maximum compression of the spring during the collision.



4. Two asteroids of equal mass in the asteroid belt collide with each other as shown in the figure. Asteroid A is initially moving with a speed of $v_{A1} = 20.0$ m/s and changes direction by 37° after the collision. Asteroid B moves after the collision in a direction that makes a 45° angle with the initial velocity of A. The asteroids may be treated as point particles.

a. Find the speeds of the asteroids after the collision.

b. Find the percentage of the total kinetic energy lost during the collision.

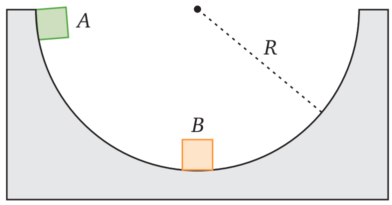


5. Point objects A and B are released on a frictionless circular track of radius R whose vertical cross section is shown in the figure. For the different mass values and collision cases given below, find the heights to which the objects rise after the collision.

a. $m_A = 2.0$ kg and $m_B = 2.0$ kg, perfectly inelastic collision.

b. $m_A = 2.0$ kg and $m_B = 1.0$ kg, perfectly inelastic collision.

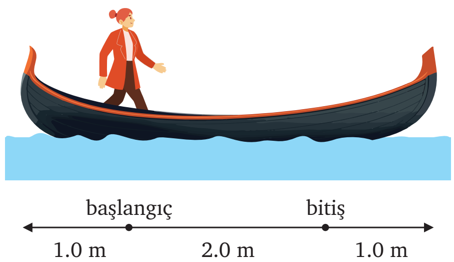
c. $m_A = 2.0$ kg and $m_B = 2.0$ kg, elastic collision.



6. A 60.0 kg woman is standing in a 90.0 kg boat that is 4.00 m long. The woman walks from a point 1.0 m from one end of the boat to a point 1.0 m from the other end. The resistance of the water to the motion of the boat will be neglected.

a. How far does the boat move during this process?

b. How far does the woman move relative to the ground?



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7. A 5.0 kg object initially at rest on a frictionless horizontal plane breaks into three pieces as a result of an internal explosion. Immediately after the explosion, the first piece ($m_1 = 1.0$ kg) moves in the $+x$ direction with a speed of 10 m/s, and the second piece ($m_2 = 2.0$ kg) moves in the $+y$ direction with a speed of 8.0 m/s. The remaining mass of the system forms the third piece.

a. Find the velocity vector of the third piece in terms of unit vectors.

b. Find the total mechanical energy released during this explosion.

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Answers

Example questions

1	a. $\vec{p}_i = -6.0\hat{\imath}$ kg · m/s b. $\vec{I} = 8.0\hat{\imath}$ N · s c. $\vec{p}_f = 2.0\hat{\imath}$ kg · m/s, $\vec{v}_f = 1.0\hat{\imath}$ m/s
2	a. $v_r = -1.35$ m/s b. $P_b = 5.4$ kg · m/s, $P_r = -5.4$ kg · m/s, $P_{initial} = P_{final} = 0$
3	a. $v = 10.6$ m/s, in the northeast direction b. $K_{initial} = 4.8 \times 10^5$ J, $K_{final} = 2.25 \times 10^5$ J $\Delta K = -2.55 \times 10^5$ J
4	$v_{A2x} = -2.0$ m/s $v_{B2x} = 3.0$ m/s
5	a. $v_{B2} = 4.47$ m/s b. $\alpha = 36.8°$, $\beta = 26.6°$
6	0.37 m
7	$x_{cm} = 0$, $y_{cm} = \frac{4R}{3\pi}$
8	a. $x_{cm} = -1.85$ m b. $d_B = 6.41$ m c. $\Delta x_{cm} = 0$ m

Your turn questions

1	a. $\Delta\vec{p} = -20\hat{\imath}$ kg·m/s b. $F_{avg} = 250$ N
2	a. $A = 15$ N/s^2 b. $\vec{J} = 130\hat{\imath}$ N·s c. $\Delta\vec{v} = 0.65\hat{\imath}$ m/s
3	a. $\vec{v}_A = -4.0\hat{\imath}$ m/s b. $U_{spring} = 24$ J
4	$v_0 = \frac{m+M}{m}\sqrt{2gh}$
5	a. $x = 360$ m b. $\Delta E = 3.20 \times 10^3$ J
6	a. $v_s = 8.77 \times 10^3$ m/s b. $F = 2.0 \times 10^5$ N

Exercises and problems

1	$\vec{p}(t = 2.0\text{ s}) = (-2.6\hat{\imath} + 3.2\hat{\jmath})\text{ kg}\cdot\text{m/s}$
2	a. $\vec{F}_{\text{net}}(t) = (1.0\text{ N/s})t\hat{\imath} + 0.50\text{ N}\hat{\jmath}$
3	a. $v_{A2} = 2.0\hat{\imath}\text{ m/s}, \quad v_{B2} = 8.0\hat{\imath}\text{ m/s}$ b. $x_{\text{max}} = 0.20\text{ m}$
4	a. $v_{A2} = 14.3\text{ m/s}, \quad v_{B2} = 12.1\text{ m/s}$ b. 12.3%
5	a. $h_s = R/4$ b. $h_s = 4R/9$ c. $h_A = 0, \quad h_B = R$
6	a. $\Delta x_{\text{boat}} = 0.80\text{ m}$, go to the video for direction b. $\Delta x_{\text{woman}} = 1.20\text{ m}$, go to the video for direction
7	a. $\vec{v}_3 = (-5.0\hat{\imath} - 8.0\hat{\jmath})\text{ m/s}$ b. $\Delta E = 203\text{ J}$

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CHAPTER 9

ROTATIONAL MOTION

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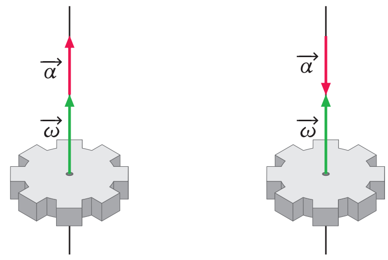
Rotational Variables

Let us define the concepts of **angular position** and **angular displacement**.

Let us define the concept of **angular velocity**. Let us express average angular velocity and instantaneous angular velocity mathematically.

Let us define the concept of **angular acceleration**. Let us express average angular acceleration and instantaneous angular acceleration mathematically.

Let us examine the figure below to better understand the concepts of angular velocity and angular acceleration.



EXAMPLE 1: The angular velocity of a disk varies according to the relation $\omega_z(t) = A + Bt^2$. Here, t is in seconds, and the constants A and B have numerical values of 4.0 and 2.0, respectively.

a. Given that the unit of the quantity ω_z is rad/s, find the units of the constants A and B.

b. Calculate the angular acceleration of the disk at $t = 0$ and $t = 3.0$ s.

Rotational Motion with Constant Angular Acceleration

Let us write the forms of the equations we obtained for motion with constant acceleration in one dimension adapted to rotational motion, and examine these equations.

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EXAMPLE 2: A wheel starting from rest rotates with constant angular acceleration. During the first 2.0 s of its motion, the wheel makes 5 revolutions. Accordingly, how many additional revolutions does the wheel make during the next 6.0 s?

Relating Linear and Angular Kinematics

Let us mathematically derive the relation between linear speed and angular speed for an object moving on a circle of radius r.

Let us mathematically derive the relation between the tangential component of the linear acceleration and the angular acceleration for an object moving on a circle of radius r. Let us recall the expression for centripetal acceleration obtained earlier.

EXAMPLE 3: A grinding wheel of radius $r = 20$ cm rotates about a fixed axis through its center. At a certain instant, the angular speed of the wheel is measured as $\omega = 5$ rad/s and its angular acceleration as $\alpha = 15$ rad/s^2.

a. Find the tangential (a_t) and centripetal (a_c) acceleration components of a point on the outer edge of the wheel.

b. Calculate the magnitude of the total linear acceleration at this point.

Energy in Rotational Motion

Let us discuss how the rotational kinetic energy of a particle is obtained.

Let us write the rotational kinetic energy of a rigid body composed of more than one particle. Then, let us define the concept of **moment of inertia**. Let us also write the kinetic energy of the body using the moment of inertia.

> **NOTE:** The moment of inertia of a body depends on the chosen axis.

