

# APPENDIX

## MATHEMATICAL SUPPLEMENT FOR GENERAL PHYSICS



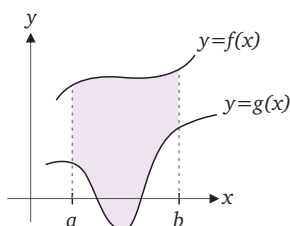
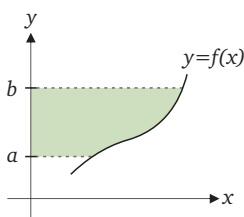
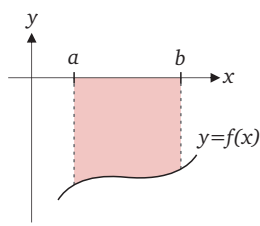
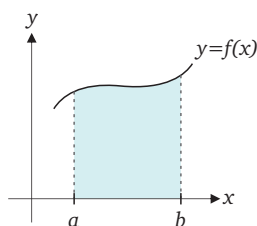
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### Calculations with Integrals

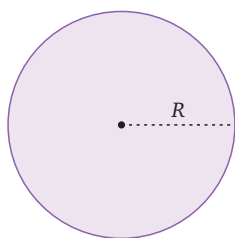
**EXAMPLE 1:** Let us find the length of the rod shown in the figure by using an integral.



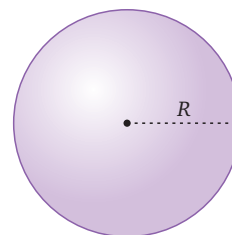
**EXAMPLE 2:** Let us write the integrals that give the areas shown in the figures.



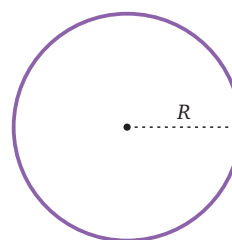
**EXAMPLE 3:** Let us find the area of a circle of radius  $R$  by integrating infinitesimal rings.



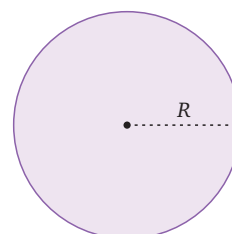
**EXAMPLE 4:** Let us find the volume of a sphere of radius  $R$  using integration.



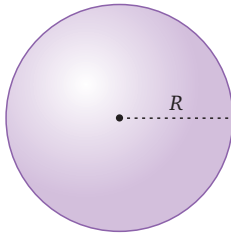
**EXAMPLE 5:** Let us find the circumference of a circle of radius  $R$  using integration.



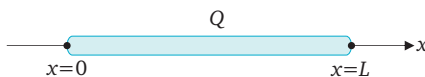
**EXAMPLE 6:** Let us find the area of a circle of radius  $R$  by integrating infinitesimal sectors.



**EXAMPLE 7:** Let us find the surface area of a sphere of radius  $R$  using integration.



**EXAMPLE 8:** The linear charge density on the rod of length  $L$  and total charge  $Q$  shown in the figure is given by  $\lambda(x) = Ax^2$ . Accordingly, let us find the constant  $A$  in terms of the given quantities.



**EXAMPLE 9:** The thickness of the disk shown is negligible. The surface mass density of this disk, which has radius  $R$  and mass  $M$ , depends on the distance from the center as  $\sigma(r) = \frac{A}{r}$ . Accordingly, let us find the constant  $A$  in terms of the given quantities.



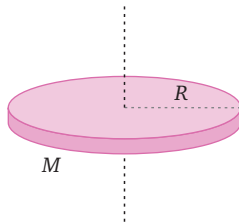
**EXAMPLE 10:** Let us find the gravitational force exerted by the uniform rod of mass  $M$  and length  $L$  shown in the figure on the object of mass  $m$ .



**EXAMPLE 11:** Let us find the moments of inertia of the uniform rod shown in the figure, which has mass  $M$  and length  $L$ , about one end and about its center.



**EXAMPLE 12:** The axis passing through the center of the uniform disk of mass  $M$  and radius  $R$  is shown in the figure. Let us find the moment of inertia of the disk about this axis.



## Answers

1  $L = \int_a^b dx = b - a$

2 a.  $A = \int_a^b f(x) dx$   
 b.  $A = -\int_a^b f(x) dx$   
 c.  $A = \int_a^b f^{-1}(y) dy$   
 d.  $A = \int_a^b [f(x) - g(x)] dx$

3  $A = \int_0^R 2\pi r dr = \pi R^2$

4  $V = \frac{4}{3} \pi R^3$

5 Circumference  $= 2\pi R$

6  $A = \int_0^R 2\pi r dr = \pi R^2$

7  $A = 4\pi R^2$

8  $A = \frac{3Q}{L^3}$

9  $A = \frac{M}{2.0\pi R}$

10  $F = \frac{GMm}{d(d+L)}$

11 a.  $I_{\text{end}} = \frac{1}{3} ML^2$   
 b.  $I_{\text{center}} = \frac{1}{12} ML^2$

12  $I = \frac{1}{2} MR^2$

# CHAPTER 1

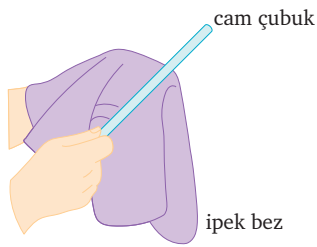
## ELECTRIC CHARGES AND ELECTRIC FIELD



### Electric Charges and Their Properties

Let's examine a few everyday examples where we can observe the existence of electric charges.

- Rubbing a glass rod with a silk cloth



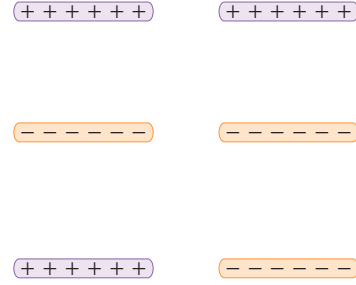
- Rubbing a plastic comb through our hair



- Experiencing a slight electric shock when touching a car door



Let's examine the interaction of rods charged with like and unlike charges.



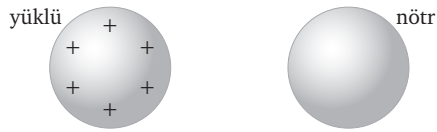
### Conservation and Quantization of Electric Charges

1. Let's state the law of conservation of charge.

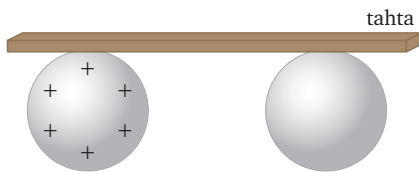
2. Let's explain the phenomenon of quantization of electric charges. Let's write the magnitude of an electron's charge (elementary charge).

## Conductors and Insulators

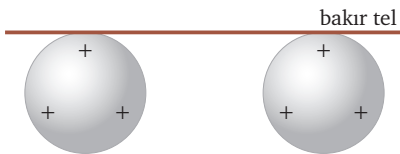
The spheres shown in the figure are conductors.



1. Let's examine the charge distribution when a piece of wood is brought into contact with both of these spheres simultaneously.

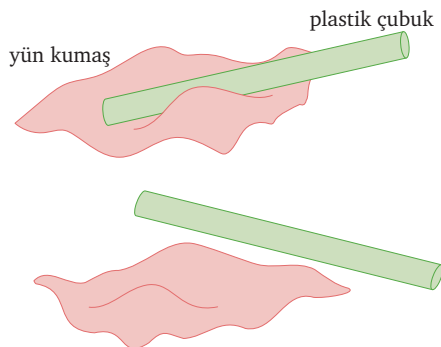


2. Let's examine the charge distribution when a copper wire is brought into contact with both of these spheres simultaneously.



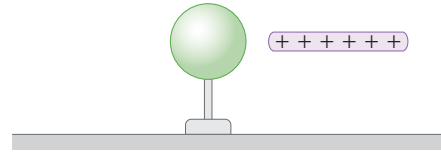
## Charging by Friction and Induction

1. Let's examine charging by friction using the following example.

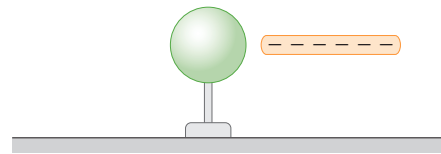


2. Let's examine charging by induction for the following three cases.

A positively charged insulating rod is brought near a conducting sphere. Let's examine the charge separation on the sphere in this case.



A negatively charged insulating rod is brought near a conducting sphere. Let us examine the charge separation on the sphere in this case.

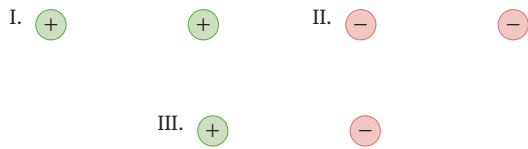


Let's discuss why pieces of paper are attracted to a comb when we bring the comb we used to brush our hair near them.

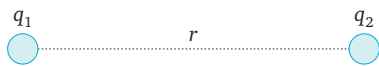


## Coulomb's Law

Let's examine the interaction between electrically charged objects for each case given in the figure.



Let's state Coulomb's law using the point charges given in the figure.

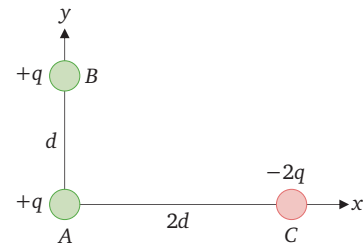


Let's examine the constants  $k$  and  $\epsilon_0$  and write down their numerical values and units.

**EXAMPLE 1:** Let's show the forces exerted by the ions on each other, whose separation and net charges are given in the figure, and find the magnitude of these forces.

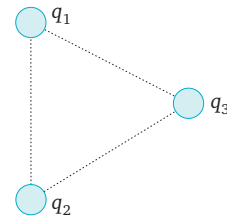


**EXAMPLE 2:** The distances between point charges A, B, and C, along with their net charges, are shown in the figure.  $q$  represents a positive quantity. Let's find the ratio of the magnitude of the electrical force exerted by object A on object B to the magnitude of the electrical force exerted by object A on object C.



## The superposition principle

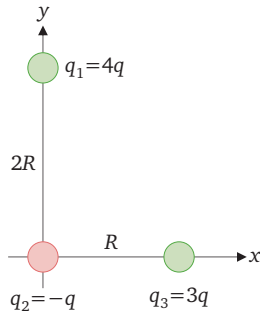
Let us explain the superposition principle for Coulomb's law.



**EXAMPLE 3:** Charged objects on the same line are shown in the figure. The magnitude of the electrical force exerted by the object with charge  $q_1$  on the object with charge  $q_3$  is  $F$ . Accordingly, let us find the net electrical force on the object with charge  $q_2$  in terms of  $F$ .



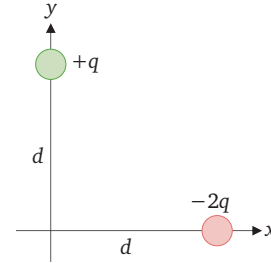
**EXAMPLE 4:** The point charges in the figure are shown in a two-dimensional standard coordinate plane. Accordingly, let us find the electrical force on the object with charge  $q_2$  using unit vectors.



### Your turn

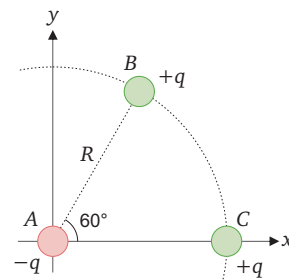
1. At a certain moment, one of two electrons exerts a force of magnitude  $0.64 \times 10^{-11}$  N on the other. Accordingly, find the distance between these two electrons. (The charge of an electron will be taken as  $1.6 \times 10^{-19}$  C.)

2. The point charges in the figure are shown in a two-dimensional standard coordinate system, where  $q$  represents a positive quantity. Find the magnitude of the electrical forces exerted by the objects on each other in terms of the given values and appropriate constants.



3. The point charges in the figure are shown in a two-dimensional standard coordinate system. Objects B and C are located on a circle of radius  $R$  centered at the origin.  $q$  represents a positive quantity, and the magnitude of the electrical force exerted by object A on object C is  $F$ . Accordingly,

- find the net electrical force acting on object A in terms of  $F$  and by using unit vectors.
- find the magnitude of the net electrical force acting on object A in terms of  $F$ .

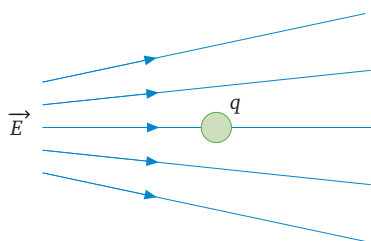


## Electric Field

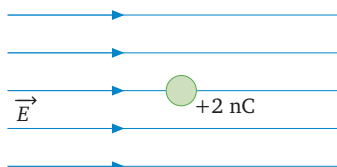
Let us discuss the physical meaning of the electric field and define the electric field.



Let us write the expression for the electric force acting on a point charge in an external electric field.

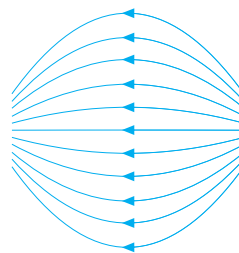


**EXAMPLE 5:** A point charge of  $+2.0 \text{ nC}$  is located within the uniform electric field shown in the figure. Given that the magnitude of the electric field is  $5 \times 10^3 \text{ N/C}$ , let us find the electrical force acting on this object.



## Electric field lines

Using the electric field lines in the figure, let us discuss what electric field lines physically represent.



Let us draw the electric field lines for the point charges in the following three different cases with equal charge magnitudes and interpret these lines physically.

- A positive point charge



- Two point charges, one positively and one negatively charged



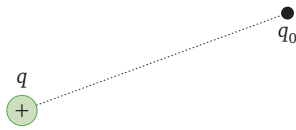
- Two negative point charges



**NOTE:** Electric field lines never cross each other.

### Electric field of a point charge

By utilizing a test charge  $q_0$  placed at any point around a positive point charge, let us obtain the magnitude of the electric field due to a point charge.

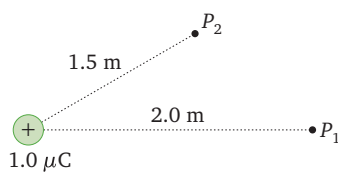


Let us define the unit vector  $\hat{r}$  used to express the direction of the electric field.

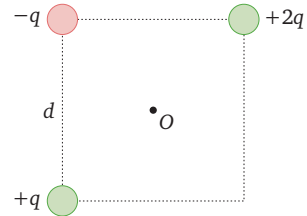
**NOTE:** The superposition principle is also valid for electric fields.

**NOTE:** There is only one net electric field at any point.

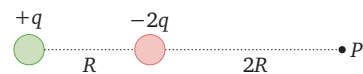
**EXAMPLE 6:** Let us show the electric fields created by the point charge, whose charge magnitude is given in the figure, at points  $P_1$  and  $P_2$  by drawing them and find the magnitudes of the electric field at these points.



**EXAMPLE 7:** The charged objects in the figure are placed at the corners of a square, and point O is the center of the square.  $q$  represents a positive quantity. Accordingly, let us find the magnitude of the net electric field at point O in terms of the given values and appropriate constants.

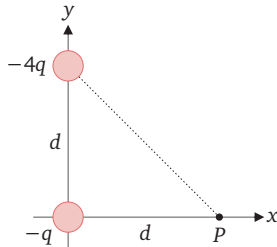


4. The point charges in the figure are on the same line, and the distance between the objects and the net charges are shown.  $q$  represents a positive quantity. Accordingly, find the magnitude of the net electric field at point  $P$  in terms of the given values and appropriate constants.

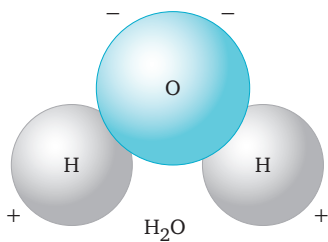


**Your turn**

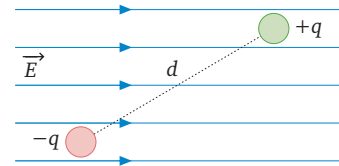
5. The point charges in the figure are shown in a two-dimensional standard Cartesian coordinate system.  $q$  represents a positive quantity. Find the net electric field at point  $P$  using the given values, appropriate physical constants, and unit vectors.

**Electric Dipoles**

Let us show what an electric dipole is by using a model of a water molecule.

**Force and torque on an electric dipole**

Let us show the forces on the point charges located in the external uniform electric field in the figure and find the magnitude of the torque on the dipole.



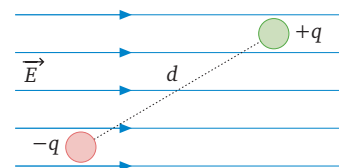
**NOTE:** A net force can act on a dipole in a non-uniform electric field.

Let us define the electric dipole moment vector.

Let us define the torque acting on a dipole located in an external uniform electric field.

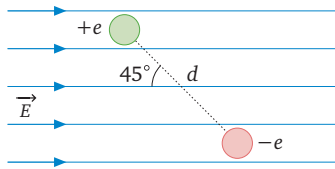
**Potential energy of an electric dipole**

Let us discuss why an electric dipole located in a uniform external electric field, as shown in the figure, will have potential energy.



- Let us define the electrical potential energy for this system.
- Let us obtain the maximum and minimum values of the electrical potential energy for this system.

**EXAMPLE 8:** An electric dipole is fixed in a uniform external electric field oriented as shown in the figure. The magnitudes of the charges are equal to the elementary charge, with  $d = 0.5 \times 10^{-8}$  m and  $E = 3 \times 10^4$  N/C. Let us find the electric dipole moment vector, the torque acting on the dipole, and the dipole's electric potential energy.

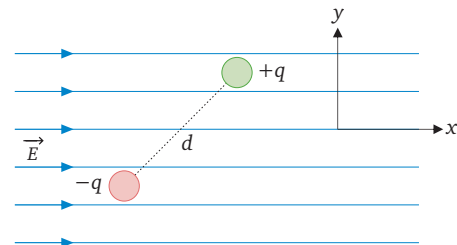


### Your turn

**6.** The electric dipole in the figure is held in a uniform electric field of magnitude  $E$ , making an angle of  $60^\circ$  with the electric field lines. Then, the electric dipole moment is oriented to be in the same direction as the electric field lines. During this process, the dipole does not gain kinetic energy. Accordingly:

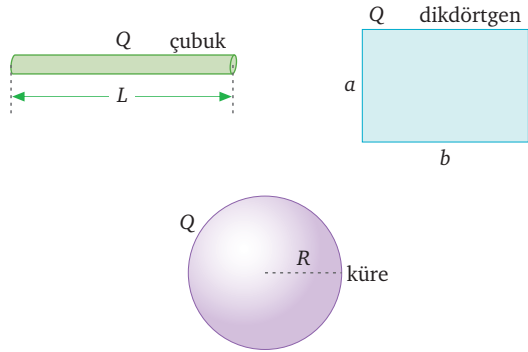
- the torque acting on the dipole in the initial case in terms of unit vectors,
- the electrical potential energy of the dipole in the initial and final cases,
- the work that must be done to bring the system from its initial state to its final state

find (a)-(c).

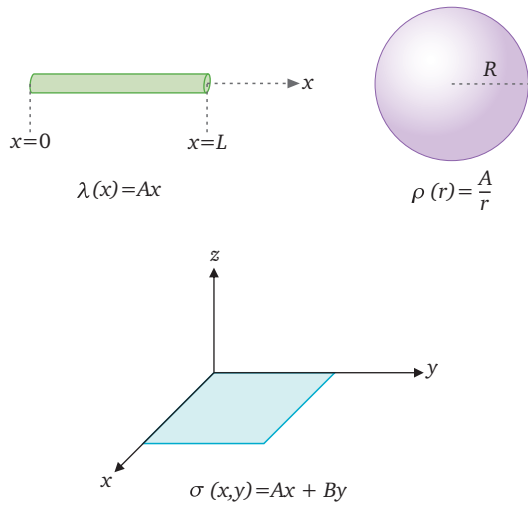


### Electric fields of continuous charge distributions

Let us examine the continuous and uniform charge distributions given below and define the concepts of linear, surface, and volume charge density for uniform charge distributions.

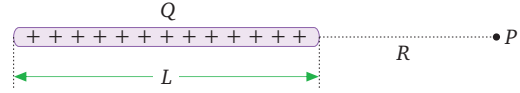


Let us examine the continuous and non-uniform charge distributions given below.

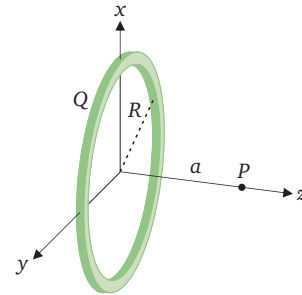


**NOTE:** In the case of a continuous charge distribution, the electric field can be obtained by integrating the contributions from infinitesimal elements.

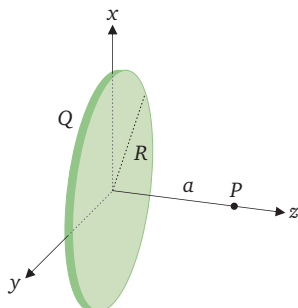
**EXAMPLE 9:** A charge  $Q$  is uniformly distributed over a rod of length  $L$  with negligible thickness, as shown in the figure. Let us find the direction and magnitude of the electric field at point  $P$ , which is at a distance  $R$  from the right end of the rod.



**EXAMPLE 10:** A charge  $Q$  is uniformly distributed over a ring of radius  $R$  with negligible thickness, as shown in the figure. The ring lies in the  $xy$  plane and is centered at the origin. Accordingly, let us find the direction and magnitude of the electric field at point  $P$ , which is at a distance  $a$  from the origin.



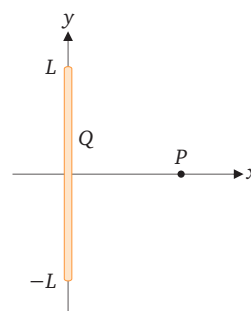
**EXAMPLE 11:** A charge  $Q$  is uniformly distributed over a disk of radius  $R$  with negligible thickness, as shown in the figure. The disk lies in the  $xy$  plane and is centered at the origin. Accordingly, let us find the direction and magnitude of the electric field at point  $P$ , which is at a distance  $a$  from the origin.



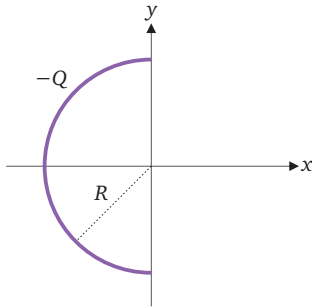
### Your turn

7. A positive charge  $Q$  is uniformly distributed over a rod of length  $2L$  with negligible thickness, as shown in the figure, and the midpoint of the rod is at the origin. The distance of point  $P$  from the origin should be taken as  $x$ . Accordingly,

- find the electric field at point  $P$  in terms of unit vectors.
- find what value the electric field at point  $P$  will approach if the value of  $x$  is much larger than  $L$ .
- find what value the electric field at point  $P$  will approach if the value of  $L$  is much larger than  $x$ . (Use  $\lambda$  for linear charge density.)

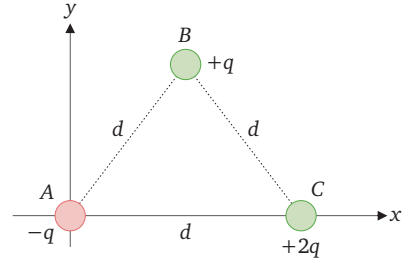


8. The center of the semi-circular object with negligible thickness shown in the figure is the origin, and a charge of  $-Q$  is uniformly distributed over the object. Accordingly, find the net electric field at the origin using unit vectors.

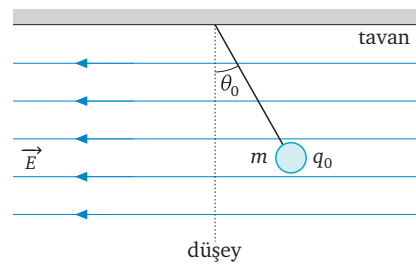


### Exercises and Problems

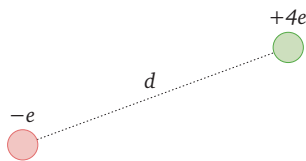
1. The charges on point charges  $A$ ,  $B$ , and  $C$  along with the distances between them are given in the figure.  $q$  represents a positive quantity. Accordingly, find the net electrical force on object  $B$  using unit vectors and calculate the magnitude of this force.



2. The charged object in the figure is suspended from the ceiling by an insulating string of negligible weight. The object is brought to equilibrium at an angle  $\theta_0$  with the vertical in a uniform electric field of magnitude  $E$ .  $q_0$  represents a positive charge quantity. Accordingly, find the sign of the object's charge and the value of  $E$  in terms of the given variables.

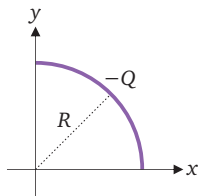


3. The point charges in the figure are  $-e$  and  $4e$ , and the distance between them is  $d$ . Accordingly, what is the distance from the point where the net electric field is zero to the object with charge  $4e$ ?

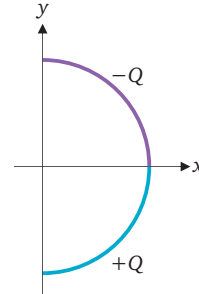


4. A charge of  $-Q$  is uniformly distributed over the quarter circle shown in the figure, and the center of the circle is the origin.  $Q$  represents a positive quantity.

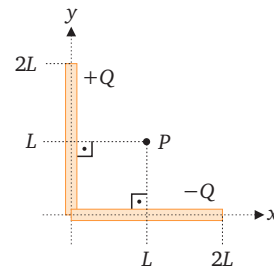
- Find the net electric field at the origin using unit vectors.
- If a point charge of  $+e$  is placed at the origin, find the electrical force acting on this object using unit vectors.



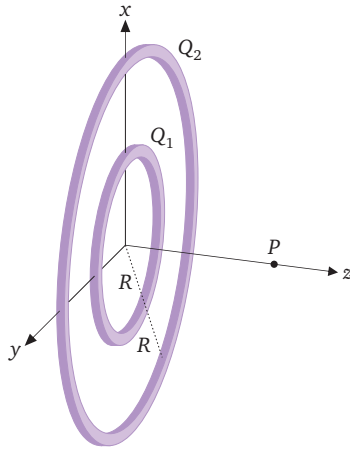
5. The semi-circle in the figure consists of two quarter-circles with charges  $+Q$  and  $-Q$ .  $Q$  represents a positive quantity and the charges are uniformly distributed over the quarter-circles. Accordingly, find the net electric field at the origin using unit vectors.



6. In the figure, charges  $+Q$  and  $-Q$  are uniformly distributed over rods of length  $2L$  with negligible thickness.  $Q$  represents a positive quantity. Accordingly, find the electric field at point P using unit vectors.

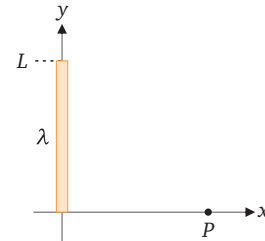


7. In the figure, circular objects of negligible thickness are in the  $xy$  plane and their centers are at the origin. Charges  $Q_1$  and  $Q_2$  are uniformly distributed over these circles. Point  $P$  can be represented as  $P = (0, 0, 2R)$ . Since the net electric field at point  $P$  is zero, obtain the relationship between  $Q_1$  and  $Q_2$ .

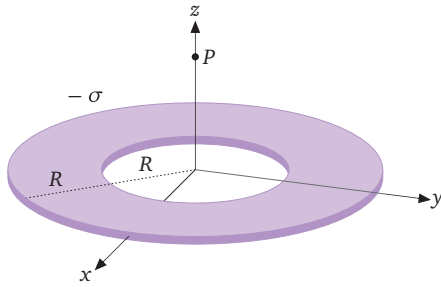


8. In the figure, positive charges are distributed on a rod of negligible thickness in the form  $\lambda(y) = \lambda_0 y$ .  $\lambda_0$  is a constant and the length of the rod is  $L$ . Given that the distance of point  $P$  from the origin is  $a$ ,

- Find the magnitude of the  $x$ -component of the electric field at point  $P$ .
- Set up an integral for the magnitude of the  $y$ -component of the electric field at point  $P$ .



9. The object in the figure was formed by removing a circular section (a hole) of radius  $R$  from the inner part of a disk of negligible thickness. The object is in the  $xy$  plane. Negative charges are uniformly distributed on the surface of the object, and the surface charge density is  $-\sigma$ .  $\sigma$  represents a positive quantity and the distance of point  $P$  from the origin is  $a$ . Accordingly, find the electric field at point  $P$  using unit vectors.



## Answers

### Example questions

1  $F = 3.2 \times 10^{-18} \text{ N}$

2  $\frac{F_{AB}}{F_{AC}} = 2.0$

3  $\vec{F}_{net} = -8.5F\hat{i}$

4  $\vec{F} = \frac{kq^2}{R^2}(3.0\hat{i} + 1.0\hat{j})$

5  $F = 1.0 \times 10^{-5} \text{ N}$

6  $E_1 = 2.3 \times 10^3 \text{ N/C}$

$E_2 = 4.0 \times 10^3 \text{ N/C}$

7  $E_{net} = \frac{2\sqrt{2}kq}{d^2}$

$\vec{p} = 0.80 \times 10^{-27} \left( -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} \right) \text{ C} \cdot \text{m}$

8  $\vec{\tau} = -1.7 \times 10^{-23} \hat{k} \text{ N} \cdot \text{m}$

$U = 1.7 \times 10^{-23} \text{ J}$

9  $\vec{E} = \frac{kQ}{R(R+L)}\hat{i}$

10  $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Qa}{(R^2+a^2)^{3/2}}\hat{k}$

11  $\vec{E} = \frac{Q}{2\pi R^2\epsilon_0} \left( 1 - \frac{a}{\sqrt{R^2+a^2}} \right) \hat{k}$

### Your turn questions

1  $6 \times 10^{-9} \text{ m}$

2  $k \frac{q^2}{d^2}$

3 a.  $\frac{3F}{2}\hat{i} + \frac{\sqrt{3}F}{2}\hat{j}$

b.  $\sqrt{3}F$

4  $\frac{7}{18}k \frac{q}{R^2}$

5  $-(1 + \sqrt{2})k \frac{q}{d^2}\hat{i} + \sqrt{2}k \frac{q}{d^2}\hat{j}$

a.  $-\frac{\sqrt{3}}{2}qdE\hat{k}$

6 b.  $U_i = -\frac{1}{2}qdE, U_f = -qdE$

c.  $W = -\frac{1}{2}qdE$

|   |                                                                           |
|---|---------------------------------------------------------------------------|
|   | a. $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{x\sqrt{x^2+L^2}} \hat{i}$ |
| 7 | b. $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{x^2}$                           |
|   | c. $E = \frac{\lambda}{2\pi\epsilon_0 x}$                                 |
| 8 | $\vec{E} = -\frac{1}{2\pi\epsilon_0} \frac{Q}{\pi R^2} \hat{i}$           |

### Exercises and problems

|   |                                                                                           |
|---|-------------------------------------------------------------------------------------------|
| 1 | $\vec{F} = \frac{kq^2}{d^2} \left(-\frac{3}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}\right)$ |
|   | $F = \frac{\sqrt{3}kq^2}{d^2}$                                                            |

|   |                                    |
|---|------------------------------------|
| 2 | Charge sign: (−)                   |
|   | $E = \frac{mg}{q_0} \tan \theta_0$ |

|   |      |
|---|------|
| 3 | $2d$ |
|---|------|

|   |                                                                    |
|---|--------------------------------------------------------------------|
| 4 | a. $\vec{E} = \frac{Q}{2\pi^2\epsilon_0 R^2} (\hat{i} + \hat{j})$  |
|   | b. $\vec{E} = \frac{Qe}{2\pi^2\epsilon_0 R^2} (\hat{i} + \hat{j})$ |

|   |                                                    |
|---|----------------------------------------------------|
| 5 | $\vec{E} = \frac{Q}{\epsilon_0 \pi^2 R^2} \hat{j}$ |
|---|----------------------------------------------------|

|   |                                                                                |
|---|--------------------------------------------------------------------------------|
| 6 | $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{\sqrt{2}L^2} (\hat{i} - \hat{j})$ |
|---|--------------------------------------------------------------------------------|

|   |                  |
|---|------------------|
| 7 | $Q_1 = -0.49Q_2$ |
|---|------------------|

|   |                                                                                                    |
|---|----------------------------------------------------------------------------------------------------|
| 8 | a. $E_x = \frac{a\lambda_0}{4\pi\epsilon_0} \left( \frac{1}{a} - \frac{1}{\sqrt{a^2+L^2}} \right)$ |
|   | b. $E_y = \frac{\lambda_0}{4\pi\epsilon_0} \int_0^L \frac{y^2}{(a^2+y^2)^{\frac{3}{2}}} dy$        |

|   |                                                                                                                       |
|---|-----------------------------------------------------------------------------------------------------------------------|
| 9 | $\vec{E} = -\frac{\sigma a}{2\epsilon_0} \left( \frac{1}{\sqrt{a^2+R^2}} - \frac{1}{\sqrt{a^2+4R^2}} \right) \hat{k}$ |
|---|-----------------------------------------------------------------------------------------------------------------------|

# CHAPTER 2

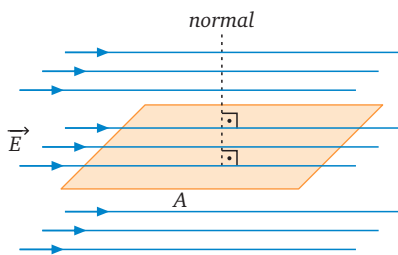
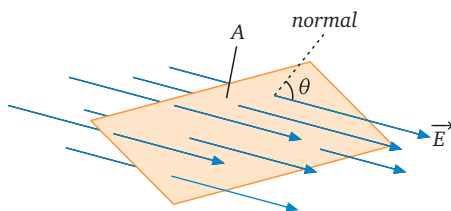
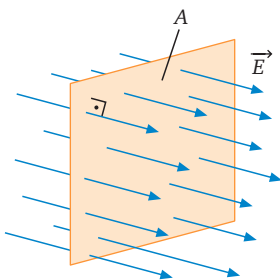
## GAUSS'S LAW



### Electric Flux

#### Electric field uniform over the surface

Let's discuss the meaning of electric flux for the uniform electric fields and planar surfaces shown in the figure.