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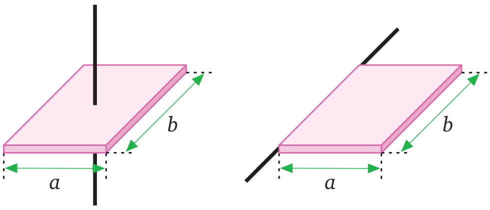
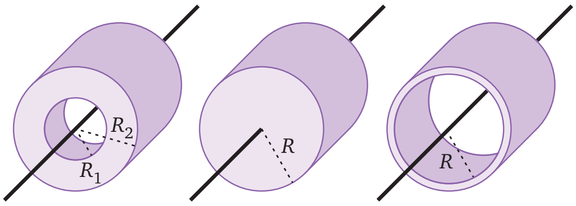
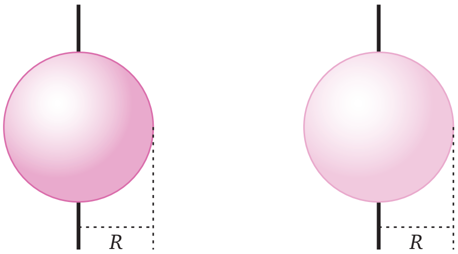
Let us write the moments of inertia of different bodies about different axes.







EXAMPLE 4: A uniform solid cylinder of mass $M = 50$ kg and radius $R = 20$ cm can rotate freely about a frictionless horizontal axis passing through its center. A constant force of magnitude $F = 20$ N is applied to the end of a light string wrapped around the cylinder. The string is pulled through a distance $x = 2.5$ m without slipping over the cylinder. Given that the cylinder is initially at rest,

a. Calculate the angular speed of the cylinder at the end of this motion.

b. What is the linear speed of the string when it has been pulled 2.5 m?

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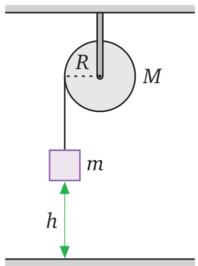
Your turn

1. A uniform solid cylinder of mass $M = 4.0$ kg and radius $R = 10$ cm can rotate freely about a frictionless horizontal axis passing through its center. A block of mass $m = 2.0$ kg is attached to the end of a light string wrapped around the cylinder. The block is released from rest at a height $h = 2.5$ m above the floor. Take the magnitude of the gravitational acceleration as $g = 10\ \mathrm{m/s^2}$. Assuming the string unwinds without slipping over the cylinder,

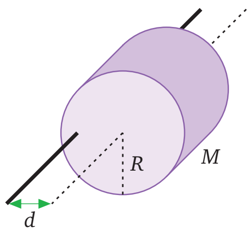
a. What is the speed of the block in m/s at the instant it hits the floor?

b. What is the angular speed of the cylinder in rad/s at the instant the block hits the floor?



Parallel-Axis Theorem

Let us explain the parallel-axis theorem using the example shown in the figure.

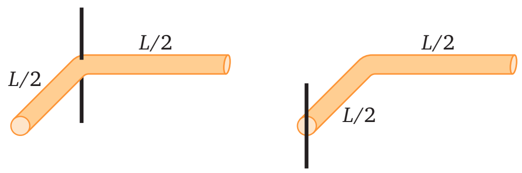


Using the moment of inertia of a thin rod about an axis through its center of mass, let us obtain its moment of inertia about an axis through one end. Then, using the moment of inertia obtained about an axis through one end of the rod, let us obtain its moment of inertia about an axis through its center.

EXAMPLE 5: A uniform thin rod of mass M and length L is bent at its midpoint so that its two parts are perpendicular to each other. For the resulting object, calculate the moment of inertia:

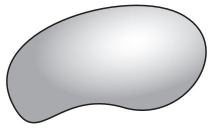
a. about an axis passing through the corner where the two parts meet and perpendicular to the plane of the object,

b. about an axis passing through one end of the rod and perpendicular to the plane of the object.

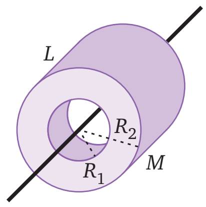


Moment of Inertia Calculations

Let us discuss how to calculate the moment of inertia of any object.



EXAMPLE 6: A hollow cylinder of mass M, length L, inner radius R_1, and outer radius R_2, with uniform mass density, is given. Using the integral method, calculate the moment of inertia of this cylinder about the symmetry axis passing through its center.



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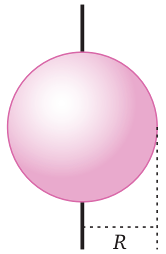
EXAMPLE 7: Calculate the moment of inertia of a sphere of mass M and radius R with uniform mass density about an axis passing through its center.

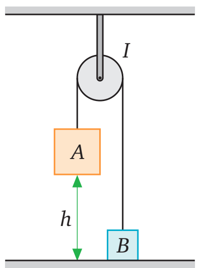


Exercises and Problems

1. In the system shown in the figure, blocks of masses $m_A = 4.0$ kg and $m_B = 2.0$ kg are connected to each other by a massless string passing over a pulley in the shape of a solid homogeneous cylinder of mass $M = 4.0$ kg and radius $R = 20$ cm. The string does not slip on the pulley. The system is released from rest when block A is at a height $h = 5.0$ m above the ground. Take the magnitude of the gravitational acceleration as $g = 10\ \mathrm{m/s^2}$. If the pulley axle is frictionless,

a. what is the speed of block A as it hits the ground?

b. what is the angular speed of the pulley at the instant block A hits the ground?



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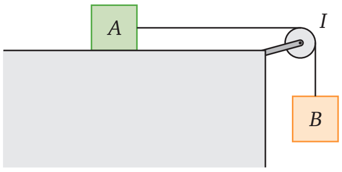
2. Block A of mass $m_A = 4.0$ kg rests on a horizontal table. The coefficient of kinetic friction between block A and the table is $\mu_k = 0.25$. Block A is connected by a cord passing over a pulley of radius $R = 10$ cm to block B of mass $m_B = 6.0$ kg. The moment of inertia of the pulley about its axis of rotation is $I = 0.02\ \text{kg}\cdot\text{m}^2$. When the system is released, block B starts moving downward. Take the magnitude of the gravitational acceleration to be $g = 10\ \text{m/s}^2$. Assuming that the cord does not slip on the pulley and that the pulley axle is frictionless:

a. What is the speed of block B when it has descended a distance $h = 3.0$ m?

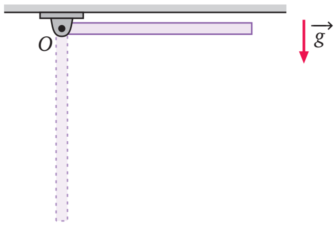
b. What is the angular speed of the pulley at this instant?



3. A uniform rod of length $L = 1.2$ m and mass $M = 3.0$ kg can rotate freely in a vertical plane about a frictionless axis passing through end O. The rod is released from rest while it is in the horizontal position, as shown in the figure. Take the magnitude of the gravitational acceleration as $g = 10\ \text{m/s}^2$. Accordingly:

a. What is the angular speed of the rod when it reaches the vertical position?

b. What is the linear speed of the lowest end of the rod when the rod reaches the vertical position?



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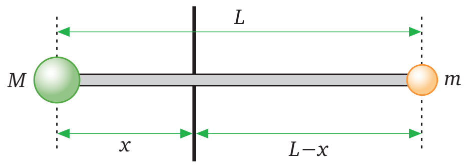
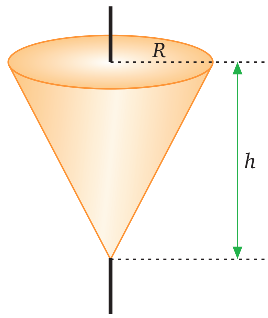
4. Objects of mass M and m, whose dimensions are negligible, are attached to the ends of a uniform thin rod of length L and mass m_{rod} as shown in the figure. The system rotates about an axis perpendicular to the rod, located a distance x from the end where the object of mass M is attached. Find the total moment of inertia of the system about this axis in terms of the given quantities.



5. Using the integral method, calculate the moment of inertia of a uniform solid cone of mass M, height h, and circular base radius R about the vertical symmetry axis passing through its center. Obtain the result in terms of the given quantities.



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Answers

Example questions

1	a. $A = 4.0\ \text{rad/s},\ B = 2.0\ \text{rad/s}^3$ b. $\alpha_z(0) = 0.0\ \text{rad/s}^2,\ \alpha_z(3.0) = 12\ \text{rad/s}^2$
2	75
3	a. $a_t = 3.0\ \text{m/s}^2,\ a_c = 5.0\ \text{m/s}^2$ b. $a = 5.8\ \text{m/s}^2$
4	a. $\omega = 10\ \text{rad/s}$ b. $v = 2.0\ \text{m/s}$
5	a. $I = \frac{1}{12}ML^2$ b. $I = \frac{5}{24}ML^2$
6	$I = \frac{1}{2}M(R_1^2 + R_2^2)$
7	$I = \frac{2}{5}MR^2$

Your turn questions

1	a. $v = 5\ \text{m/s}$ b. $\omega = 50\ \text{rad/s}$

Exercises and problems

1	a. $v = 5.0\ \text{m/s}$ b. $\omega = 25\ \text{rad/s}$
2	a. $v = 5.0\ \text{m/s}$ b. $\omega = 50\ \text{rad/s}$
3	a. $\omega = 5.0\ \text{rad/s}$ b. $v = 6.0\ \text{m/s}$
4	$I = Mx^2 + m(L-x)^2 + m_{\text{rod}}\left(\frac{1}{12}L^2 + \left(\frac{L}{2} - x\right)^2\right)$
5	$I = \frac{3}{10}MR^2$

CHAPTER 10

DYNAMICS OF ROTATIONAL MOTION

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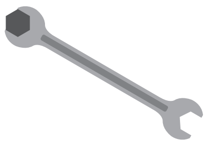
Torque

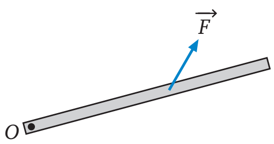
Let us define the concept of torque and examine the factors on which the magnitude of torque depends. Let us write the SI unit of torque.



NOTE: Torque is always defined with respect to a reference point.

Let us examine what it means for torque to be positive or negative.

Let us express mathematically the magnitude of the torque produced by a force acting on a rod. Then, let us define torque as a vector.



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EXAMPLE 1: A cyclist, while riding uphill, applies a downward vertical force of magnitude $F = 500$ N to the right pedal. The length of the crank arm is $r = 20$ cm, and at the instant the force is applied, the crank arm makes an angle of 37° with the horizontal plane. What is the magnitude, in N · m, of the torque produced by the cyclist about the center of the pedal sprocket at this instant?

EXAMPLE 2: The force acting on an object located in the xy plane is given by $\vec{F} = (3.0\hat{\imath} + 6.0\hat{\jmath})$ N. The position vector of the point at which the force is applied relative to the origin is $\vec{r} = (2.0\hat{\imath} - 4.0\hat{\jmath})$ m.

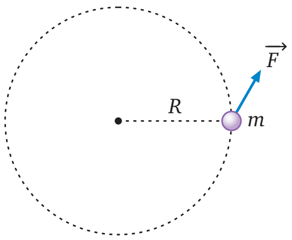
a. Make a rough sketch showing the vectors $\vec{r}$ and $\vec{F}$ and the origin.

b. Use the right-hand rule to predict the direction of the torque.

c. Calculate the torque vector $\vec{\tau}$ produced by the force about the origin and check whether it agrees with your prediction in part (b).

Torque and Angular Acceleration

Let us derive the torque produced by a force acting on a particle and the relationship between this torque and the particle's angular acceleration.



Let us derive the relationship between the net torque acting on a rigid object and the angular acceleration of the object.

EXAMPLE 3: A uniform solid disk of mass $M = 4.0$ kg and radius $R = 50$ cm is free to rotate about a fixed frictionless horizontal axis through its center. A string wrapped around the rim of the disk is pulled horizontally with a constant force of magnitude $F = 10$ N.

a. What is the magnitude of the torque produced by the force about the axis of rotation, in $\mathrm{N \cdot m}$?
b. What is the angular acceleration of the disk, in $\mathrm{rad/s^2}$?
c. What is the magnitude of the acceleration of the pulled string, in $\mathrm{m/s^2}$?

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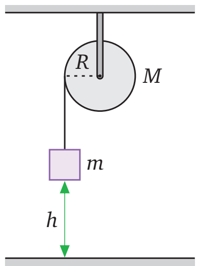
EXAMPLE 4: A uniform solid cylinder of mass $M = 4.0$ kg and radius $R = 10$ cm is free to rotate about a frictionless horizontal axis through its center. A block of mass $m = 2.0$ kg is attached to the end of a light string wrapped around the cylinder. The block is released from rest from a height $h = 2.5$ m above the ground. Take the magnitude of the gravitational acceleration to be $g = 10\ \mathrm{m/s^2}$. If the string unwinds from the cylinder without slipping:

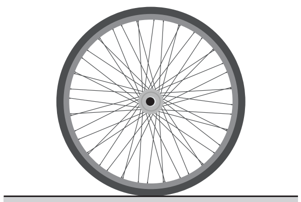
a. What is the magnitude of the downward acceleration of the block, in $\mathrm{m/s^2}$?
b. What is the magnitude of the angular acceleration of the cylinder, in $\mathrm{rad/s^2}$?
c. What is the speed of the block at the instant it hits the ground, in m/s?



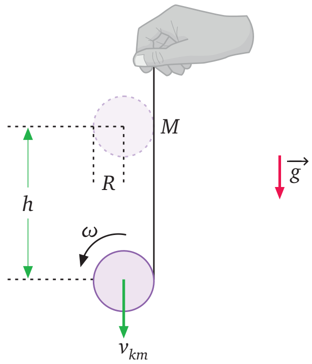
Rigid-Body Rotation About a Moving Axis

Let us examine how to obtain the kinetic energy of a body that undergoes both rotational and translational motion.

Let us examine the condition for rolling without slipping. Let us examine the instantaneous velocities, relative to the ground, of different points on a wheel undergoing this motion.



EXAMPLE 5: A uniform solid disk of mass M and radius R is released from rest while the end of a string wound around it is held fixed by hand. The disk moves downward as it rotates without slipping on the string. Using the principle of conservation of energy, find the linear speed v_{cm} of the disk's center of mass, in terms of the gravitational acceleration g and h, at the instant when the center of mass of the disk has moved downward by a vertical distance h.



Let us discuss how we can treat the dynamics of bodies that undergo translational and rotational motion together.

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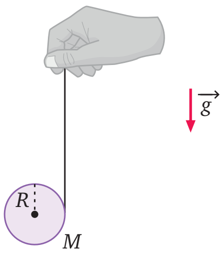
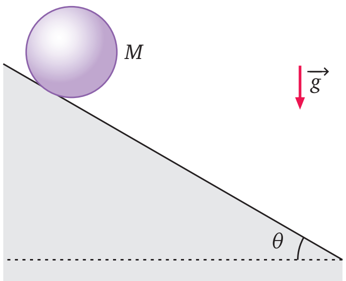
Let us carry out a dynamical analysis of the yo-yo example that we considered through energy. Let us compare the results we obtain.



Let us carry out a detailed analysis of the motion of a sphere of mass M and radius R that starts from rest and rolls without slipping down an inclined plane.



Your turn

1. A thin hollow spherical shell of mass $M = 5.0$ kg is rolling without slipping upward on an inclined plane that makes an angle $\alpha = 37°$ with the horizontal. The sphere is launched upward from the lower end of the inclined plane with an initial speed of magnitude $v_0 = 6$ m/s. Take the magnitude of the gravitational acceleration as $g = 10$ m/s^2.

a. Determine the direction of the static friction force acting on the sphere.

b. What is the magnitude of the deceleration of the center of mass of the sphere in m/s^2?

c. What is the minimum value that the coefficient of static friction μ_s between the surface and the sphere can have so that rolling without slipping is maintained?

d. How many meters above its initial vertical level can the sphere rise at most?

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Work and Power in Rotational Motion

Let us derive the relationship between the work done by a force on an object undergoing rotational motion and torque.

Let us adapt the work-kinetic energy theorem to rotational motion.

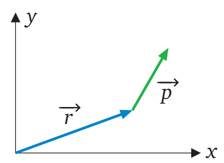
Let us express the instantaneous power for rotational motion mathematically.

EXAMPLE 6: A uniform solid disk of mass $M = 4.0$ kg and radius $R = 50$ cm can rotate freely about a frictionless fixed axis passing through its center. A force of constant magnitude $F = 10$ N is applied tangentially to the outer rim of the disk. When the disk has rotated through $\Delta\theta = 20$ rad under the action of this force:

a. How much work, in J, is done by the force on the disk?
b. Using the work-kinetic energy theorem, find the angular speed of the disk at this instant.
c. What is the instantaneous power delivered to the disk by the force at this instant, in W?

Angular Momentum

Let us define the concept of angular momentum for a particle. Let us examine the unit of angular momentum.



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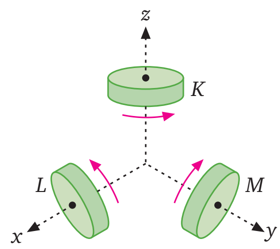
Let us examine the time derivative of angular momentum.

Let us derive the angular momentum of a rigid object.

Let us examine the relationship between the net torque on a rigid object and the time rate of change of its angular momentum.

Let us discuss what it means for angular momentum to be a vector quantity.



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EXAMPLE 7: The rotating part of a centrifuge used in a laboratory has a moment of inertia about its axis of rotation of $I = 4.0\ \text{kg}\cdot\text{m}^2$. When the device starts from rest, its angular speed increases according to the relation $\omega_z = (1.5\ \text{rad/s}^3)t^2$.