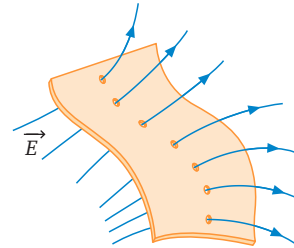


**Area vector** ( $\vec{A}$ ) Let's define it.

For the special case given above, **Electric flux** ( $\Phi$ ) let's define it as a dot product.

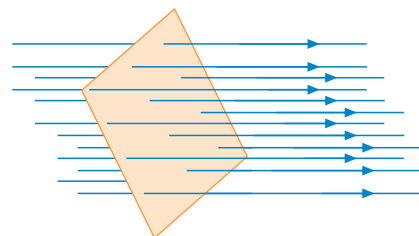
#### Electric field not uniform over the surface

Let's discuss how the electric flux can be calculated for the electric field and surface shown in the figure and write the general definition of electric flux.



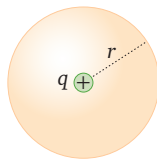
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**EXAMPLE 1:** The side lengths of the rectangle in the figure are 1 m and 2 m. The angle between the surface normal and the uniform electric field is  $30^\circ$ , and the magnitude of the electric field is 5 N/C. Determine the electric flux through the surface.



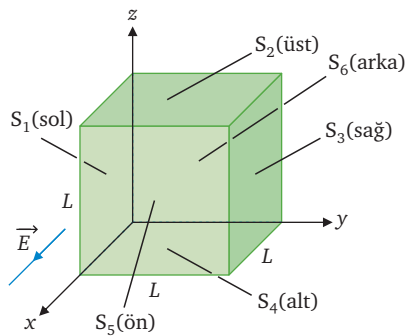
**EXAMPLE 2:** Determine the electric flux through a surface with area vector  $\vec{A} = 0.5\hat{i} \text{ m}^2$  in a region where the electric field is  $\vec{E} = (3\hat{i} - 2\hat{j}) \text{ N/C}$ .

**EXAMPLE 3:** A positively charged point charge is located at the center of the spherical surface in the figure. Determine the electric flux through this spherical surface.



### Your turn

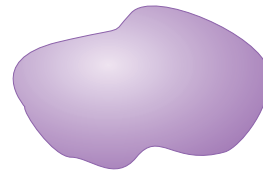
1. The length of one edge of the cube in the figure is  $L$ . The uniform electric field in the medium is in the  $+x$  direction with magnitude  $E_0$  and covers a region large enough to enclose the entire cube. Accordingly, calculate the electric flux for each surface of the cube and find the total electric flux for the cube.



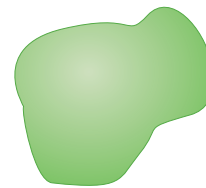
2. Find the electric flux for a surface with area vector  $\vec{A} = (\hat{i} + 2\hat{k}) \text{ m}^2$  in an environment where the electric field is  $\vec{E} = (-2\hat{i} + 2\hat{j} + 4\hat{k}) \text{ N/C}$ .

### Gauss's Law

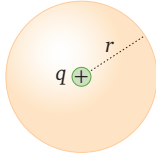
Let's discuss what the electric flux will be for a closed surface when there is no net electric charge inside it.



Let's write **Gauss's law**.

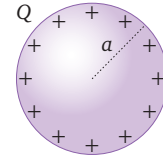


Let's apply Gauss's law to a spherical surface drawn around a positively charged point charge, and show that the result agrees with the one obtained from Coulomb's law.



**EXAMPLE 4:** Positive electric charges with a total of  $Q$  have been transferred to the solid conducting sphere of radius  $a$  shown in the figure. The distance of any point from the center is expressed by  $r$ .

- Let's find the electric field at any point in the inner region of the sphere.
- Let's find the electric field at any point in the outer region of the sphere.
- Let's plot the electric field as a function of  $r$ .



### Applications of Gauss's Law

Let's examine charge distributions in conductors and insulators in electrostatic equilibrium.



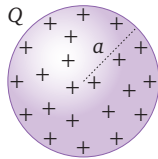
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**EXAMPLE 5:** Positive electric charges with a total of  $Q$  are uniformly distributed throughout the volume of the solid insulating sphere of radius  $a$  shown in the figure. The distance of any point from the center is expressed by  $r$ .

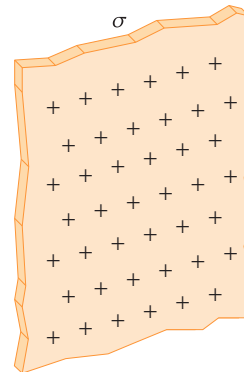
- Let's find the electric field at any point in the inner region of the sphere.
- Let's find the electric field at any point in the outer region of the sphere.
- Let's compare the results obtained in a and b for the case where  $r = a$ .
- Let's plot the electric field as a function of  $r$ .



**EXAMPLE 6:** Negative electric charges are uniformly distributed on the insulating rod of negligible thickness and physically infinite length shown in the figure. The linear charge density is  $-\lambda$ , where  $\lambda$  represents a positive quantity. Accordingly, let's find the direction and magnitude of the electric field at any point around the rod.



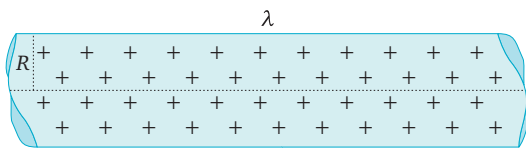
**EXAMPLE 7:** Positive electric charges are uniformly distributed on the insulating planar plate of negligible thickness and physically infinite size shown in the figure. The surface charge density on the plate is  $\sigma$ . Accordingly, let's find the direction and magnitude of the electric field at any point at a distance  $r$  from the plate.



**Your turn**

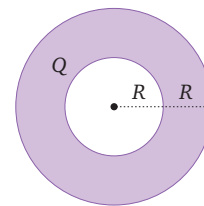
3. A solid conducting body in the form of a physically infinitely long cylinder is given in the figure. Positive electric charges on the body are uniformly distributed and the linear charge density is  $\lambda$ . The distance of any point to the cylinder axis is expressed by  $r$ .

- Find the electric field at any point in the inner region of the cylinder.
- Find the electric field at any point in the outer region of the cylinder.
- Plot the electric field as a function of  $r$ .



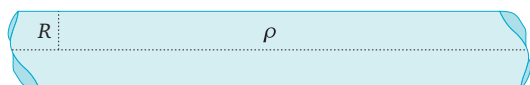
4. In the insulating spherical shell shown in the figure, there is a cavity of radius  $R$  in the inner part. Positive charge  $Q$  is uniformly distributed throughout the volume of the spherical shell. The distance of any point from the center is expressed by  $r$ .

- Find the volume charge density for the spherical shell.
- Find the electric field intensity for points at a distance  $r < R$  from the center.
- Find the electric field intensity for points at a distance  $r > 2R$  from the center.
- Find the electric field intensity for points at a distance  $R < r < 2R$  from the center.
- Compare the results obtained in c and d for points at a distance  $r = 2R$  from the center.



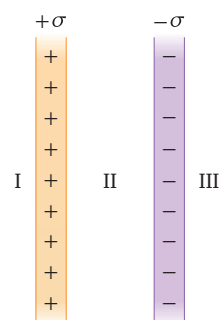
5. A physically infinitely long, solid insulating cylinder is shown in the figure. Positive electric charges are distributed uniformly throughout the volume of the cylindrical object with a volume charge density  $\rho$ . The radius of the cylinder is  $R$ , and the distance of any point from the cylinder axis is denoted by  $r$ .

- Find the electric field at any point in the inner region of the cylinder.
- Find the electric field at any point in the outer region of the cylinder.
- Compare the results obtained in a and b for the case where  $r = R$ .
- Plot the electric field as a function of  $r$ .



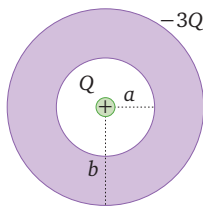
6. The plates of negligible thickness in the figure are physically infinite in size and positioned parallel to each other. The surface charge densities on the plates are  $\sigma$  and  $-\sigma$  respectively, and the charges are distributed uniformly on the plates.

- Find the direction and magnitude of the electric field in region I.
- Find the direction and magnitude of the electric field in region II.
- Find the direction and magnitude of the electric field in region III.

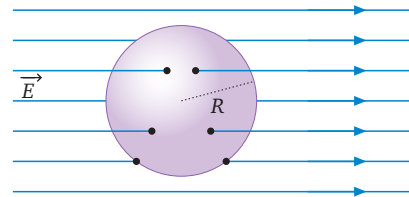


7. The conducting spherical shell in the figure has an inner radius  $a$  and an outer radius  $b$ , and there is a charged point object at the center of the shell. The point object has a net electric charge of  $Q$  and the spherical shell has a net electric charge of  $-3Q$ . The distance of any point from the center is denoted by  $r$ .

- Find the electric field magnitude in the region  $r < a$ .
- Find the electric field magnitude in the region  $a < r < b$ .
- Find the charge density on the outer surface of the spherical shell.
- Find the electric field magnitude in the region  $r > b$ .

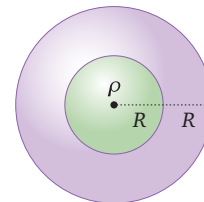


2. The sphere in the figure is located within an external electric field. Only this external electric field exists on the sphere surface. Accordingly, find the net electric charge inside the sphere.



3. Around the insulating sphere of radius  $R$  in the figure, there is a conducting spherical shell with an inner radius  $R$  and an outer radius  $2R$ . The charge density of the insulating sphere is homogeneous and denoted by  $\rho$ . The net charge of the shell is zero. The distance of any point from the center is denoted by  $r$ .

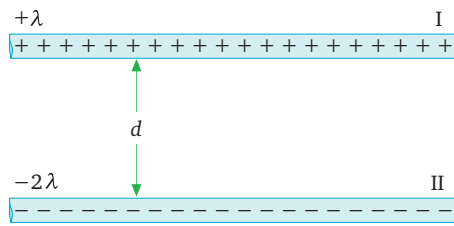
- Find the electric field magnitude in the region  $r < R$ .
- Find the electric field magnitude in the region  $R < r < 2R$ .
- Find the charge density on the inner surface of the spherical shell.
- Find the electric field in the region  $r > 2R$ .



## Exercises and Problems

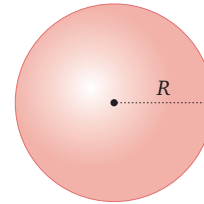
1. A point object with a charge amount  $Q_0$  is placed at the geometric center of a cube with a net charge of zero. The length of one edge of the cube is  $L$ . Accordingly, find the electric flux for each surface of the cube.

4. Two infinitely long, negligible-thickness rods are parallel to each other, with the linear charge densities as shown in the figure. What is the distance from the points where the electric field is zero to the second rod, in terms of  $d$ ?



5. In the insulating sphere shown in the figure, positive charges are distributed as a function of  $r$  and given as  $\rho(r) = ar^2$ .  $a$  is a constant.  $r$  represents the distance of any point to the center.

- Find the electric field at any point in the outer region of the sphere.
- Find the electric field at any point in the inner region of the sphere.
- Compare the results obtained in a and b when  $r = R$ .



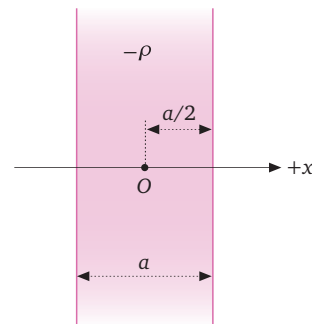
6. In the physically infinitely long cylinder shown in the figure, positive charges are distributed as a function of  $r$  and given as  $\rho(r) = ar$ .  $a$  is a constant.  $r$  represents the distance of any point to the center.

- Find the electric field at any point in the outer region of the cylinder.
- Find the electric field at any point in the inner region of the cylinder.
- Compare the results obtained in a and b when  $r = R$ .



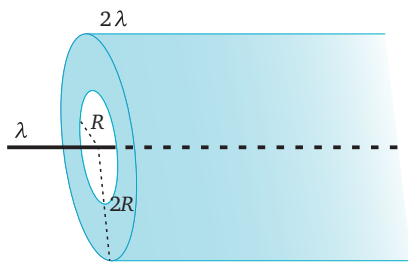
7. The cross-section of a very large, insulating, and negatively charged plate is shown in the figure. Negative charges are uniformly distributed throughout the volume of the plate with a charge density of  $-\rho$ .  $\rho$  represents a positive quantity.

- Find the direction and magnitude of the electric field at points with coordinate  $x = a/4$ .
- Find the direction and magnitude of the electric field at points with coordinate  $x = a/2$ .
- Find the direction and magnitude of the electric field at points with coordinate  $x = a$ .



8. The linear net charge density of the insulating thin rod shown in the figure is  $\lambda$  and the net charge density of the conducting cylindrical shell is  $2\lambda$ . Charges are uniformly distributed along the rod and on the shell surfaces. The rod is at the center of the shell, and  $r$  represents the distance of any point to the cylinder axis.

- Find the electric field magnitude in the region  $r < R$ .
- Find the linear charge density on the outer surface of the shell.
- Find the electric field magnitude in the region  $R < r < 2R$ .
- Find the electric field magnitude in the region  $r > 2R$ .



## Answers

### Example questions

1  $\Phi_E = 8.7 \text{ N} \cdot \text{m}^2/\text{C}$

2  $\Phi_E = 1.5 \text{ N} \cdot \text{m}^2/\text{C}$

3  $\Phi_E = \frac{q}{\epsilon_0}$

4 a.  $E = 0$

b.  $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

c. Go to the video.

5 a.  $E = \frac{1}{4\pi\epsilon_0} \frac{Qr}{a^3}$

b.  $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$

c. Go to the video.

d. Go to the video.

6  $\vec{E} = -\frac{\lambda}{2\pi\epsilon_0 r} \hat{r}$

7  $E = \frac{\sigma}{2\epsilon_0}$ , for the direction, go to the video.

### Your turn questions

$\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4 = 0$

$\Phi_5 = E_0 L^2$

$\Phi_6 = -E_0 L^2$

$\Phi_{\text{total}} = 0$

2  $6 \text{ N} \cdot \text{m}^2/\text{C}$

3 a. 0

b.  $E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$

c. Go to video.

4 a.  $\rho = \frac{3Q}{28\pi R^3}$

b.  $E = 0$

c.  $E = \frac{Q}{4\pi\epsilon_0 r^2}$

d.  $E = \frac{1}{4\pi\epsilon_0} \frac{Q(r^3 - R^3)}{7R^3 r^2}$

e. Go to video.

- 5
- $E = \frac{\rho r}{2\epsilon_0}$
  - $E = \frac{\rho R^2}{2\epsilon_0 r}$
  - Go to video.
  - Go to video.

- 6
- $E = 0$
  - $E = \frac{\sigma}{\epsilon_0}$ , go to video for direction.
  - $E = 0$

- 7
- $E = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$
  - $E = 0$
  - $\sigma = -\frac{Q}{2\pi b^2}$
  - $\frac{1}{2\pi\epsilon_0} \frac{Q}{r^2}$

### Exercises and problems

1  $\Phi = \frac{Q_0}{6\epsilon_0}$

2  $q_{\text{in}} = 0$

- 3
- $E = \frac{\rho r}{3\epsilon_0}$
  - $E = 0$
  - $\sigma = -\frac{\rho R}{3}$
  - $\frac{\rho R^3}{3\epsilon_0 r^2}$

4  $r = 2d$

- 5
- $E = \frac{aR^5}{5\epsilon_0 r^2}$
  - $E = \frac{ar^3}{5\epsilon_0}$
  - Go to the video.

- 6
- $E = \frac{aR^3}{3\epsilon_0 r}$
  - $E = \frac{ar^2}{3\epsilon_0}$
  - Go to the video.

- 7
- $E = \frac{\rho a}{4\epsilon_0}$ , in the  $-x$  direction
  - $E = \frac{\rho a}{2\epsilon_0}$ , in the  $-x$  direction
  - $E = \frac{\rho a}{2\epsilon_0}$ , in the  $-x$  direction

- 8
- $E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$
  - $3\lambda$
  - $E = 0$
  - $E = \frac{3}{2\pi\epsilon_0} \frac{\lambda}{r}$

# CHAPTER 3

## ELECTRIC POTENTIAL



### Electric Potential Energy and Potential Difference

#### Electric potential energy

Let's derive the work done by electric forces when a charge is moved from one point to another in a uniform electric field. Through this work, let's discuss the concept of electric potential energy.

To better understand the topic, let's make a comparison with gravitational potential energy.

#### Electric potential and potential difference

Let's define the concept of electric potential and discuss why such a definition is necessary.

To better understand the topic, let's make a comparison using gravitation.

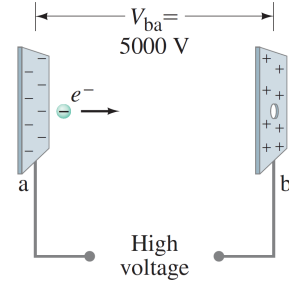
**NOTE:** Concepts such as potential and potential energy are defined relative to a reference point. On the other hand, potential energy difference and potential difference are independent of the choice of reference.