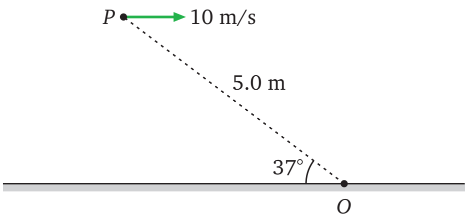
a. Write a function of time for the angular momentum L_z of the device and calculate its value at $t = 4.0$ s.

b. Write a function of time for the net torque τ_z acting on the device and find its value at $t = 4.0$ s.

EXAMPLE 8: A stone of mass $m = 4.0$ kg located at point P in the figure has a velocity of magnitude $v = 10$ m/s in the horizontal direction. At this instant, the stone's distance from point O is $r = 5.0$ m. Take the magnitude of the gravitational acceleration as $g = 10$ m/s^2.

a. At this instant, what are the magnitude and direction of the stone's angular momentum about point O?

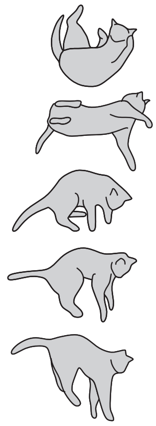
b. If the only force acting on the stone is its weight, what are the magnitude and direction of the rate of change of the stone's angular momentum at this instant?



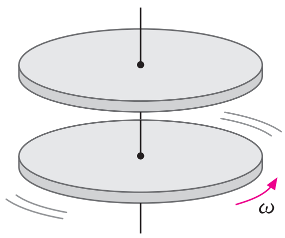
Conservation of Angular Momentum

Let us examine what conservation of angular momentum means. Let us discuss under what conditions the angular momentum of a system is not conserved.

Let us discuss how cats dropped while upside down manage to land on their feet.



Let us discuss what happens as a result of contact between two disks, one at rest and the other rotating.



Let us discuss what happens when a person spinning freely on a rotating stool moves the weights in their hands away from their body.

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EXAMPLE 9: A solid, uniform, disk-shaped platform of mass $M = 1.0 \times 10^2$ kg and radius $R = 2.0$ m is rotating about a frictionless vertical axis through its center with angular speed $\omega_1 = 2.0$ rad/s. A student of mass $m = 50$ kg standing at the outer edge of the platform walks on the rotating platform to a point at a distance $r = 1.0$ m from the axis of rotation. Treat the student as a point particle.

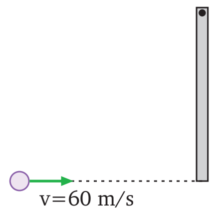
a. What is the angular speed of the system in rad/s when the student reaches the final position?

b. By how much does the rotational kinetic energy of the system change during this process?

EXAMPLE 10: A uniform rod of mass $M = 3.0$ kg and length $L = 2.0$ m is free to rotate about a frictionless vertical axis passing through one end. The rod is initially at rest. A piece of putty of mass $m = 0.50$ kg is thrown in a horizontal plane with a speed $v = 60$ m/s in a direction perpendicular to the rod. The putty moves on the frictionless horizontal plane and strikes and sticks to the free end of the rod.

a. What is the common angular speed of the system immediately after the collision in rad/s?

b. Is kinetic energy conserved in this collision?



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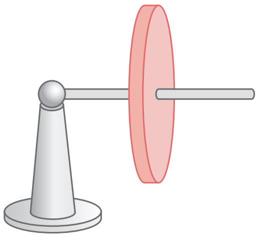
Gyroscopes and Precession

Let us examine the operating principle of gyroscopes and explain the phenomenon of precession.



EXAMPLE 11: A solid uniform disk (gyroscope wheel) of radius $R = 40$ cm and mass $M = 2.0$ kg is attached to one end of a horizontal axle passing through its center. The disk rotates about the axle with a constant angular speed $\omega = 25$ rad/s. The other end of the axle is placed on a support point located a distance $r = 20$ cm from the disk's center of mass. When the system is released, it undergoes precessional motion in the horizontal plane due to the gravitational torque. Take the magnitude of the gravitational acceleration as $g = 10\ \mathrm{m/s^2}$.

a. What is the magnitude of the angular momentum of the disk about its own axis in $\mathrm{kg \cdot m^2/s}$?

b. What is the precessional angular speed Ω of the system in rad/s?

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Exercises and Problems

1. A uniform thin rod of mass $M = 6.0$ kg and length $L = 2.0$ m is in horizontal equilibrium on a frictionless horizontal axle passing through its midpoint. Pieces of putty of negligible size and mass $m = 2.0$ kg are attached to both ends of the rod. Suddenly, the piece of putty at the right end detaches and falls, while the piece of putty at the left end remains attached to the rod. When the system is released, it begins to rotate due to the weight of the mass on the left side. Take the magnitude of the gravitational acceleration as $g = 10\ \mathrm{m/s^2}$.

a. What is the magnitude of the angular acceleration of the rod immediately after the mass on the right falls, in $\mathrm{rad/s^2}$?

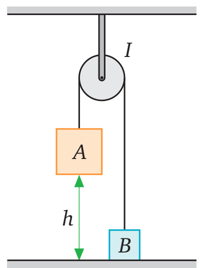
b. What is the angular speed of the rod when it reaches the vertical position, in rad/s?

c. How does the angular acceleration of the rod change as it moves from the horizontal position to the vertical position? Explain.

2. In the system shown in the figure, blocks of masses $m_A = 4.0$ kg and $m_B = 2.0$ kg are connected to each other by a light cord passing over a uniform solid cylindrical pulley of mass $M = 4.0$ kg and radius $R = 20$ cm. The cord does not slip on the pulley. The system is released from rest when block A is at a height $h = 5.0$ m above the ground. Take the magnitude of the gravitational acceleration as $g = 10\ \mathrm{m/s^2}$. Assuming that the pulley axle is frictionless:

a. What is the magnitude of the acceleration of block A?

b. What is the angular speed of the pulley at the instant block A hits the ground?

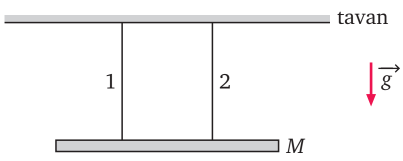


3. A uniform rod of mass $M = 4.0$ kg and length $L = 3.0$ m is attached to the ceiling by two vertical light cords fastened at points a distance $d = 1.0$ m inward from each end, and the rod is aligned horizontally. Suddenly, cord 2 breaks. Take the magnitude of the gravitational acceleration as $g = 10\ \mathrm{m/s^2}$.

a. What is the magnitude of the angular acceleration of the rod immediately after the cord breaks, in $\mathrm{rad/s^2}$?

b. What is the magnitude of the linear acceleration of the center of mass of the rod immediately after the cord breaks, in $\mathrm{m/s^2}$?

c. What is the magnitude of the tension in cord 1 immediately after the cord breaks, in N?



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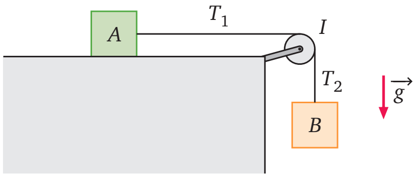
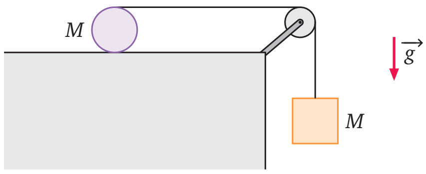
4. Block A of mass $m_A = 4.0$ kg rests on a table for which the coefficient of kinetic friction with the horizontal surface is $\mu_k = 0.25$. It is connected by a cord passing over a pulley of radius $R = 10$ cm to block B of mass $m_B = 6.0$ kg. The moment of inertia of the pulley about its axis of rotation is given as $I = 0.025\ \mathrm{kg \cdot m^2}$. When the system is released, block B starts moving downward. Take the magnitude of the gravitational acceleration as $g = 10\ \mathrm{m/s^2}$. Assuming that the cord does not slip on the pulley and that the pulley axle is frictionless, what are the cord tensions T_1 and T_2?



5. A cord is wrapped around a uniform solid disk of mass M and radius R that rests on a horizontal surface. The cord passes over a massless, frictionless pulley and is attached to a block of mass M. When the system is released, the block moves downward while the disk rolls without slipping on the horizontal surface. The magnitude of the gravitational acceleration is g.

a. Find the magnitude a of the acceleration of the block in terms of the given quantities.

b. Find the tension T in the cord.