Let's examine the electric potential energies of positive and negative charges in a uniform electric field formed by charged, infinitely large parallel plates.

Let's examine the relationship between potential difference and the electric field in a uniform electric field.

**EXAMPLE 1:** The electron in the figure is accelerated from plate a toward plate b.

- Determine the change in the electron's electric potential energy.
- Find the electron's final speed.



### Relationship between electric field and electric potential

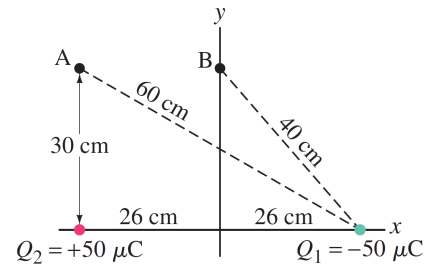
In a region where an electric field exists, let's determine the work done by the electric forces in moving a charge from point A to point B.

Let's express the potential difference between points A and B in terms of the electric field.

**EXAMPLE 2:** Let's find the potential difference between points  $A$  and  $B$  near a point charge  $Q$ .

**EXAMPLE 3:** The positions of two different charges are given in the figure.

- Find the electric potential at point  $A$ .
- Find the electric potential at point  $B$ .
- Find the potential difference between points  $A$  and  $B$ .



### Electric potential of point charges

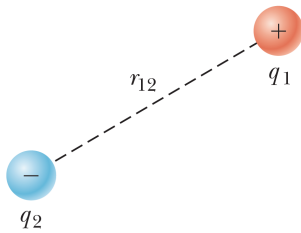
Let's discuss how we define the electric potential of a point charge.

**NOTE:** To define the potential of a point charge at a certain point, we can take the potential at infinity to be zero.

**NOTE:** When finding the total electric potential at a point due to multiple charges, we can algebraically sum the individual potentials.

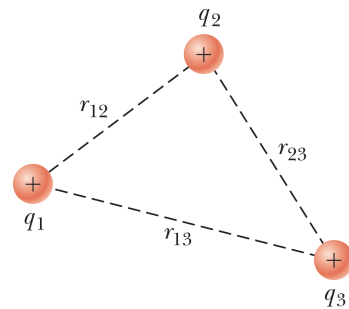
### Electric potential energy of point charges

Let's determine the electric potential energy of the system consisting of two charges shown in the figure.



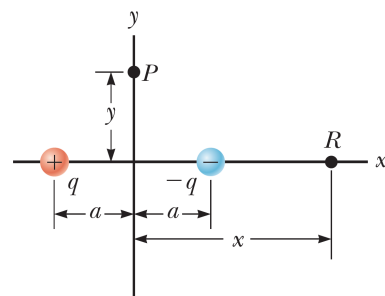
Let's discuss the physical implications of charges being negative or positive in terms of electric potential energy.

**EXAMPLE 4:** Let's find the electric potential energy of the system consisting of three charges shown in the figure.



### Your turn

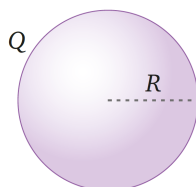
1. An electric dipole is shown in the figure.
  - a. Find the electric potential of the dipole at point  $P$  in terms of the given parameters.
  - b. Find the electric potential of the dipole at point  $R$  in terms of the given parameters.
  - c. Find the electric potential energy of the dipole in terms of the given parameters.



### Obtaining the electric potential from the electric field

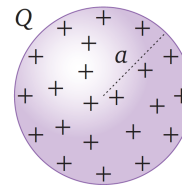
Let's examine how the electric potential is obtained using the electric field through an illustrative example.

**EXAMPLE 5:** For a conducting sphere of radius  $R$  carrying total charge  $Q$ , find how the electric potential varies with distance from the center. Sketch the graph of electric potential as a function of radial distance.



### Your turn

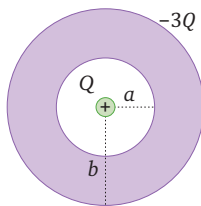
2. A positive electric charge  $Q$  is uniformly distributed throughout the volume of an insulating solid sphere of radius  $a$ . Determine the electric potential as a function of distance from the center and plot it.



**Your turn**

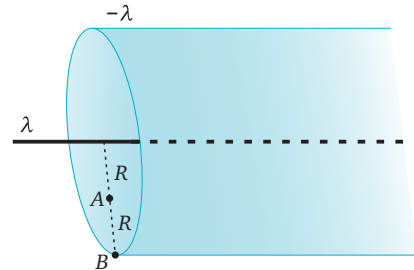
3. In the figure, a conducting spherical shell has inner radius  $a$  and outer radius  $b$ , with a charged point object located at its center. The point object has charge  $Q$  and the spherical shell has a net charge  $-3Q$ . The distance from the center is denoted by  $r$ . Assume the electric potential at infinity is zero.

- Find the electric potential for the region  $r > b$ .
- Find the electric potential for the region  $b > r > a$ .
- Find the electric potential for the region  $a > r$ .
- Sketch the graph of the electric potential as a function of  $r$ .
- Calculate the potential difference between the points  $r = a/2$  and  $r = 2b$ .



4. In the figure, an insulating thin rod has a linear charge density  $\lambda$ , and a conducting cylindrical shell has a net linear charge density  $-\lambda$ . The charges are uniformly distributed along the infinitely long rod and over the surface of the shell.

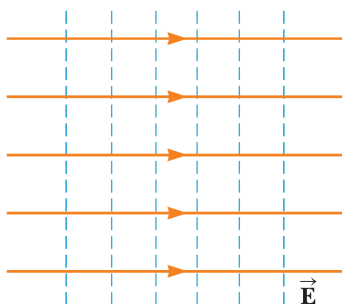
- Find the electric potential at point  $B$ .
- Find the electric potential at point  $A$ .
- Find the potential difference between points  $A$  and  $B$ .



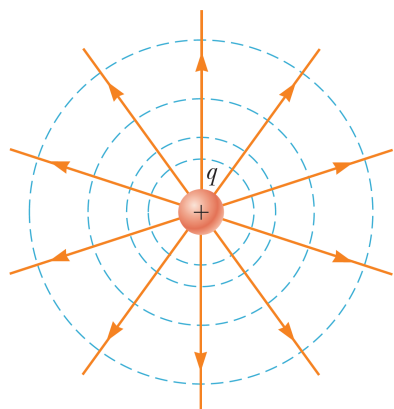
### Obtaining the electric field from the electric potential

Using the figures below, let's define the concept of an **equipotential surface**.

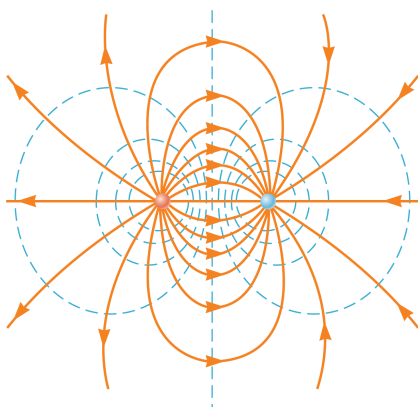
- Uniform electric field between parallel plates.



- Electric field around a point charge.



- Electric field produced by two charges.

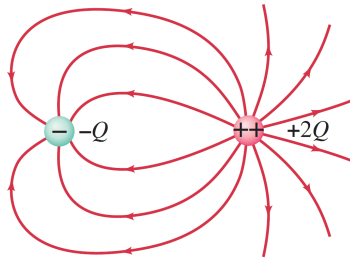


In a region where an electric field exists, let's examine the relationship between potential difference and electric field over an infinitesimal displacement. From this, let's derive how the electric field can be obtained from the electric potential.

Using our findings, let's examine the relationship between the potential and the electric field of a point charge.

Write how to obtain the electric field from a given potential in the Cartesian coordinate system.

**EXAMPLE 6:** The electric field around two point charges is shown in the figure. Let's draw the equipotential curves around the charges.

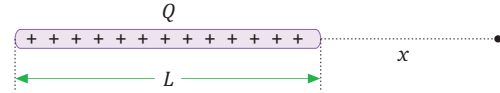


**EXAMPLE 7:** Given that  $V_0$ ,  $A$ , and  $B$  are constants, the electric potential in the Cartesian coordinate system is  $V(x, y, z) = V_0 + Axy + Bx^2z$ .

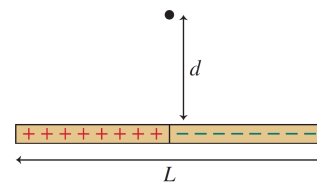
- Obtain the electric field vector field  $\vec{E}(x, y, z)$  for every point in space.
- Determine the electric field at the point  $(x, y, z) = (1, 1, 2)$ .

### Potential of an arbitrary charge distribution

A total positive charge  $Q$  is uniformly distributed along a rod of length  $L$ . Let's find the electric potential at a distance  $x$  from the rod. Using this potential, derive the electric field at that point.

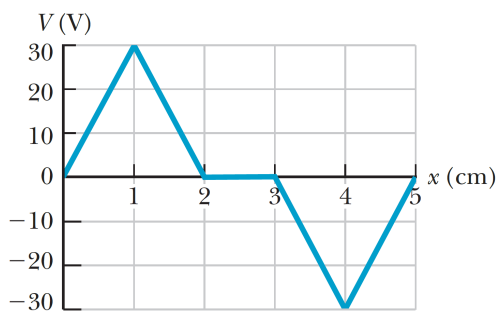


**EXAMPLE 8:** One half of the rod carries a uniform charge  $+Q$ , while the other half carries  $-Q$ . Find the electric potential at the specified point. Identify the equipotential surface passing through this point and determine the direction of the electric field.



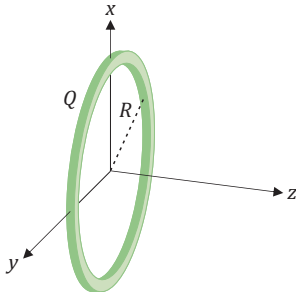
### Your turn

5. An electric field exists along the  $x$ -axis in a certain region. The position dependence of the electric potential is shown in the graph. Sketch the graph of the  $x$ -component of the electric field.



**EXAMPLE 9:** A positive charge  $Q$  is uniformly distributed on a thin ring of radius  $R$ .

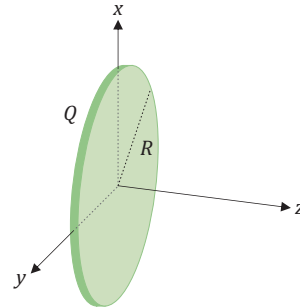
- Determine the electric potential on the  $z$ -axis as a function of position.
- Using the result from part (a), find the electric field on the  $z$ -axis as a function of position.



### Your turn

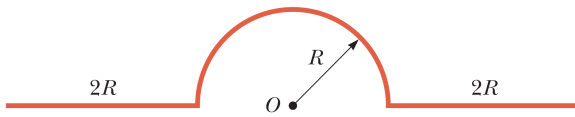
**6.** A total positive charge  $Q$  is uniformly distributed on a thin disk of radius  $R$ .

- Determine the electric potential on the  $z$ -axis as a function of position.
- Using the result from part (a), determine the electric field on the  $z$ -axis as a function of position.



## Your turn

7. A wire with linear charge density  $\lambda$  is bent as shown. Determine the electric potential at point  $O$ .



## Answers

## Example problems

1

- a.  $\Delta U = -8.0 \times 10^{-16} \text{ J}$   
b.  $v = 4.2 \times 10^7 \text{ m/s}$

2

$$V_B - V_A = kQ \left( \frac{1}{r_B} - \frac{1}{r_A} \right)$$

a.  $V_A = 7.5 \times 10^5 \text{ V}$

3

b.  $V_B = 0 \text{ V}$

c.  $\Delta V_{AB} = -7.5 \times 10^5 \text{ V}$

4

$$U = k \left( \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right)$$

5

$$r < R \implies V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R}$$

$$r \geq R \implies V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

6

Go to the video.

7

a.  $\vec{E}(x, y, z) = -(Ay + 2.0Bxz)\hat{i} - Ax\hat{j} - Bx^2\hat{k}$

b.  $\vec{E}(1.0, 1.0, 2.0) = -(A + 4.0B)\hat{i} - A\hat{j} - B\hat{k}$

8

Go to the video.

9

a.  $V = \frac{kQ}{\sqrt{R^2 + z^2}}$

b.  $\vec{E} = \frac{kQz}{(R^2 + z^2)^{3/2}} \hat{k}$

## Your turn questions

1

a.  $V_p = 0$

b.  $V_R = -\frac{2kqa}{x^2 - a^2}$

c.  $U = -\frac{kq^2}{2a}$

2

$$r < a \implies V = \frac{1}{4\pi\epsilon_0} \frac{Q}{2a} \left( 3 - \frac{r^2}{a^2} \right)$$

$$r \geq a \implies V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r}$$

3

a.  $V = -\frac{2kQ}{r}$

b.  $V = -\frac{2kQ}{b}$

c.  $V = \frac{kQ}{r} - \frac{kQ}{a} - \frac{2kQ}{b}$

d. Go to the video.

e.  $\Delta V = \frac{kQ}{b} - \frac{kQ}{a}$

4

a.  $V_B = 0$  (if the shell is chosen as the reference point)

b.  $V_A = \frac{\lambda}{2\pi\epsilon_0} \ln(2)$

c.  $\Delta V = \frac{\lambda}{2\pi\epsilon_0} \ln(2)$

5

Go to the video.

6

a.  $V = \frac{Q}{2\pi R^2 \epsilon_0} (\sqrt{R^2 + z^2} - |z|)$

b.  $\vec{E} = \frac{Q}{2\pi R^2 \epsilon_0} \left( 1 - \frac{z}{\sqrt{R^2 + z^2}} \right) \hat{k}$

7

$$V = k\lambda(2.0 \ln(3.0) + \pi)$$

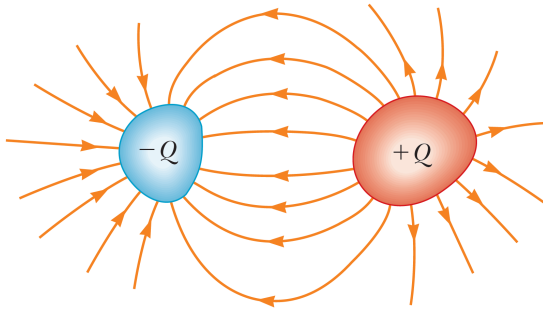
# CHAPTER 4

## CAPACITANCE AND DIELECTRICS



### Definition of Capacitance

Let us define the concept of **capacitance**. Let us write the equation for capacitance and examine its unit.



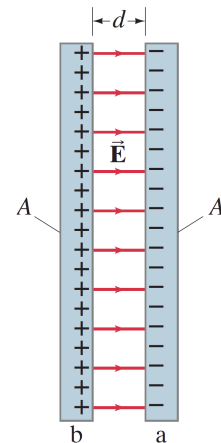
Let us examine some examples of capacitors used as circuit elements.



### Determination of Capacitance

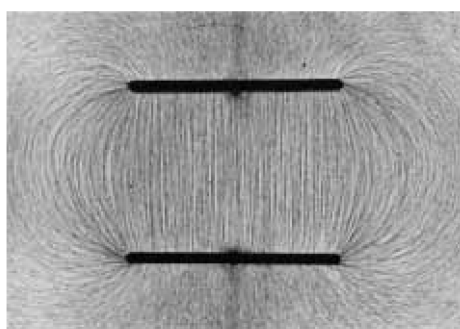
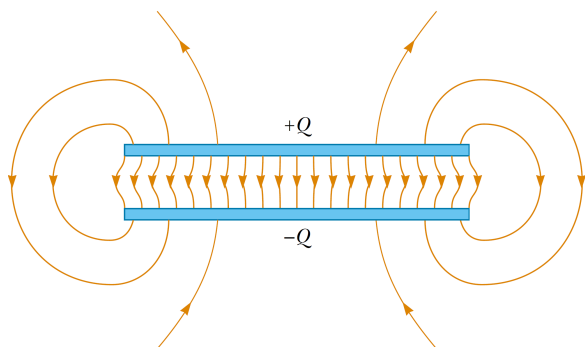
#### Parallel-Plate Capacitor

Let us derive the capacitance of a capacitor consisting of parallel plates.



**NOTE:** The capacitance of a capacitor depends on its physical properties.

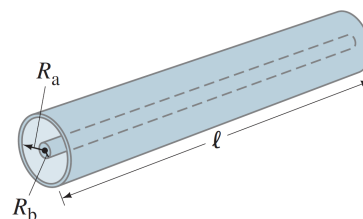
Let us discuss whether the electric field between charged parallel plates is uniform or not.



**NOTE:** In many problems, for the sake of simplicity, the electric field between parallel plates is assumed to be uniform.

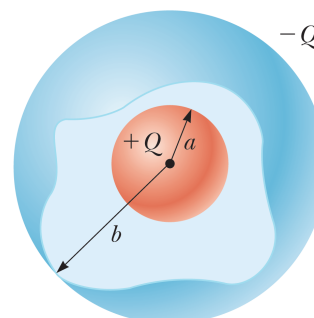
## Cylindrical Capacitor

Let us derive the capacitance of a cylindrical capacitor whose dimensions are given as shown in the figure.



## Spherical Capacitor

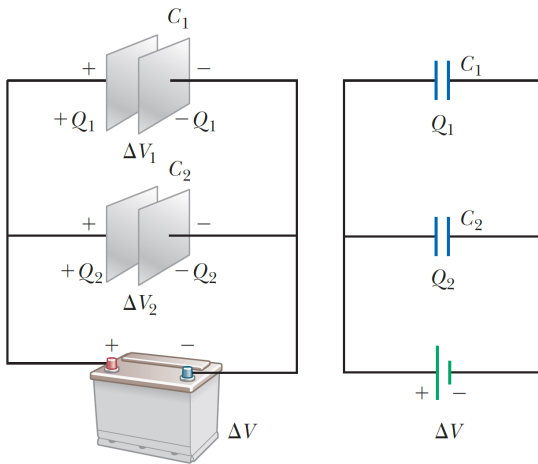
Let us derive the capacitance of a spherical capacitor whose dimensions are given as shown in the figure.



## Capacitors in Series and Parallel

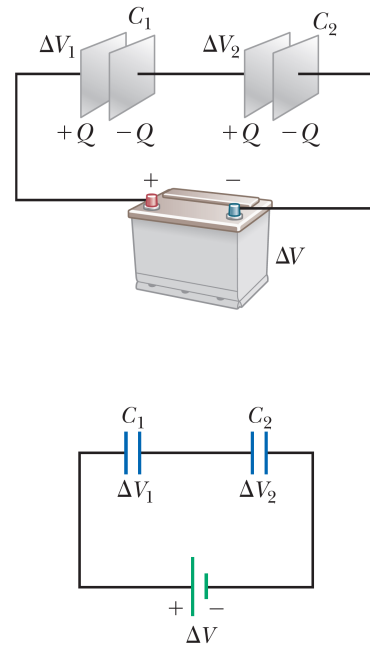
### Parallel Connection

Let us examine how to obtain the equivalent capacitance when multiple capacitors are connected in parallel.



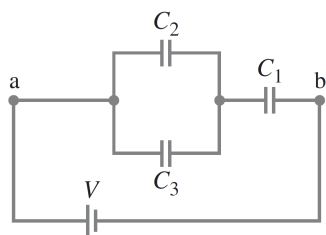
### Series Connection

Let us examine how to obtain the equivalent capacitance when multiple capacitors are connected in series.



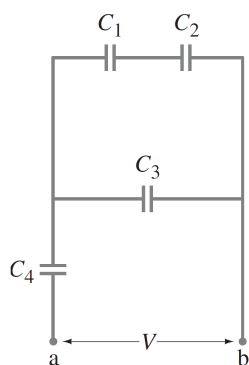
**EXAMPLE 1:** In the circuit shown, the capacitances of the capacitors are given as  $C_1 = C_2 = C_3 = C$ .

- Find the equivalent capacitance of the circuit.
- Compare the amounts of charge on the capacitors.

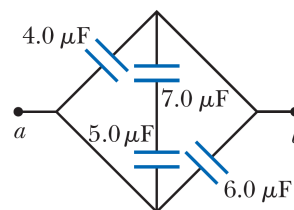


### Your turn

- In the circuit segment shown, the capacitances of the capacitors are given as  $C_1 = C_2 = C_3 = C_4 = C$ . Find the equivalent capacitance of the circuit.

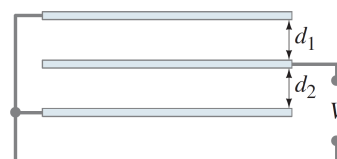


- Find the equivalent capacitance between points  $a$  and  $b$  in the circuit segment shown.



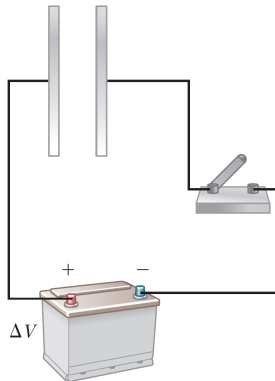
- A circuit is formed by placing three plates of surface area  $A$  parallel to one another as shown. The distances between the plates and the voltage of the source are given in the figure.

- Are the two resulting capacitors connected in parallel or in series?
- Obtain the equivalent capacitance in terms of the given quantities.



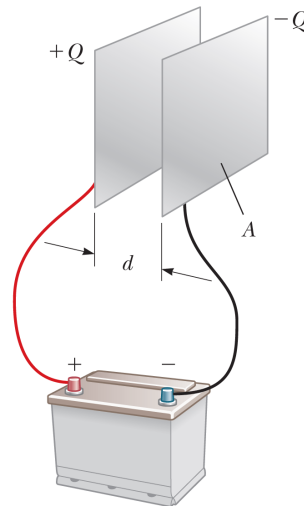
## Energy Stored in a Charged Capacitor

Let us calculate the work required to place a charge of magnitude  $Q$  on an uncharged capacitor. Let us obtain the electric potential energy stored in a charged capacitor.



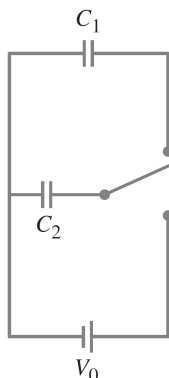
**EXAMPLE 2:** In the circuit shown, a capacitor made of parallel plates is charged while connected to a constant-voltage source.

- Write the electric potential energy stored in the capacitor in terms of the given quantities.
- Determine how this energy changes if the distance  $d$  is increased.
- Determine how the stored electric potential energy changes if the source is removed from the circuit and the distance  $d$  is increased.



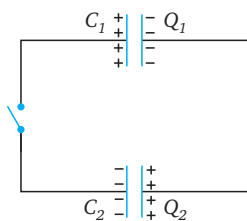
### Connecting Charged Capacitors

Let us examine what happens when a charged capacitor is connected to an uncharged capacitor.



**EXAMPLE 3:** Two charged capacitors are connected by wires as shown in the figure. The charges and capacitances of the capacitors are given as  $Q_1 = 2Q_2 = 2Q$  and  $C_1 = 2C_2 = 2C$ .

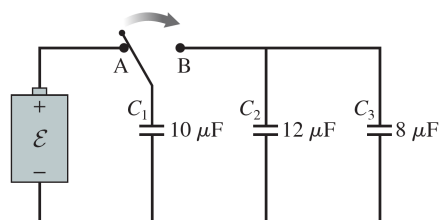
- If the switch is closed, what will the final charges on the capacitors be?
- If the switch is closed after reconnecting capacitor  $C_2$  in reverse without allowing it to discharge, what will the final charges on the capacitors be?



### Your turn

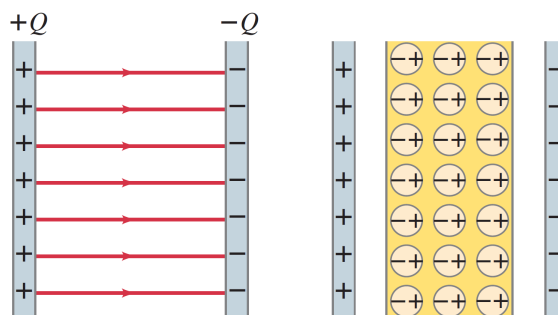
4. In the circuit shown, the switch is initially at position A and  $C_1$  is charged. Then the switch is moved to position B and enough time is allowed to pass. In this case, the potential difference across  $C_1$  becomes 4 V.

- Find the emf of the source.
- Determine how the total energy stored in the capacitors changes when the switch is moved from position A to position B.

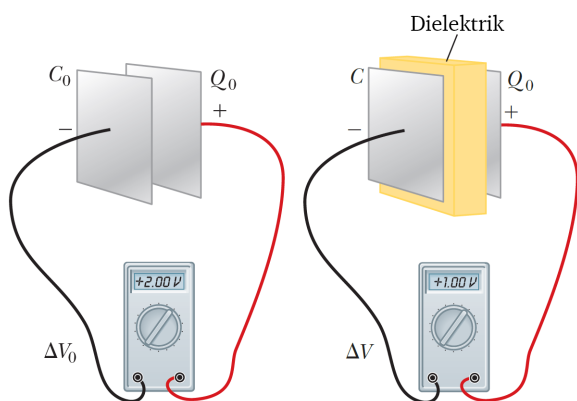


### Capacitors with Dielectrics

Let us examine the effect of a dielectric material inside a capacitor. Let us show how the capacitance changes in the presence of this material.

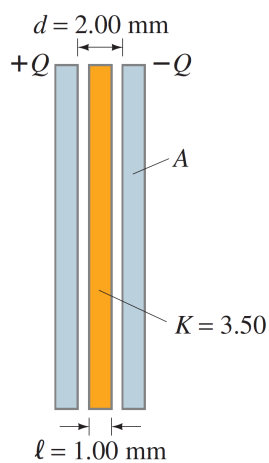


Let us discuss what results are obtained when a dielectric material is placed between the plates of a capacitor.

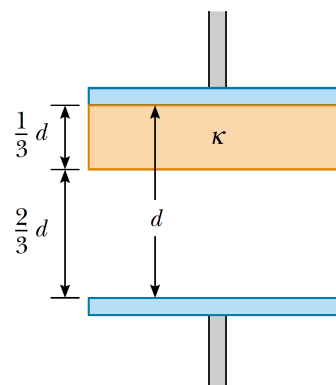


**EXAMPLE 4:** The capacitance of the capacitor shown is  $C$  when there is no dielectric material inside it.

- If the dielectric material shown filled the entire interior region, what would the capacitance be in terms of  $C$ ?
- If the dielectric material filled the interior region as shown in the figure, what would the capacitance be in terms of  $C$ ?

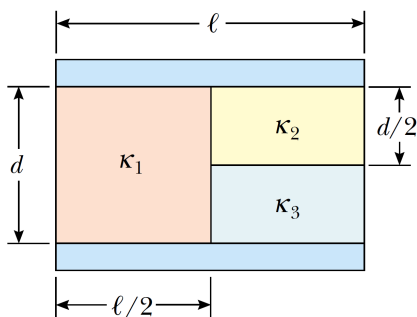


**EXAMPLE 5:** Part of the space between the plates of a capacitor of plate area  $A$  is filled as shown with a material having dielectric constant  $\kappa$ . Find the capacitance of this new capacitor.



## Your turn

5. Materials with different dielectric constants are placed inside a capacitor whose plate area is  $A$ , as shown in the figure. Given that  $2\kappa_1 = 2\kappa_2 = \kappa_3 = 2\kappa$ , find the capacitance of this new capacitor.



## Answers

## Example questions

1

a.  $C_{eq} = \frac{2.0}{3.0} C$

b.  $Q_1 = 2Q_2 = 2Q_3$

2

a.  $U = \frac{\epsilon_0 A V^2}{2d}$

b. The energy decreases.

c. The energy increases.

3

a.  $Q_1 = 0.67Q$

$Q_2 = 0.33Q$

b.  $Q_1 = 2.0Q$

$Q_2 = 1.0Q$

4

a.  $C' = 3.5C$

b.  $C_{eq} = 1.6C$

5

$C = \frac{3\kappa}{1+2\kappa} \frac{\epsilon_0 A}{d}$

## Your turn questions

1

$C_{eq} = 0.60C$

2

$C_{eq} = \frac{155}{12} \mu F$

3

a. They are connected in parallel.

b.  $C_{eq} = \epsilon_0 A \frac{d_1 + d_2}{d_1 d_2}$

4

a.  $\mathcal{E} = 12 \text{ V}$

b.  $\Delta U = -4.8 \times 10^{-4} \text{ J}$  (Energy decreases)

5

$C = \frac{7}{6} \frac{\kappa \epsilon_0 A}{d}$

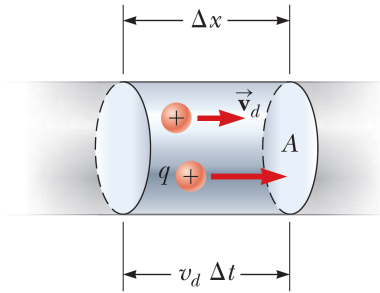
# CHAPTER 5

## CURRENT AND RESISTANCE

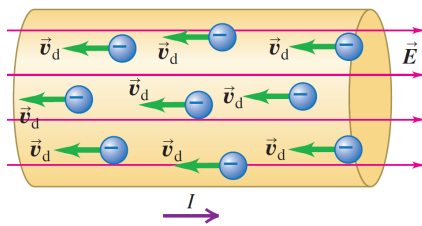


### Electric Current

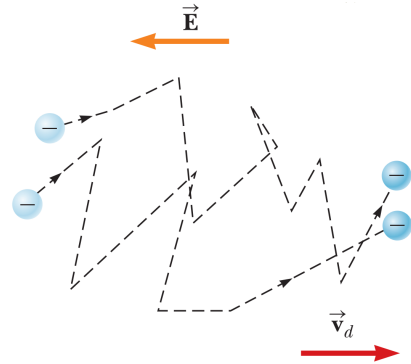
Let us define the concept of current. Let us express current with an equation and examine the unit of current.



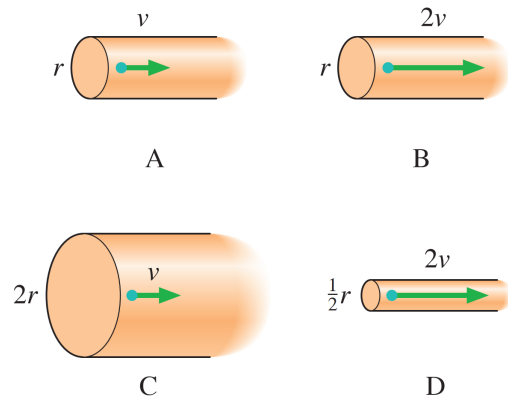
Let us examine how electric current forms inside a metal in terms of the electric field and electric potential.



Let us examine the concept of **drift speed** using a conductor in the presence of an electric field.



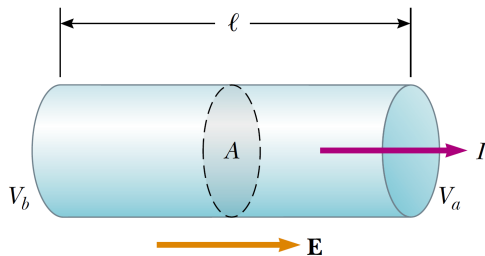
**EXAMPLE 1:** All of the cylindrical wires in the figures are made of the same metal. The drift speeds of the electrons and the radii of the wires are as shown. Compare the electric currents in these wires.



## Resistance and Ohm's Law

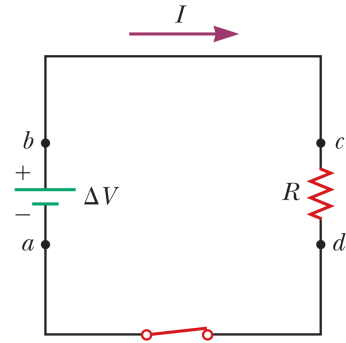
Let us examine the resistance of a conducting wire carrying current by following the steps below.

- Let us define the **current density** inside the conductor.
- Let us obtain the electric field inside the conductor from the potential difference.
- By explaining **Ohm's law**, let us obtain the relation between the current in the conductor and the electric field.
- Let us define the concept of **conductivity**.
- Let us define the concept of **resistance** and examine its unit.
- Let us define the concept of **resistivity**.



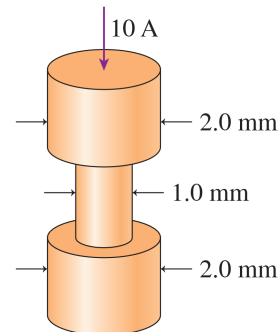
## Ohm's law

Let us state Ohm's law once again by using the circuit shown in the figure.



**EXAMPLE 2:** A cylindrical conductor whose thickness varies from place to place is shown in the figure. A current of 10 A passes through the conductor, whose conductivity is  $\sigma$ . For conductor segments of equal length, compare the following quantities.

- Electric current
- Current density
- Electric field strength
- Potential difference
- Resistance



**Your turn**

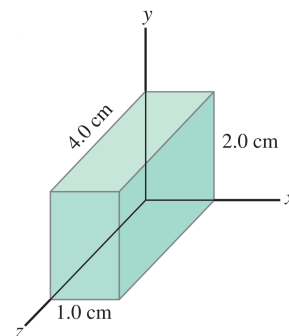
1. An electric current  $I$  flows through conducting wire segments whose dimensions are completely identical. The relation between the conductivities of the wires is given as  $\sigma_1/\sigma_2 = 2$ . For the wire segments, compare the following quantities.

- Electric current
- Current density
- Electric field strength
- Resistivity
- Resistance
- Potential difference between the ends

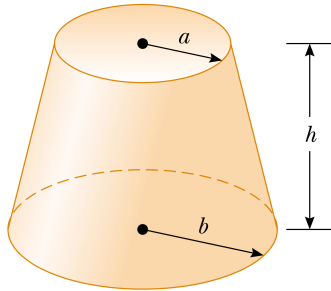


2. A silver prism with resistivity  $\rho$  is given in the figure. Find the resistance of this object when current is made to pass through it in different directions.