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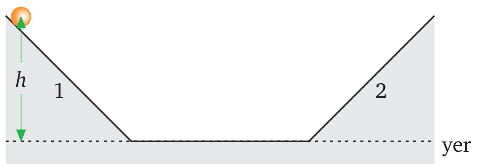
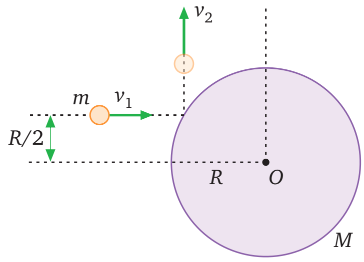
6. A uniform solid sphere of mass M and radius R is released from rest from the top of a ramp at a point a height h above the ground. In the vertical cross section of the ramp shown in the figure, section 1 is rough and section 2 is frictionless. Mechanical energy is conserved throughout the motion of the sphere. The magnitude of the gravitational acceleration is g. What is the maximum height the sphere can reach on ramp section 2?



7. A uniform disk of mass M and radius R is free to rotate about a frictionless vertical axis through point O. The disk is initially at rest. A particle of mass m approaches the disk with speed v_1 along a line a distance $R/2$ from the center of the disk, as shown in the figure. After striking the disk, the particle rebounds in a direction perpendicular to its incident direction with speed $v_2 = \frac{\sqrt{3}}{5}v_1$.

a. Find the angular speed ω of the disk immediately after the collision in terms of the given quantities (m, M, R, v_1).

b. If the particle had stuck to the disk, what would the common angular speed ω_{common} of the system be?



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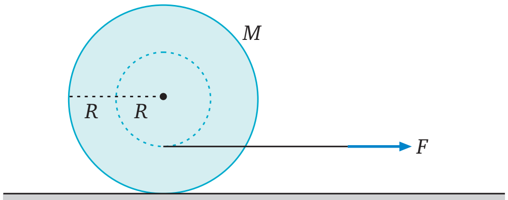
8. A uniform spool of mass $M = 10$ kg, outer radius $2R = 0.40$ m, and inner radius $R = 0.20$ m rests on a horizontal surface. A cord wrapped around the inner cylinder of the spool is pulled by a constant horizontal force of magnitude $F = 12$ N to the right, tangent to the spool at its lowest point. The friction between the spool and the surface is sufficiently large, and the spool rolls without slipping. Model the spool as a cylinder whose moment of inertia about the axis through its center of mass is $I_{cm} = \frac{1}{2}M(2R)^2$.

a. What are the magnitude and direction of the static friction force exerted by the surface on the spool?

b. What is the magnitude of the acceleration of the center of mass of the spool in m/s^2?

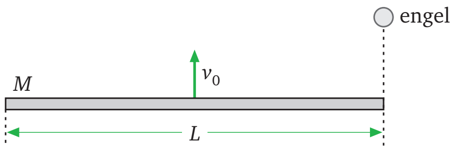


9. A uniform thin rod of mass M and length L moving without rotating on a frictionless horizontal surface is shown in the figure. The center of mass of the rod is moving with a speed of magnitude v_0. The right end of the rod strikes a fixed obstacle. The collision lasts for a very short time, and immediately after the collision, the direction of the velocity of the center of mass of the rod does not change, but its magnitude is reduced by half.

a. What is the angular speed ω of the rod about its center of mass immediately after the collision?

b. What is the change ΔK in the total kinetic energy of the system during the collision?

c. What are the instantaneous linear speeds v_{left} and v_{right} of the left and right ends of the rod, respectively, relative to the ground immediately after the collision?



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Answers

Example questions

1	$80\ \mathrm{N\cdot m}$
2	a. Go to the video b. $+\hat{z}$, ⊙ c. $\vec{\tau} = 24\hat{k}\ \mathrm{N\cdot m}$
3	a. $\tau = 5.0\ \mathrm{N\cdot m}$ b. $\alpha = 10\ \mathrm{rad/s^2}$ c. $a = 5.0\ \mathrm{m/s^2}$
4	a. $a = 5.0\ \mathrm{m/s^2}$ b. $\alpha = 50\ \mathrm{rad/s^2}$ c. $v = 5.0\ \mathrm{m/s}$
5	$v_{\mathrm{cm}} = \sqrt{\frac{4gh}{3}}$
6	a. $W = 100\ \mathrm{J}$ b. $\omega = 20\ \mathrm{rad/s}$ c. $P = 100\ \mathrm{W}$
7	a. $L_z(t) = (6.0\ \mathrm{kg\cdot m^2/s^3})t^2$, $L_z(4.0) = 96\ \mathrm{kg\cdot m^2/s}$ b. $\tau_z(t) = (12\ \mathrm{N\cdot m/s})t$, $\tau_z(4.0) = 48\ \mathrm{N\cdot m}$
8	a. $120\ \mathrm{kg\cdot m^2/s}$, ⊗ b. $160\ \mathrm{kg\cdot m^2/s^2}$, ⊙
9	a. $\omega_2 = 3.2\ \mathrm{rad/s}$ b. $\Delta K = 4.8 \times 10^2\ \mathrm{J}$
10	a. $\omega = 10\ \mathrm{rad/s}$ b. It is not conserved.
11	a. $L = 4.0\ \mathrm{kg\cdot m^2/s}$ b. $\Omega = 1.0\ \mathrm{rad/s}$

Your turn questions

1	a. It is directed uphill. b. $a = 3.6\ \mathrm{m/s^2}$ c. $\mu_s = 0.30$ d. $h_{\max} = 3.0\ \mathrm{m}$

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Exercises and problems

1	a. $\alpha = 5.0\ \mathrm{rad/s^2}$ b. $\omega = 3.2\ \mathrm{rad/s}$ c. Decreases
2	a. $a = 2.5\ \mathrm{m/s^2}$ b. $\omega = 25\ \mathrm{rad/s}$
3	a. $\alpha = 5.0\ \mathrm{rad/s^2}$ b. $a_{\mathrm{cm}} = 2.5\ \mathrm{m/s^2}$ c. $T_1 = 30\ \mathrm{N}$
4	$T_1 = 26\ \mathrm{N}, \quad T_2 = 36\ \mathrm{N}$
5	a. $a_{\mathrm{block}} = \frac{8}{11}g$ b. $T = \frac{3}{11}Mg$
6	$h_{\mathrm{final}} = \frac{5}{7}h$
7	a. $\omega = \frac{2mv_1}{5MR}$ b. $\omega_{\mathrm{common}} = \frac{mv_1}{R(M+2m)}$
8	a. $f_f = 8.0\ \mathrm{N}$ (to the left) b. $a = 0.40\ \mathrm{m/s^2}$
9	a. $\omega = \frac{3v_0}{L}$ b. $\Delta K = 0$ c. $v_{\mathrm{left}} = 2v_0, \quad v_{\mathrm{right}} = v_0$

CHAPTER 11

GRAVITATION

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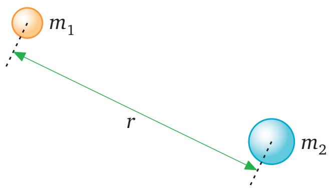
Newton's Law of Universal Gravitation

Let us explain Newton's law of universal gravitation. Let us express the law mathematically.



Let us discuss how the gravitational effects of spherically symmetric objects differ from those of objects without spherical symmetry.

Şekil I

Şekil II

Let us discuss the importance of the gravitational force in the universe.

Your turn

1. The mass of one of the small spheres in a Cavendish balance is $m_1 = 0.010$ kg, and the mass of the nearest large sphere is $m_2 = 0.50$ kg. The distance between the centers of the spheres is measured to be $r = 0.050$ m. Take the universal gravitational constant to be $G = 6.7 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.

a. What is the magnitude F_g of the gravitational force that the spheres exert on each other, in newtons?

b. What is the magnitude a_1 of the instantaneous acceleration of the small sphere (m_1) toward the large sphere due to the gravitational force, in $\mathrm{m/s^2}$?

c. What is the magnitude a_2 of the instantaneous acceleration of the large sphere (m_2) toward the small sphere due to the gravitational force, in $\mathrm{m/s^2}$?

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Weight

Let us explain the concept of weight and derive the free-fall acceleration at Earth's surface.

Your turn

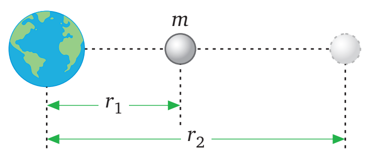
2. A geological sample whose weight on Earth's surface is $W_D = 588$ N is taken to a newly discovered planet X, which has mass $M_X = 3.00 \times 10^{24}$ kg and radius $R_X = 4.00 \times 10^6$ m. Take the magnitude of the free-fall acceleration at Earth's surface to be $g = 9.80\ \mathrm{m/s^2}$ and the universal gravitational constant to be $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.

a. What is the magnitude g_X of the free-fall acceleration at the surface of planet X, in $\mathrm{m/s^2}$?

b. What is the magnitude W_X of the weight of this sample at the surface of planet X, in newtons?

Gravitational Potential Energy

Let us calculate the work done by the gravitational force when an object of mass m is moved from a point at a distance r_1 from the center of Earth to a point at a distance r_2. Then, using this work, let us define the gravitational potential energy of the object.