- in the  $x$  direction
- in the  $y$  direction
- in the  $z$  direction

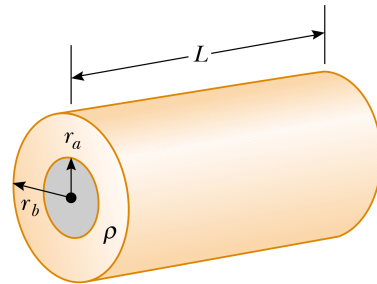


3. An object with the dimensions shown in the figure is made of a conducting material with resistivity  $\rho$ . Assume that the current is distributed uniformly across any cross section of this material. Calculate the resistance between the two ends of this object in terms of the given quantities.



4. A hollow cylinder with the dimensions shown in the figure is made of a conducting material with resistivity  $\rho$ .

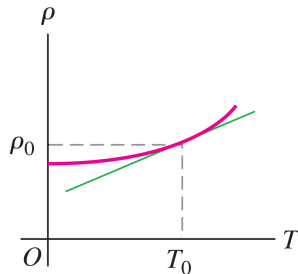
- When a potential difference is applied across the two ends of the cylinder, find the resistance of the cylinder in terms of the given quantities.
- When a potential difference is applied between the inner and outer surfaces of the cylinder, find the resistance of the cylinder in terms of the given quantities.



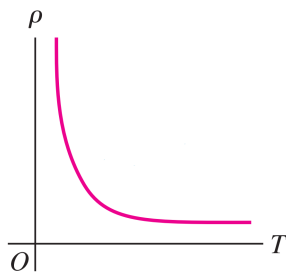
## Relationship Between Temperature and Resistivity

Let us examine the relationship between the resistivity of different materials and temperature.

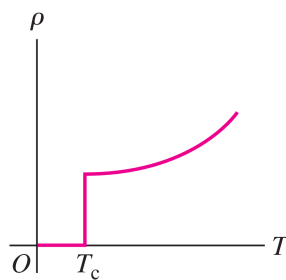
- For a metal



- For a semiconductor

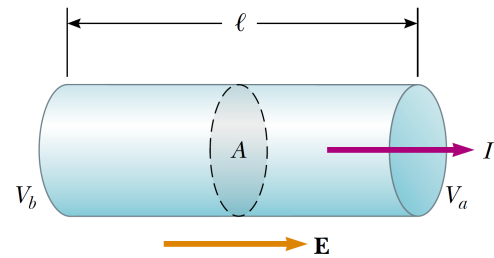


- For a superconductor



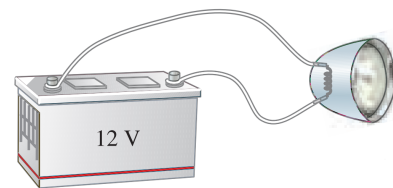
## Electric Power

Let us obtain how much power is consumed when current passes through a conductor.



**EXAMPLE 3:** A 36 W lamp is operated by a source with a voltage of 12 V. The resistance of the wires connected to the lamp is negligible.

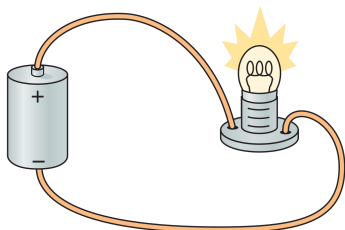
- Find the current through the lamp.
- Find the resistance of the lamp.



## Your turn

5. The resistance of a light bulb designed to operate under 120 V is given as  $360 \Omega$ .

- Find the power consumed by the lamp while it is operating.
- Find the current through the lamp while it is operating.



## Answers

## Example questions

1  $I_C > I_B > I_A > I_D$

a.  $I_a = I_b = I_c = 10 \text{ A}$

b.  $J_b = 4.0J_a = 4.0J_c$

2 c.  $E_b = 4.0E_a = 4.0E_c$

d.  $V_b = 4.0V_a = 4.0V_c$

e.  $R_b = 4.0R_a = 4.0R_c$

a.  $I = 3.0 \text{ A}$

3 b.  $R = 4.0 \Omega$

## Your turn questions

a.  $I_1 = I_2$

b.  $J_1 = J_2$

c.  $E_2 = 2.0E_1$

1 d.  $\rho_2 = 2.0\rho_1$

e.  $R_2 = 2.0R_1$

f.  $V_2 = 2.0V_1$

a.  $R_x = 0.13\rho$

2 b.  $R_y = 0.50\rho$

c.  $z = 2.0\rho$

3  $R = \frac{\rho h}{\pi ab}$

a.  $R = \frac{\rho L}{\pi(r_b^2 - r_a^2)}$

4 b.  $R = \frac{\rho}{2\pi L} \ln\left(\frac{r_b}{r_a}\right)$

a.  $P = 40 \text{ W}$

5 b.  $I = 0.33 \text{ A}$

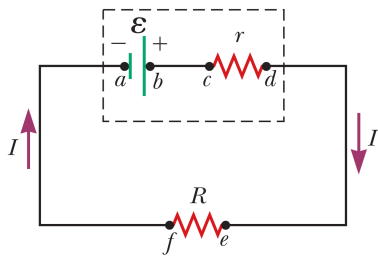
# CHAPTER 6

## DIRECT-CURRENT CIRCUITS

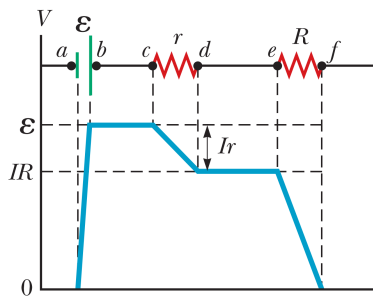


### Electromotive Force

Let us define the concept of **electromotive force** for a battery. Let us determine the potential difference between the terminals of a battery with internal resistance from the circuit shown.

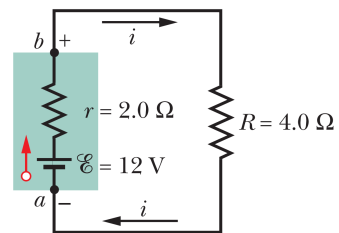


Let us compare the electric potentials at the points shown in the circuit.



**EXAMPLE 1:** In the circuit shown, the colored part represents the internal structure of the source.

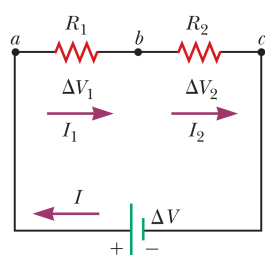
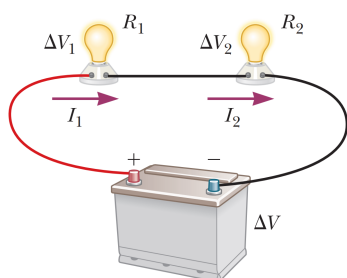
- Find the current in the circuit.
- Find the potential difference between the terminals of the source.
- Find the potential difference across the  $4\ \Omega$  resistor.



## Resistors in Series and Parallel

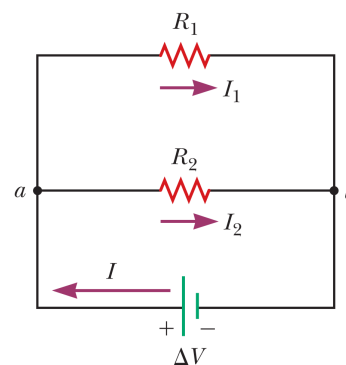
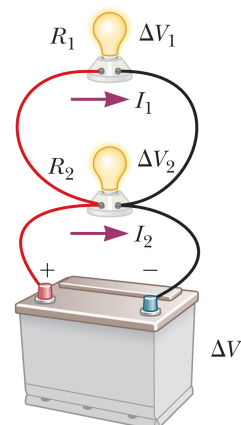
### Series Connection

Let us obtain the equivalent resistance for a circuit in which the resistors are connected in series as shown.



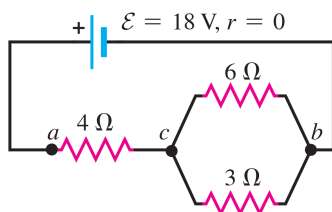
### Parallel Connection

Let us obtain the equivalent resistance for a circuit in which the resistors are connected in parallel as shown.



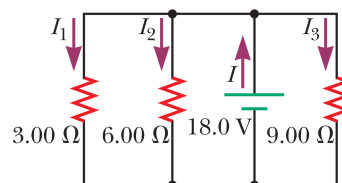
**EXAMPLE 2:** The circuit shown is constructed with an ideal source whose internal resistance is neglected and three different resistors.

- Find the equivalent resistance of the circuit.
- Find the currents in all branches.
- Find the potential differences between the given points.

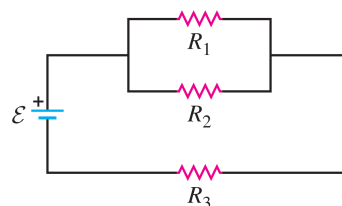


### Your turn

1. In the circuit shown, the internal resistance of the source and the resistance of the wires are negligible. Accordingly, what are the currents  $I$ ,  $I_1$ ,  $I_2$ , and  $I_3$  in amperes?

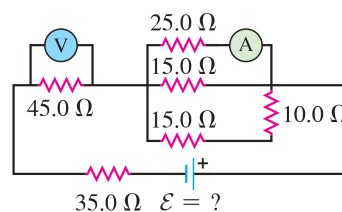


2. In the circuit shown, the internal resistance of the source and the resistance of the wires are negligible. The emf of the source is 27 V. The resistance  $R_1 = 6 \Omega$  and the current through it is 2 A. If the current through resistor  $R_3$  is 3 A, what are the resistances of  $R_2$  and  $R_3$  in  $\Omega$ ?



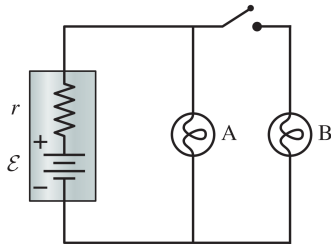
3. In the circuit shown, the internal resistance of the source and the resistance of the wires are negligible. The current through the ammeter is measured as 0.3 A.

- Find the reading of the voltmeter.
- Find the emf of the source.



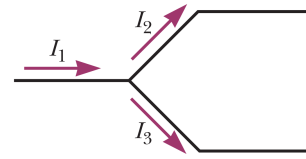
4. A circuit is constructed as shown using a source with an emf of 6 V and an internal resistance of  $2\ \Omega$ , together with two lamps each having a resistance of  $8\ \Omega$ .

- Find the potential difference across the terminals of the source when the switch is open.
- Find the potential difference across the terminals of the source when the switch is closed.
- Discuss how closing the switch affects the brightness of the lamps depending on whether the internal resistance is present or absent.

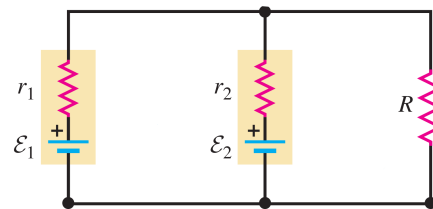


## Kirchhoff's Rules

Let us discuss Kirchhoff's junction rule. Let us write the corresponding equation for the example in the figure.

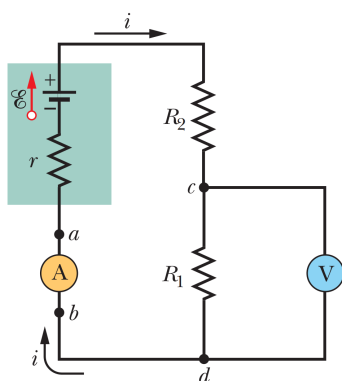


Let us discuss Kirchhoff's loop rule. Let us write the corresponding equations for the example in the figure.



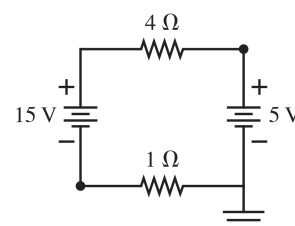
## Voltmeter and Ammeter

Let us examine how the voltmeter and ammeter work and how they should be connected in a circuit.



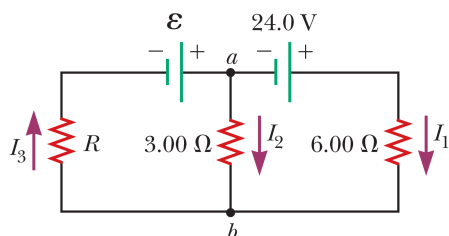
When applying Kirchhoff's loop rule, let us examine how to include a source and a resistor in the potential equations when we encounter them.

Let us examine the effect of **grounding** in the circuit shown. Let us determine the electric potential at each corner.

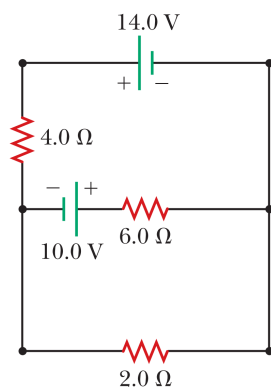


**EXAMPLE 3:** The circuit shown is constructed using sources with negligible internal resistance and three resistors. It is given that  $I_1 = 3 \text{ A}$ .

- Find the potential difference between points a and b.
- Find the values of  $I_2$  and  $I_3$ .
- If  $R = 16 \Omega$ , find  $\mathcal{E}$ .



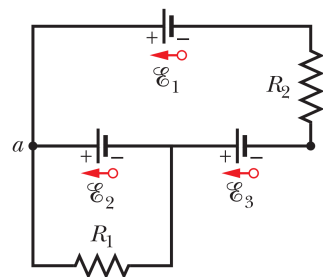
**EXAMPLE 4:** The circuit shown is constructed using sources with negligible internal resistance and three resistors. Find the currents in all branches of the circuit.



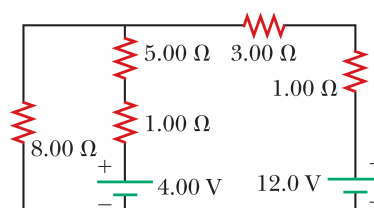
### Your turn

**5.** The circuit shown in the figure is constructed using sources with negligible internal resistance and two resistors. It is given that  $\mathcal{E}_1 = 10 \text{ V}$ ,  $\mathcal{E}_2 = 12 \text{ V}$ ,  $\mathcal{E}_3 = 14 \text{ V}$ ,  $R_1 = 6 \Omega$ , and  $R_2 = 8 \Omega$ .

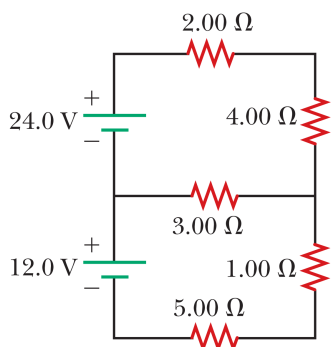
- Find the potential difference between points a and b.
- Find the currents passing through the resistors.



**6.** The circuit shown in the figure is constructed using sources with negligible internal resistance and five resistors. Find the currents in all branches of the circuit.



7. The circuit shown in the figure is constructed using sources with negligible internal resistance and five resistors. Find the currents in all branches of the circuit.

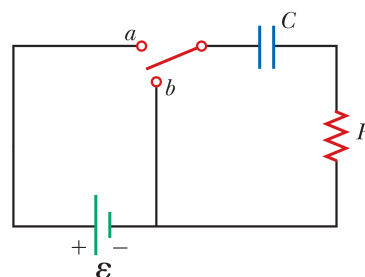


## RC Circuits

### Charging of a Capacitor

Let us discuss what happens during the charging of an initially uncharged capacitor. Then let us carry out the following steps.

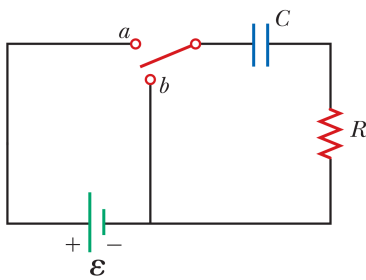
- Derive the time-dependent equation for the charge on the capacitor.
- Derive the time-dependent equation for the current through the circuit.
- Plot the graphs of the quantities we obtain.
- Discuss what happens initially and in the final state during charging.
- Define the concept of the **time constant** and examine its unit.



### Discharging of a Capacitor

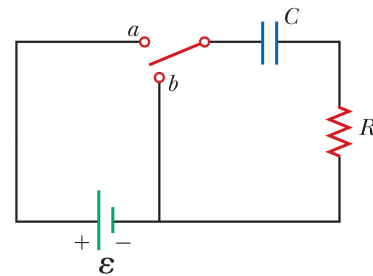
Let us discuss what happens during the discharge of a charged capacitor. Then let us carry out the following steps.

- Derive the time-dependent equation for the charge on the capacitor.
- Derive the time-dependent equation for the current through the circuit.
- Plot the graphs of the quantities we obtain.
- Discuss what happens initially and in the final state during discharging.
- Define the concept of the **time constant**.



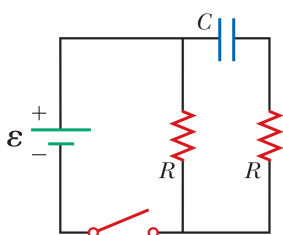
**EXAMPLE 5:** In the circuit shown, constructed with a source whose internal resistance is negligible, the capacitor is initially uncharged. It is given that  $\mathcal{E} = 10 \text{ V}$ ,  $R = 5 \text{ k}\Omega$ , and  $C = 2 \text{ mF}$ .

- Calculate the time constant for the charging and discharging cases of the circuit.
- Let the switch be moved to position a at  $t = 0$ . Write the time-dependent equations for the charge stored on the capacitor and the current through the circuit.
- Find the electric potential energy stored in the capacitor at  $t = 20 \text{ s}$ .
- After waiting long enough, the switch is moved to position b. Obtain the time-dependent equations for the charge and the current.



**EXAMPLE 6:** In the circuit shown, which is constructed with a source whose internal resistance is neglected, the capacitor is initially uncharged.

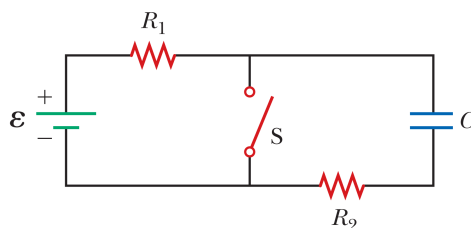
- Find the current through the source in terms of the given quantities at the instant the switch is closed.
- Find the current through the source in terms of the given quantities after waiting a long time.
- Discuss what happens if the switch is opened again after waiting a long time.
- After waiting a long time and then opening the switch again, find in terms of the given quantities the time required for the charge on the capacitor to decrease to half of its value.



### Your turn

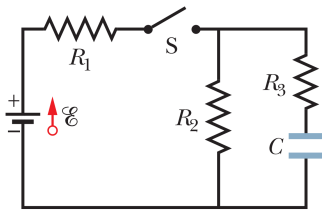
**8.** In the circuit shown, which is constructed with a source whose internal resistance is neglected, after a long time, the capacitor is fully charged.  $\mathcal{E} = 20 \text{ V}$ ,  $R_1 = 100 \text{ k}\Omega$ ,  $R_2 = 50 \text{ k}\Omega$ , and  $C = 10 \text{ }\mu\text{F}$  are given.

- Find the time constant before the switch is closed.
- Find the time constant after the switch is closed.
- Suppose the switch is closed at  $t = 0$ . Find, as functions of time, the currents through the resistors after the switch is closed.
- Find the time required for the charge on the capacitor to decrease to one-tenth of its value.

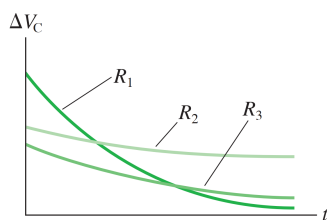


9. In the circuit shown, which is constructed with a source whose internal resistance is neglected,  $\mathcal{E} = 6\text{ V}$ ,  $R_1 = R_2 = R_3 = 10\text{ k}\Omega$ , and  $C = 6\text{ mF}$  are given. The switch is closed at  $t = 0$ .

- Find the currents through the resistors at  $t = 0$ .
- Find the currents through the resistors after waiting a long time.
- Find the charge on the capacitor after waiting a long time.



10. A charged capacitor is discharged separately through the resistors  $R_1$ ,  $R_2$ , and  $R_3$ . As the capacitor discharges, the time dependence of its potential is as shown in the graph. Rank  $R_1$ ,  $R_2$ , and  $R_3$  from greatest to least.



## Answers

### Example questions

- |   |                                                                                                                                                                                                                                                                                  |
|---|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1 | a. $i = 2.0\text{ A}$<br>b. $V_{ab} = 8.0\text{ V}$<br>c. $V_R = 8.0\text{ V}$                                                                                                                                                                                                   |
| 2 | a. $R = 6.0\ \Omega$<br>b. $I_{4\Omega} = 3.0\text{ A}$ $I_{6\Omega} = 1.0\text{ A}$ $I_{3\Omega} = 2.0\text{ A}$<br>c. $V_{ac} = 12\text{ V}$ $V_{cb} = 6.0\text{ V}$ $V_{ab} = 18\text{ V}$                                                                                    |
| 3 | a. $V_{ab} = 6.0\text{ V}$<br>b. $I_2 = -2.0\text{ A}$ $I_3 = 1.0\text{ A}$<br>c. $\mathcal{E} = 10\text{ V}$                                                                                                                                                                    |
| 4 | $I_{4.0\Omega} = 3.0\text{ A}$<br>$I_{6.0\Omega} = 2.0\text{ A}$<br>$I_{2.0\Omega} = 1.0\text{ A}$                                                                                                                                                                               |
| 5 | a. $\tau = 10\text{ s}$<br>b. $q(t) = 2.0 \times 10^{-2}(1 - e^{-t/10})\text{ C}$<br>$i(t) = 2.0 \times 10^{-3}e^{-t/10}\text{ A}$<br>c. $U = 7.3 \times 10^{-2}\text{ J}$<br>d. $q(t) = 2.0 \times 10^{-2}e^{-t/10}\text{ C}$<br>$i(t) = -2.0 \times 10^{-3}e^{-t/10}\text{ A}$ |
| 6 | a. $i = \frac{2\mathcal{E}}{R}$<br>b. $i = \frac{\mathcal{E}}{R}$<br>c. Go to the video.<br>d. $t = 2.0RC \ln(2)$                                                                                                                                                                |

### Your turn questions

- |   |                                                                                           |
|---|-------------------------------------------------------------------------------------------|
| 1 | $I_1 = 6.0\text{ A}$<br>$I_2 = 3.0\text{ A}$<br>$I_3 = 2.0\text{ A}$<br>$I = 11\text{ A}$ |
| 2 | $R_2 = 12\ \Omega$<br>$R_3 = 5.0\ \Omega$                                                 |

3

a.  $V = 49.5 \text{ V}$

b.  $\mathcal{E} = 95.5 \text{ V}$

4

a.  $V = 4.8 \text{ V}$

b.  $V = 4.0 \text{ V}$

c. Go to the video.

5

$I_{4.0 \Omega} = 3.0 \text{ A}$

$I_{6.0 \Omega} = 2.0 \text{ A}$

$I_{2.0 \Omega} = 1.0 \text{ A}$

6

Left Branch:  $I = 0.85 \text{ A}$

Middle Branch:  $I = 0.46 \text{ A}$

Right Branch:  $I = 1.3 \text{ A}$

7

$I_{\text{upper}} = 3.5 \text{ A}$

$I_{\text{lower}} = 2.5 \text{ A}$

$I_{\text{middle}} = 1.0 \text{ A}$

8

a.  $\tau_1 = 1.5 \text{ s}$

b.  $\tau_2 = 0.50 \text{ s}$

c.  $i_1 = 2.0 \times 10^{-4} \text{ A}$     $i_2(t) = (4.0 \times 10^{-4})e^{-2.0t} \text{ A}$

d.  $t = 1.2 \text{ s}$

9

a.  $I_1 = 4.0 \times 10^{-4} \text{ A}$     $I_2 = 2.0 \times 10^{-4} \text{ A}$

$I_3 = 2.0 \times 10^{-4} \text{ A}$

b.  $I_1 = 3.0 \times 10^{-4} \text{ A}$     $I_2 = 3.0 \times 10^{-4} \text{ A}$     $I_3 = 0 \text{ A}$

c.  $Q = 1.8 \times 10^{-2} \text{ C}$

10

$R_2 > R_3 > R_1$

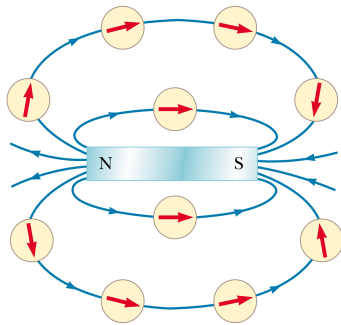
# CHAPTER 7

## MAGNETIC FIELD AND MAGNETIC FORCE

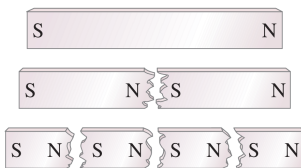


### Magnets and Magnetic Fields

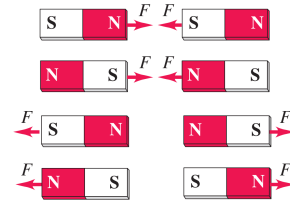
Let us examine the magnetic field formed around a bar magnet. Let us discuss what happens to compass needles placed around the magnet. Let us define the magnetic poles.



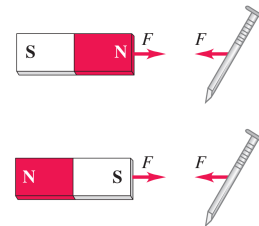
Let us examine what happens when a bar magnet is divided. Let us discuss the internal structure of the magnet.



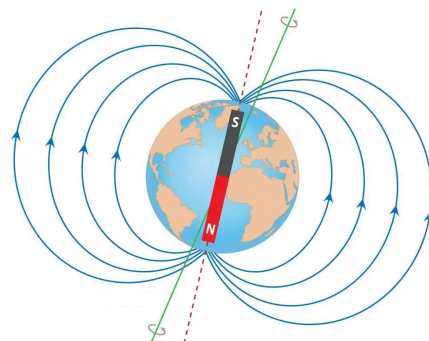
Let us examine the interaction between unlike poles of magnets.



Let us examine the interaction between a magnet and an iron nail.

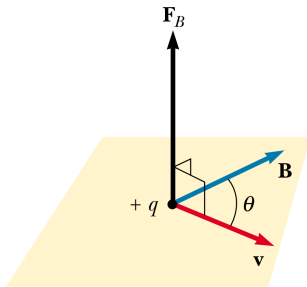


Let us examine Earth's magnetic field and its magnetic poles.

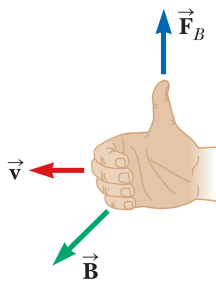


## Magnetic Force on a Charge

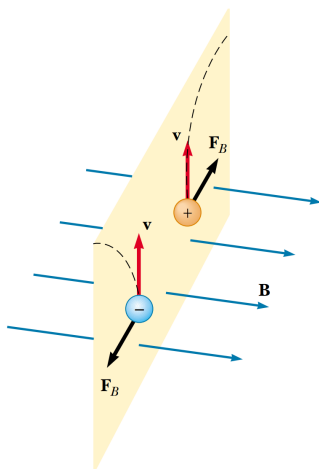
Let us discuss what the magnitude of the magnetic force on a moving charge depends on. Let us derive the unit of the magnetic field in terms of other units.



Let us examine how the direction of the magnetic force on the charge is determined.



Let us examine how the sign of the charge changes the direction of the magnetic force. Let us discuss under which conditions no magnetic force acts on a charge.



**EXAMPLE 1:** In a region where an electron has velocity  $\vec{v} = 2\hat{i}$  km/s, the magnetic field is given as  $\vec{B} = (\hat{i} + 2\hat{j})$  mT. Find the magnetic force acting on the electron.

### Your turn

1. The magnetic force acting on a point particle with charge  $2\ \mu\text{C}$  is given as  $\vec{F} = (-6\hat{j} + 8\hat{k})$  mN. If the particle's velocity is 1 km/s in the  $+x$  direction and is perpendicular to the magnetic field, find the magnetic field.

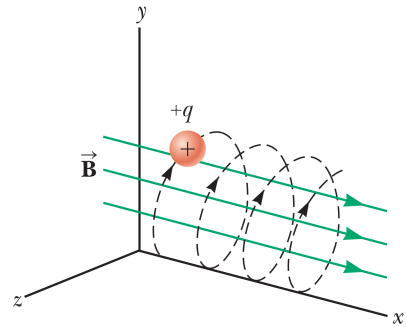
### Motion of a Charged Particle in a Uniform Magnetic Field

Let us examine the motion of a charged particle entering a region where there is a magnetic field of magnitude  $B$  directed into the page. Let us obtain the following quantities for the particle.

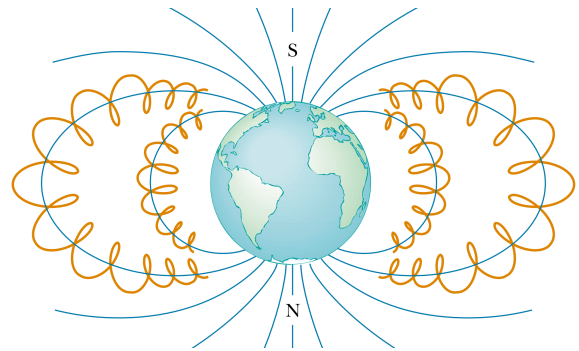
- Radius of the trajectory
- Period and frequency
- Angular speed



Let us examine the motion of a charged particle in a uniform magnetic field when it has a velocity component in the same direction as the magnetic field.



Let us examine how Earth's magnetic field affects cosmic rays.



**EXAMPLE 2:** A particle with charge  $q$  enters a region of uniform magnetic field of magnitude  $B$  with momentum of magnitude  $p$ . The particle's kinetic energy is  $K$ . Assuming that the motion is perpendicular to the magnetic field, find the period of the particle's circular motion in terms of the given quantities.

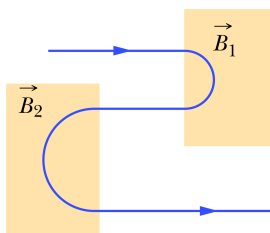
### Your turn

2. An electron moves in a circular path of radius 10 cm in a region of uniform magnetic field of magnitude  $1,20 \times 10^{-4}$  T. Given that the motion of the particle is perpendicular to the magnetic field, find the speed of the electron.

### Your turn

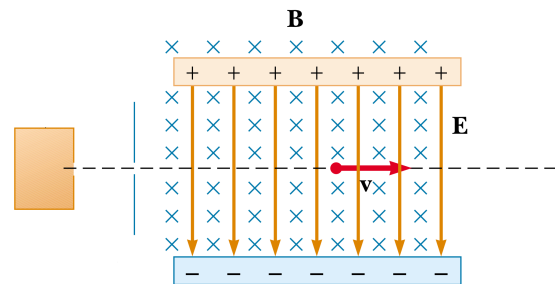
3. The figure shows the trajectory followed by a proton in two different regions of uniform magnetic field. In both regions, the proton follows a semicircular path.

- Compare the magnitudes of the magnetic fields in the regions.
- Determine the directions of the magnetic fields in the regions.
- Compare the time the proton spends in these two regions.

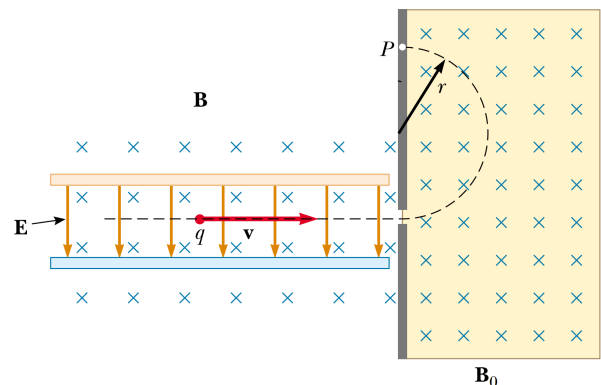


### Applications involving charged particles moving in a magnetic field

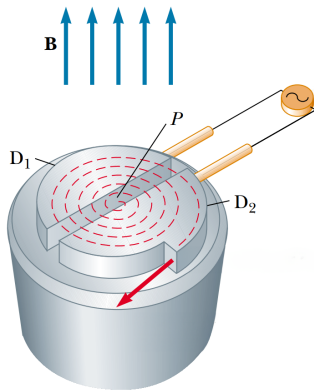
Let us define the **Lorentz force** and examine the operating principle of a **velocity selector**.



Let us examine how the **mass spectrometer** technique works.



Let us examine the working principle of the particle accelerator called the **Cyclotron**.

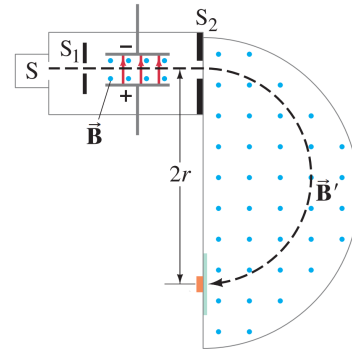


**EXAMPLE 3:** A stationary electron is accelerated in the  $+x$  direction through a potential difference  $V_1$ . As the accelerated electron passes between the parallel plates shown in the figure, it follows a straight trajectory with constant speed. The electric field between the parallel plates is in the  $+y$  direction. Find the magnetic field that must exist between the plates for the electron to follow a straight trajectory.



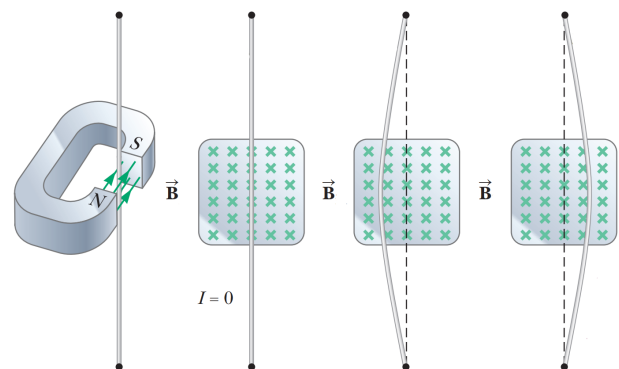
### Your turn

4. Of two ions with the same mass, one is doubly ionized and the other is triply ionized. Find the relationship between the radii of the trajectories of these two ions in a mass spectrometer.