Let us examine the relationship between gravitational potential energy and the gravitational force.

Let us show that the work done by the gravitational force on an object moving around a planet is independent of the path.

NOTE: The work done by conservative forces is independent of the path.

EXAMPLE 1: A probe is launched vertically upward from the surface of an airless planet of mass $M = 5.0 \times 10^{24}$ kg and radius $R = 6.0 \times 10^6$ m. Neglect the effect of the planet's rotation and the gravitational attraction of other celestial bodies. Take the universal gravitational constant to be $G = 6.7 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.

a. What must be the minimum initial launch speed v_1 of the probe so that it can reach a height $h = R$ (one radius) above the planet's surface, in m/s?

b. What must be the minimum initial launch speed v_{esc} required for the probe not to return to the planet, in m/s?

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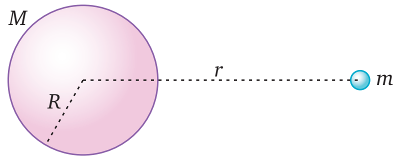
Let us discuss the concept of escape speed and derive the escape speed for an object launched from Earth's surface.

Let us discuss the potential energy expression $U = mgy$, which we previously obtained for points near Earth's surface, in terms of the results we have now obtained.

Let us explain what the **superposition principle** means for gravitation.

Spherical Mass Distributions

We had accepted without proof that the gravitational force between two spherical mass distributions can be calculated as if all their masses were concentrated at their centers. Now let us prove this.



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Let us show that the gravitational force exerted by a uniform spherical shell on a mass located inside it is zero.

Let us express, in its most general form, the gravitational force exerted on a point object by an object with a spherically symmetric mass distribution.

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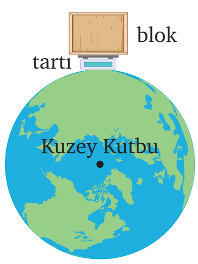
Assuming that Earth has a uniform mass distribution, let us derive the gravitational force that Earth exerts on a point mass located inside it.

Gravitation at Earth's Surface

Let us discuss why the magnitude of an object's weight is not the same everywhere on Earth's surface.

- Earth's mass is not distributed uniformly.
- Earth is not a sphere.
- Earth is rotating.



Satellite Motion

Let us examine the orbits of satellites revolving around Earth.

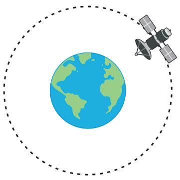
Let us examine a satellite revolving around Earth in a circular orbit.

a. Let us derive the satellite's orbital speed.

b. Let us derive the satellite's orbital period.

c. Let us examine the relationship between the period and the radius of the circular orbit.

d. Let us derive the satellite's kinetic, potential, and mechanical energy.



EXAMPLE 2: The International Space Station (ISS) revolves in a circular orbit at a height $h = 420$ km above Earth's surface. Take Earth's mass to be $M_D = 5.97 \times 10^{24}$ kg, its radius to be $R_D = 6.37 \times 10^6$ m, and the universal gravitational constant to be $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.

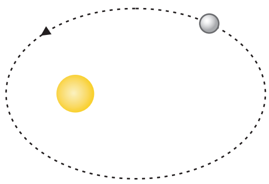
a. What is the station's orbital speed v in m/s?

b. How many minutes does it take the station to complete one revolution around Earth?

Kepler's Laws and the Motion of Planets

Let us examine Kepler's three laws separately.

First Law: Each planet moves in an elliptical orbit with the Sun at one focus.



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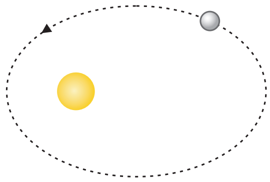
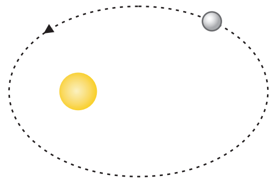
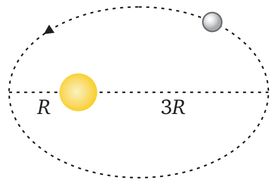
Second Law: A line drawn from the Sun to any planet sweeps out equal areas in equal time intervals.



Third Law: The square of the orbital period of any planet orbiting the Sun is proportional to the cube of the semimajor axis length of its orbit.



Let us analyze the motion of a planet at some special points in its elliptical orbit.



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EXAMPLE 3: Halley's Comet moves around the Sun in a highly elongated elliptical orbit. The comet's closest and farthest distances from the Sun along its orbit are given as $r_p = 8.8 \times 10^7$ km and $r_a = 5.3 \times 10^9$ km, respectively. In addition, the mass of the Sun is given as $M = 2.0 \times 10^{30}$ kg, and the universal gravitational constant is given as $G = 6.7 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.

a. Calculate the semimajor axis length a of the orbit.

b. Calculate the eccentricity e of the orbit.

c. Calculate the period T of the comet in years.

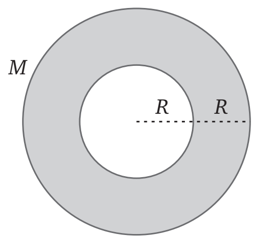
Exercises and Problems

1. A uniform (constant-density) spherical shell with inner radius R, outer radius $2R$, and total mass M is located in space. A point object of mass m is placed at various points a distance r from the center of the sphere. For each of the following regions, find the magnitude of the gravitational force exerted on the object in terms of the given quantities (M, m, R, r) and the universal gravitational constant G.

a. $r < R$ (Region in the cavity)

b. $R < r < 2R$ (Region inside the shell)

c. $r > 2R$ (Region outside the shell)



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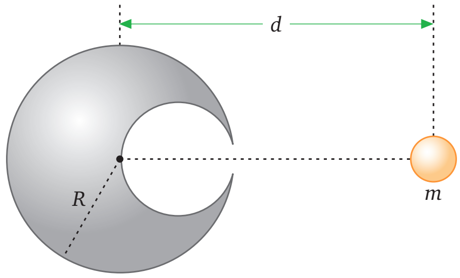
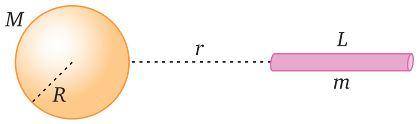
2. A spherical piece whose diameter is equal to the radius of a uniform solid sphere of mass M and radius R is removed from the sphere as shown in the figure. The center of the removed piece, the center of the original sphere, and the point mass m lie on the same axis. What is the magnitude of the gravitational force exerted on the point mass m, which is a distance d from the center of the sphere?



3. A uniform sphere of mass M and radius R and a uniform thin rod of length L and mass m are placed as shown in the figure. The distance between the objects is r.

a. Using integration, write the gravitational potential energy of the system as a function of r.

b. Using the potential energy function obtained, find the magnitude of the gravitational force exerted on the object of mass m.



4. A communications satellite is placed in a circular orbit in Earth's equatorial plane such that its orbital period is equal to the period of Earth's rotation about its own axis ($T = 24$ h). Take the mass of Earth to be $M_D = 5.97 \times 10^{24}$ kg and the universal gravitational constant to be $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.

a. What is the orbital radius r of the satellite (its distance from the center of Earth) in meters?

b. What is the linear speed v of the satellite in its orbit, in m/s?

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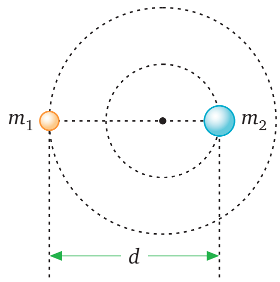
5. Two spherically symmetric planets with no atmospheres have the same average density, but the radius of planet B is twice the radius of planet A. A small satellite of mass m_A orbits just above the surface of planet A in a circular orbit with speed v_A. A small satellite of mass m_B orbits just above the surface of planet B in a circular orbit with speed v_B. What is the relationship between the orbital speeds of these two satellites?

6. A communications satellite of mass m, initially at rest on the surface of the Earth of mass M_D and radius R_D, is to be placed in a circular orbit at a height d above Earth's surface. What is the minimum work required to put the satellite into this orbit in terms of the given quantities? The rotation of Earth about its own axis and frictional forces in the atmosphere will be neglected.

7. Two stars with masses $m_1 = m$ and $m_2 = 2m$, respectively, revolve in circular orbits about their common center of mass under the influence of their mutual gravitational attraction, with the distance between them equal to d. The stars will be treated as point masses. The system is isolated from other celestial bodies.

a. Find the orbital period of the stars in terms of the given quantities.

b. Find the magnitude of the total angular momentum of the system about its center of mass in terms of the given quantities.

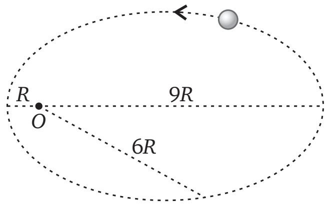


8. The orbit of a planet of mass m revolving in an elliptical orbit around a black hole of mass M at point O is shown in the figure. The planet's closest distance to the black hole (periapsis) is R, and its farthest distance (apoapsis) is $9R$. Assume that the system is isolated from other celestial bodies. The universal gravitational constant is G.

a. In units of R, what is the semimajor axis a of the orbit?

b. What is the ratio of the planet's maximum speed to its minimum speed in the orbit, (v_{max}/v_{min})?

c. Find the planet's linear speed at the instant when its distance from the black hole is $r = 6R$, in terms of G, M, and R.



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Answers

Example questions

1	a. $v_1 = 7.47 \times 10^3$ m/s b. $v_{esc} = 1.06 \times 10^4$ m/s
2	a. $v = 7.66 \times 10^3$ m/s b. $T = 93$ min
3	a. $a = 2.7 \times 10^9$ km b. $e = 0.97$ c. $T = 76$ yr

Your turn questions

1	a. $F_g = 1.3 \times 10^{-10}$ N b. $a_1 = 1.3 \times 10^{-8}$ m/s^2 c. $a_2 = 2.7 \times 10^{-10}$ m/s^2
2	a. $g_X = 12.5$ m/s^2 b. $W_X = 750$ N

Exercises and problems

1	a. $F = 0$ b. $F = \frac{GMm(r^3 - R^3)}{7r^2R^3}$ c. $F = \frac{GMm}{r^2}$
2	$F = GMm\left(\frac{1}{d^2} - \frac{1}{8(d-R/2)^2}\right)$
3	a. $U(r) = -\frac{GMm}{L}\ln\left(\frac{R+r+L}{R+r}\right)$ b. $F = -\frac{GMm}{(R+r)(R+r+L)}$
4	a. $r = 4.2 \times 10^7$ m b. $v = 3.05 \times 10^3$ m/s
5	$v_B = 2v_A$
6	$W = GM_Dm\left(\frac{1}{R_D} - \frac{1}{2(R_D+d)}\right)$
7	a. $T = 2\pi\frac{d^{3/2}}{\sqrt{3Gm}}$ b. $L_{system} = 2m\sqrt{\frac{Gmd}{3}}$
8	a. $a = 5R$ b. $\frac{v_{max}}{v_{min}} = 9$ c. $v = \sqrt{\frac{2GM}{15R}}$

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CHAPTER 12

PERIODIC MOTION

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Oscillatory Motion

Let us examine oscillatory motion. Let us define the concepts of *amplitude*, *period*, *frequency*, and *angular frequency*. Let us write the mathematical relationship among them.

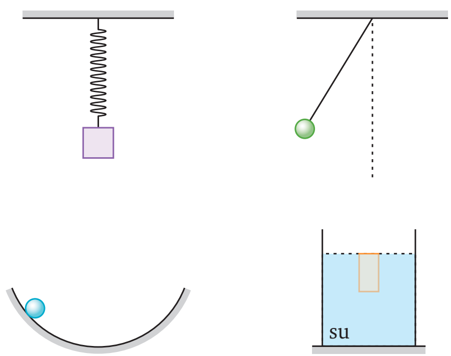
EXAMPLE 1: The ultrasonic transmitter used in an automobile parking sensor vibrates by emitting sound waves at a frequency of $f = 4.0 \times 10^4$ Hz to detect obstacles in its surroundings.

a. How long does each vibration of the transmitter last (what is the period of the motion)?

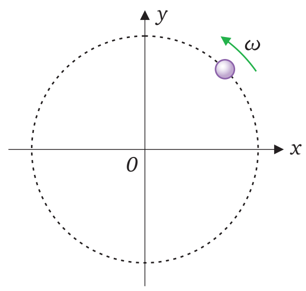
b. What is the angular frequency (ω) of this vibration?

Simple Harmonic Motion

Let us define simple harmonic motion. Let us obtain the acceleration as a function of position for this motion.



Let us show the relationship between simple harmonic motion and uniform circular motion. Let us express the period and frequency for simple harmonic motion mathematically.



NOTE: In simple harmonic motion, the period is independent of the amplitude.

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EXAMPLE 2: In an experiment performed on a frictionless horizontal air table to determine the force constant of a spring, it is found that a force of magnitude 15 N applied to the spring stretches it by 0.050 m. In the next part of the experiment, a block of mass 0.30 kg is attached to the end of this spring. When the system is at its equilibrium position, the block is pulled 0.12 m to the right and released from rest, causing it to undergo simple harmonic motion.

a. Calculate the force constant (k) of the spring.

b. Find the angular frequency (ω), frequency (f), and period (T) of the system.

Let us write the equation for position as a function of time for simple harmonic motion. Let us draw the graph of the equation we obtained.

Using the equation for position as a function of time, let us obtain the velocity and acceleration.

Let us examine how to determine the phase angle and amplitude for known initial conditions.

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EXAMPLE 3: On a frictionless horizontal surface, a block of mass $m = 0.50$ kg attached to the end of an ideal spring with spring constant $k = 50$ N/m undergoes simple harmonic motion. At $t = 0$, the block is located a distance $x_0 = 0.060$ m from the equilibrium point and is moving with velocity $v_{0x} = -0.80$ m/s (toward the equilibrium point).

a. Find the period (T), angular frequency (ω), and amplitude (A) of the motion.

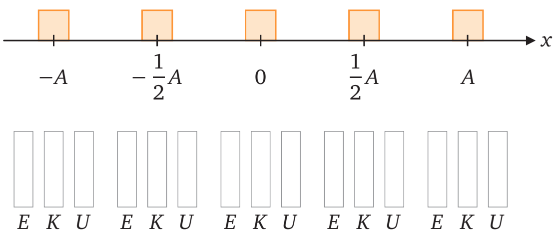
b. Write the block's position as a function of time in the form $x(t) = A\cos(\omega t + \phi)$ by substituting the numerical values.

Energy in Simple Harmonic Motion

Let us examine the conservation of energy for simple harmonic motion. Let us obtain the total mechanical energy of the system.

Using the conservation of mechanical energy in simple harmonic motion, let us obtain the speed of the object at a particular position.

Let us complete the energy graphs of the moving object at different points.



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EXAMPLE 4: A block of mass 2.0 kg attached to an ideal spring with spring constant 8.0×10^2 N/m on a frictionless horizontal surface undergoes simple harmonic motion. The amplitude of the motion is measured to be 0.20 m.

a. Find the magnitudes of the maximum speed (v_{max}) and maximum acceleration (a_{max}) of the block.

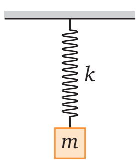
b. Find the speed (v) and the magnitude of the acceleration (a) when the block is 0.10 m from the equilibrium position.

c. When the block is at this position ($x = 0.10$ m), find the total mechanical energy (E), potential energy (U), and kinetic energy (K) of the system.

Applications of Simple Harmonic Motion

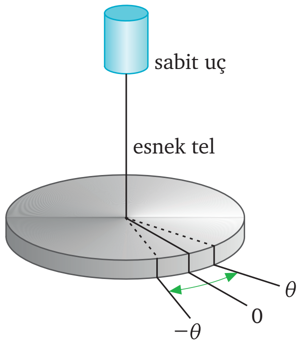
Vertical spring oscillator

Let us examine the properties of a spring oscillator moving along a vertical axis.



Angular simple harmonic motion

Let us examine a system that undergoes angular simple harmonic motion and its properties.



EXAMPLE 5: A thin, uniform metal disk of mass 4.0 kg and radius 0.10 m is suspended from the end of a long wire passing through its center. When the disk is twisted slightly about the axis of the wire and released, it undergoes simple harmonic motion with a period of 0.50 s.

a. Calculate the moment of inertia of the disk about the axis of rotation.

b. Find the torsion constant of the wire.

Molecular Vibrations

As an example, let us examine the van der Waals interaction between atoms. The potential energy function for this interaction has been obtained empirically as

$$U(r) = U_0\left[\left(\frac{R_0}{r}\right)^{12} - 2\left(\frac{R_0}{r}\right)^{6}\right]$$

.

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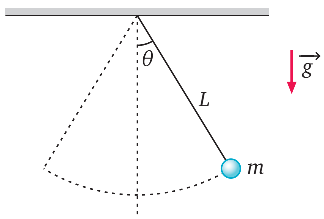
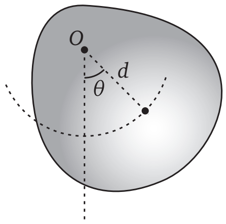
The Simple Pendulum

Let us examine the simple pendulum and discuss under what conditions it undergoes simple harmonic motion. Let us derive the period of the simple pendulum for simple harmonic motion.



The Physical Pendulum

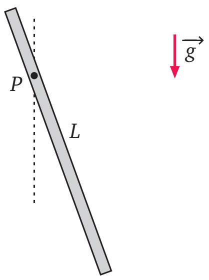
Let us examine the simple harmonic motion of the physical pendulum.



EXAMPLE 6: A uniform, thin rod of mass M and length L can rotate freely in a vertical plane about a frictionless pivot passing through point P, located a distance $L/4$ from one end. The rod is displaced through a small angle from its equilibrium position and released, so that it undergoes simple harmonic motion. The magnitude of the gravitational acceleration is g.

a. Find the moment of inertia of the rod about the given axis of rotation in terms of the given quantities.

b. Find the period of the oscillatory motion for small angular displacements in terms of the given quantities.



Damped Oscillations

Let us examine damped oscillatory motion in which the damping force is proportional to the velocity.

Let us explain the concepts of *critical damping*, *overdamping*, and *underdamping*.

Let us examine how the mechanical energy changes for damped oscillation.

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EXAMPLE 7: In a laboratory experiment, a block of mass $m = 0.50$ kg is attached to a spring with force constant $k = 50$ N/m. The block oscillates in a liquid, and the damping constant is measured to be $b = 0.10$ kg/s.

a. What is the period of this motion?

b. How long does it take for the amplitude of oscillation to decrease to half its initial value?

c. How long does it take for the mechanical energy to decrease to half its initial value?

Forced Oscillations and Resonance

Let us discuss what happens when a periodic force acts on a system undergoing oscillatory motion. Let us explain the resonance condition for such an oscillation.

EXAMPLE 8: A periodic external force of the form $F(t) = F_m \cos(\omega_d t)$ is applied to a damped spring–mass system with spring constant k, mass m, and damping constant b. For this forced oscillation, the expression for the amplitude (A) is given by

$$A = \frac{F_m}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$

Here, F_m is the constant amplitude of the external oscillating force acting on the object. ω_d is the driving angular frequency.

At resonance (assuming that the damping is small and the driving frequency is equal to the natural frequency of the system),

a. What is the amplitude (A) of the oscillating object?

b. What is the velocity amplitude (v_m) of the oscillating object?

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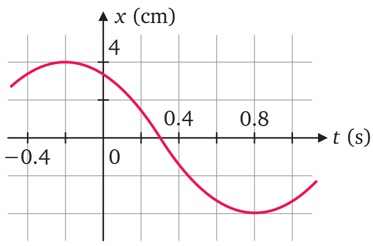
Exercises and Problems

1. The graph of position as a function of time for a particle undergoing simple harmonic motion is shown in the figure. For the motion, write the equation for position as a function of time in the form $x(t) = A\cos(\omega t + \phi)$, with all constants given as numerical values in SI units.

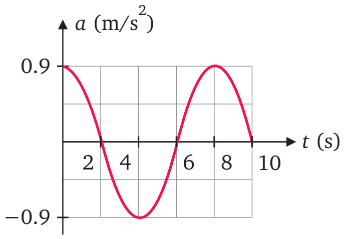


2. On a frictionless horizontal surface, an object attached to the end of an ideal spring with spring constant $k = 5.0\,\mathrm{N/m}$ undergoes simple harmonic motion. The graph of the object's acceleration as a function of time is shown in the figure.

a. Find the mass of the object.

b. Find the amplitude of the motion.

c. Find the magnitude of the maximum force exerted by the spring on the object.



3. On a frictionless horizontal surface, a 2.0 kg object attached to the end of an ideal spring undergoes simple harmonic motion. The velocity of the object is given by

$$v(t) = -(6.0\,\mathrm{m/s})\sin[(3.0\,\mathrm{rad/s})t + \pi/3]$$

.

a. Find the period of the motion.

b. Find the amplitude of the motion.

c. Find the maximum magnitude of the object's acceleration.

d. Find the force constant of the spring.

4. The amplitude of motion of an object undergoing simple harmonic motion on a frictionless horizontal surface is measured to be $A = 0.20$ m.

a. When the object is $x = 0.12$ m from the equilibrium position, what is the ratio of the system's potential energy to its total mechanical energy (U/E)?

b. When the object is at this position $(x = 0.12\,\mathrm{m})$, what is the ratio of its kinetic energy to the total mechanical energy (K/E)?

c. What is the distance from the equilibrium position at which the system's kinetic energy is equal to its potential energy?

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5. On a frictionless horizontal surface, the frequency of a 0.50 kg object undergoing simple harmonic motion at the end of an ideal spring is determined to be 2.0 Hz. When the object is $x = 0.050$ m from the equilibrium position, the magnitude of its velocity is measured to be $v = 0.40\,\mathrm{m/s}$.

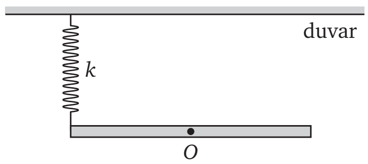
a. Find the total mechanical energy of the system.

b. Find the amplitude of the motion.

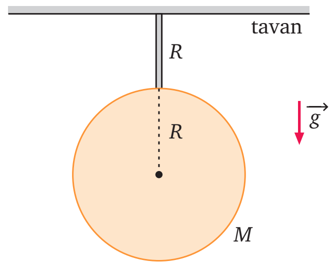
6. A uniform rod of mass M and length L is free to rotate on a frictionless horizontal surface about a vertical, frictionless axis through its center of mass (point O). A spring with spring constant k is attached horizontally between one end of the rod and a fixed wall. When the rod is in its equilibrium position, it is parallel to the wall. When the rod is rotated slightly from its equilibrium position and released, it undergoes simple harmonic motion.

a. When the rod is rotated through a very small angle θ from its equilibrium position, write the magnitude of the restoring torque acting on the system in terms of θ, k, and L.

b. Find the period of this oscillatory motion in terms of the given quantities.



7. A uniform disk of mass M and radius R is attached to the end of a massless, rigid rod of length R, as shown in the figure. The other end of the rod is suspended from the ceiling, and the system can oscillate without friction in a vertical plane about this point. The magnitude of the gravitational acceleration is g. Find the period of the oscillatory motion the system undergoes when it is displaced slightly from its equilibrium position and released, in terms of the given quantities.



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8. A 2.0 kg block suspended from a spring with spring constant $k = 1.0 \times 10^2$ N/m oscillates in a viscous liquid. The damping force exerted by the liquid on the block is proportional to the velocity ($F = -bv$), and the damping constant is given as $b = 6.0$ kg/s. The block is pulled some distance from its equilibrium position and released.

a. Calculate the undamped natural angular frequency (ω_0) and the angular frequency in the damped case (ω').

b. Find the time required for the amplitude of oscillation to decrease to one-tenth of its initial value.

Answers

Example questions

1	a. $T = 2.5 \times 10^{-5}$ s b. $\omega = 2.5 \times 10^{5}$ rad/s
2	a. $k = 3.0 \times 10^{2}$ N/m b. $\omega = 32$ rad/s, $f = 5.0$ Hz, $T = 0.20$ s
3	a. $T = 0.63$ s, $\omega = 10$ rad/s, $A = 0.10$ m b. $x(t) = (0.10\ \text{m})\cos[(10\ \text{rad/s})t + 0.93]$
4	a. $v_{\max} = 4.0$ m/s, $a_{\max} = 80$ m/s^2 b. $v = 3.5$ m/s, $a = 40$ m/s^2 c. $E = 16$ J, $U = 4.0$ J, $K = 12$ J
5	a. $I = 0.020\ \text{kg}\cdot\text{m}^2$ b. $\kappa = 3.2\ \text{N}\cdot\text{m/rad}$
6	a. $I = \frac{7}{48}ML^2$ b. $T = 2\pi\sqrt{\frac{7L}{12g}}$
7	a. $T = 0.63$ s b. $t = 6.9$ s c. $t = 3.5$ s
8	a. $A = \frac{F_m}{b\omega_d}$ b. $v_m = \frac{F_m}{b}$

Exercises and problems

1	$x(t) = (0.04\ \text{m})\cos[(\pi\ \text{rad/s})\,t + \pi/5]$
2	a. $m = 8.1$ kg b. $A = 1.5$ m c. $F_{max} = 7.3$ N
3	a. $T = 2.1$ s b. $A = 2.0$ m c. $a_{max} = 18$ m/s^2 d. $k = 18$ N/m
4	a. 0.36 b. 0.64 c. $x = 0.14$ m
5	a. $E = 0.14$ J b. $A = 0.059$ m
6	a. $\tau = \frac{kL^2}{4}\theta$ b. $T = 2\pi\sqrt{\frac{M}{3k}}$
7	$T = 3\pi\sqrt{\frac{R}{g}}$
8	a. $\omega_0 = 7.1$ rad/s, $\omega' = 6.9$ rad/s b. $t = 1.5$ s

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