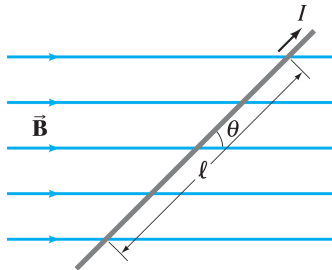


### Magnetic Force on a Current-Carrying Conductor

Let us examine how the magnetic force acts on a conductor carrying current and placed in a uniform magnetic field. Let us discuss the quantities on which this force depends and express this force mathematically.

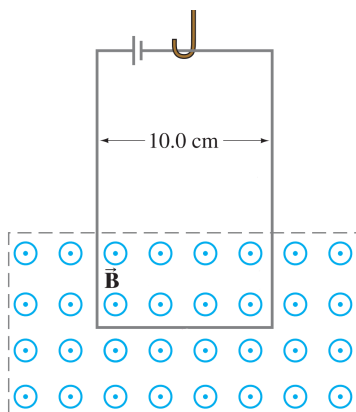


**EXAMPLE 4:** The magnitude of the magnetic force acting on the  $\ell = 20$  cm long portion of the wire shown, which carries a current of  $I = 0.40$  A, is  $F = 8 \times 10^{-5}$  N. Given that  $\theta = 30^\circ$ , find the magnitude of the uniform magnetic field in the region.

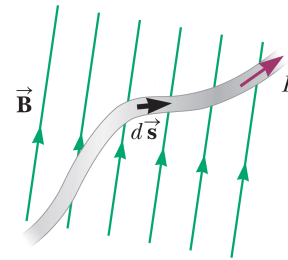


### Your turn

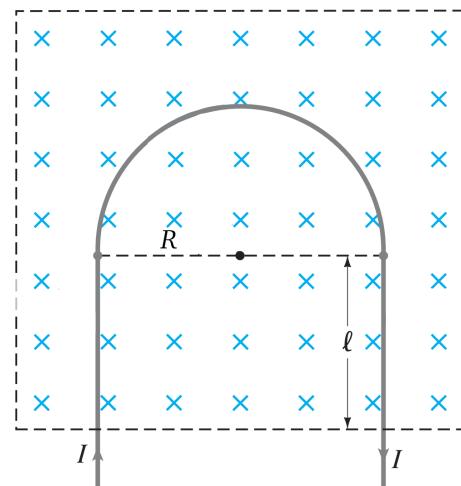
5. The magnitude of the net magnetic force acting on the circuit of negligible weight shown, which is attached to the end of a hook, is to be measured. The magnetic field is confined to a limited region, is directed out of the page, is uniform, and has a magnitude of 20 mT. If the current through the circuit is 0.3 mA, calculate the net magnetic force acting on the circuit.



Let us derive the magnetic force acting on a segment of wire carrying current and placed in a magnetic field.

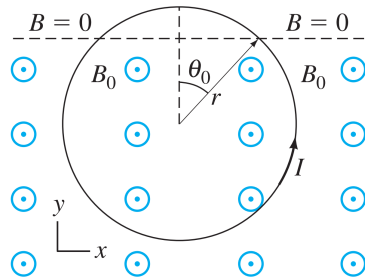


**EXAMPLE 5:** Find the net magnetic force acting on the wire shown in a magnetic field of magnitude  $B$ .

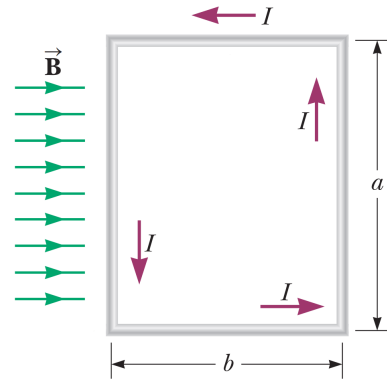


**Your turn**

6. Part of the circular wire loop shown lies in a region of uniform magnetic field. Derive the magnetic force acting on the wire in terms of the quantities given in the figure.

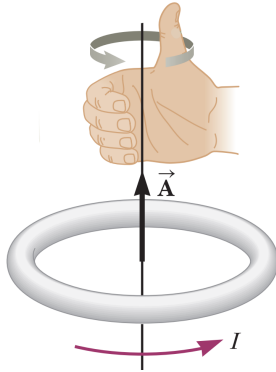
**Torque Acting on a Current Loop in a Uniform Magnetic Field**

Let us calculate the magnetic torque acting on a current loop placed in a uniform magnetic field. Let us discuss what this torque depends on and express it mathematically.



### Magnetic Dipole Moment

Let us discuss the concept of magnetic dipole moment and express it mathematically. Using this definition, let us derive again the torque acting on a current-carrying loop in a uniform  $\vec{B}$  magnetic field.

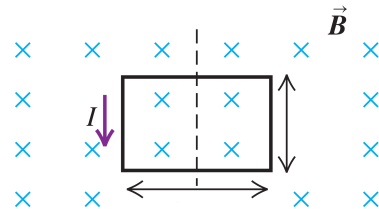


**NOTE:** As the number of turns of the wire increases, the magnetic dipole moment also increases.

Let us discuss the potential energy of a magnetic dipole placed in a magnetic field and write the related mathematical expression. Let us examine the magnitudes of the maximum and minimum potential energies.

**EXAMPLE 6:** A current of 2 A passes through a rectangular wire whose side lengths are 40 cm and 60 cm. At the location of the wire, the magnetic field is uniform, directed into the page, and has a magnitude of 0.5 T.

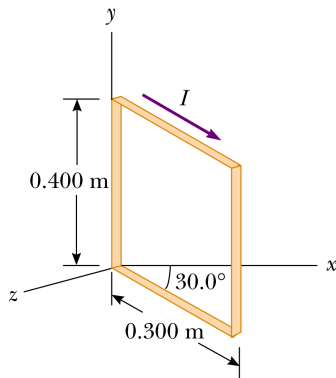
- Find the net magnetic force acting on the rectangle.
- Find the magnitude of the magnetic dipole moment of the rectangle.
- Find the magnitude of the magnetic torque acting on the rectangle.
- If the rectangle is rotated by  $30^\circ$  about the axis shown in the figure, with its left side moving into the page and its right side moving out of the page, find the magnitudes of the magnetic force and torque.
- Find the potential energies of the rectangle before and after it is rotated.



## Your turn

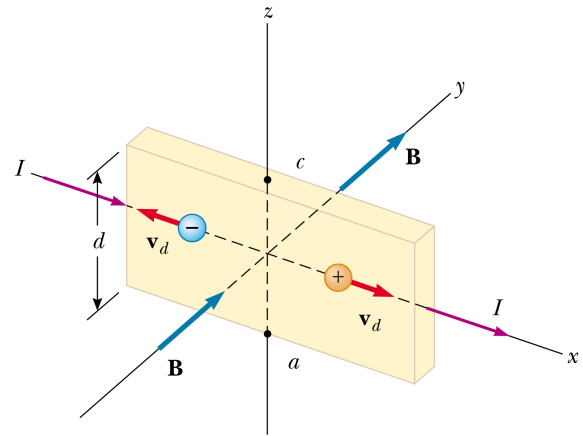
7. A current of 5 A passes through the rectangular wire loop whose side lengths are given in the figure. At the location of the wire, the magnetic field is uniform, directed along the  $+x$  axis, and has a magnitude of 0.2 T.

- Find the net magnetic force acting on the rectangle.
- Find the magnetic dipole moment of the rectangle.
- Find the magnetic torque acting on the rectangle.
- Find the potential energy of the rectangle.



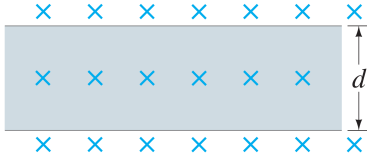
## Hall Effect

Let us examine the Hall effect that occurs when a current-carrying conductor is placed in a region containing a magnetic field. By examining the potential difference that forms between points *a* and *c*, let us define the **Hall voltage**.



**EXAMPLE 7:** A rectangular piece of metal has a width of  $d = 3 \text{ cm}$  and a thickness of  $500 \mu\text{m}$ . The metal carries a current of  $40 \text{ A}$  and is placed in a uniform magnetic field in a uniform magnetic field of magnitude  $0.2 \text{ T}$ , and a Hall voltage of  $6 \mu\text{V}$  is obtained.

- Find the magnitude of the Hall field inside the metal.
- Find the drift speed of the conduction electrons inside the metal.
- Find the free-electron density inside the metal.



## Answers

### Example questions

1  $\vec{F} = -6.4 \times 10^{-19} \hat{k} \text{ N}$

2  $T = \frac{\pi p^2}{KqB}$

3  $\vec{B} = \frac{v_2}{d} \sqrt{\frac{m_e}{2eV_1}} \hat{k}$

4  $B = 2.0 \times 10^{-3} \text{ T}$

5  $F_{\text{net}} = 2.0 \cdot I \cdot R \cdot B \hat{j}$

a.  $F_{\text{net}} = 0 \text{ N}$

b.  $\mu = 0.48 \text{ A} \cdot \text{m}^2$

6 c.  $\tau = 0 \text{ N} \cdot \text{m}$

d.  $F_{\text{net}} = 0 \text{ N}$   $\tau = 0.12 \text{ N} \cdot \text{m}$

e.  $U_{\text{initial}} = -0.24 \text{ J}$   $U_{\text{final}} = -0.21 \text{ J}$

a.  $E_H = 2.0 \times 10^{-4} \text{ V/m}$

7 b.  $v_d = 1.0 \times 10^{-3} \text{ m/s}$

c.  $n = 1.67 \times 10^{28} \text{ m}^{-3}$

### Your turn questions

1  $\vec{B} = (4.0\hat{j} + 3.0\hat{k}) \text{ T}$

2  $v = 2.1 \times 10^6 \text{ m/s}$

a.  $B_1 > B_2$

3 b.  $B_1 : \odot$   $B_2 : \otimes$

c.  $t_2 > t_1$

4  $\frac{r_1}{r_2} = \frac{3}{2}$

5  $F_{\text{net}} = 6.0 \times 10^{-7} \text{ N}$

6  $\vec{F} = -2IrB_0 \sin \theta_0 \hat{j}$

a.  $\vec{F}_{\text{net}} = 0$

b.  $\vec{\mu} = 0.30\hat{i} - 0.52\hat{k} \text{ A} \cdot \text{m}^2$

7 c.  $\vec{\tau} = -0.10\hat{j} \text{ N} \cdot \text{m}$

d.  $U = -0.060 \text{ J}$

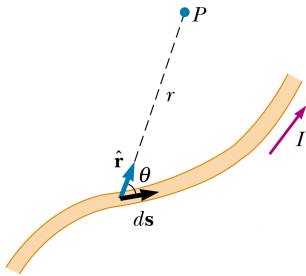
# CHAPTER 8

## SOURCES OF MAGNETIC FIELD



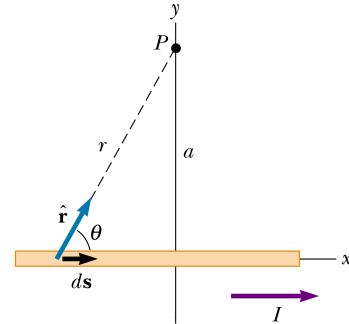
### Biot-Savart Law

Let's discuss the Biot-Savart law and write the relevant mathematical expression.



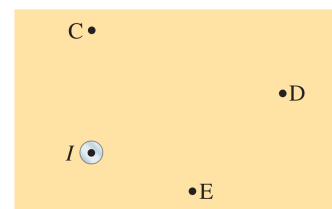
Let's write the magnetic field created by a moving charge using the Biot-Savart law.

Let's obtain the magnetic field at a point at a distance  $a$  from an infinitely long straight wire carrying current  $I$ .

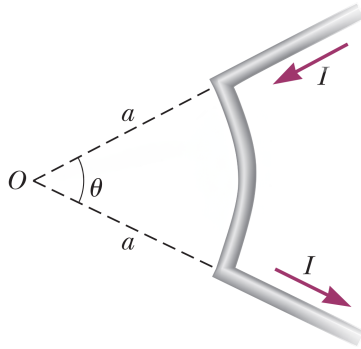


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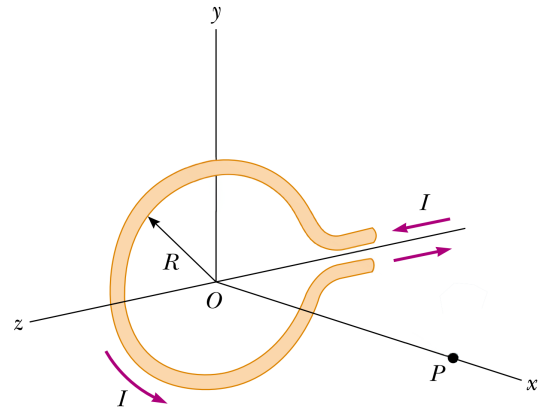
**EXAMPLE 1:** An infinitely long linear wire perpendicular to the page plane carries a current  $I$  out of the page. Find the directions of the magnetic fields at points  $C$ ,  $D$ , and  $E$  and compare their magnitudes.



Let's obtain the magnetic field created at point  $O$  by the wire segment shown, through which current  $I$  flows, using the Biot-Savart law.

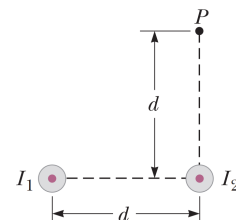


**EXAMPLE 2:** Obtain the magnetic field generated by the circular wire shown in the figure, carrying a current  $I$ , at point  $P$ . Discuss what the magnetic field would be if point  $P$  were moved to the origin.

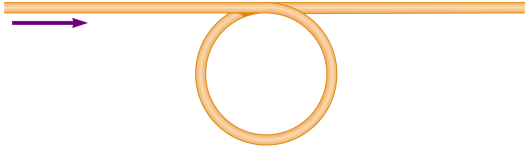


### Resultant magnetic field

Let us examine the principle of superposition for the magnetic field through the following example.

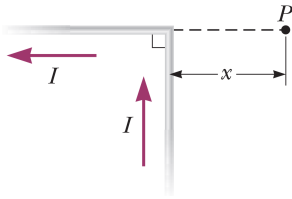


**EXAMPLE 3:** The radius of the loop part of the infinitely long wire shown in the figure, carrying a current  $I$ , is  $R$ . Accordingly, find the magnetic field at the center of this part.

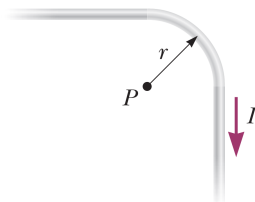


### Your turn

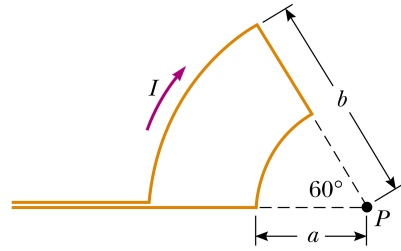
1. Obtain the magnetic field generated by the infinitely long wire shown in the figure, carrying a current  $I$ , at point  $P$ .



2. Obtain the magnetic field generated at point  $P$  by the infinitely long wire through which a current  $I$  passes as shown in the figure.

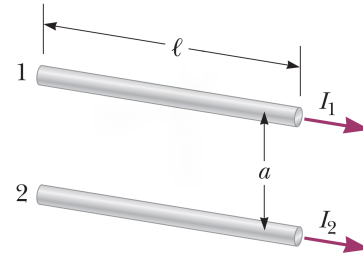


3. Obtain the magnetic field generated at point  $P$  by the wire through which a current  $I$  passes as shown in the figure.

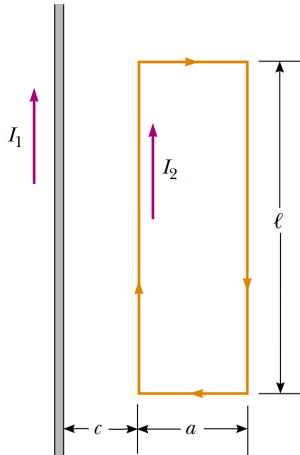


### Magnetic Forces Between Two Current-Carrying Wires

Let's obtain the magnetic force that two current-carrying wires exert on each other. Let's discuss the relationship between the directions of the forces and the directions of the currents.



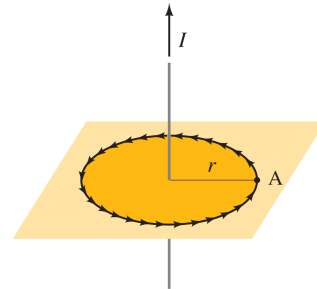
**EXAMPLE 4:** Find the net magnetic force exerted by an infinitely long straight wire carrying current  $I_1$  on a rectangular current loop carrying current  $I_2$ . The straight wire is positioned parallel to the long sides of the rectangular loop.



## Ampère's Law

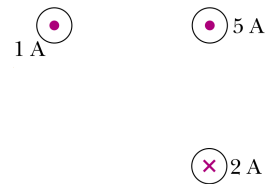
Let's explain Ampère's law and write the relevant mathematical expression.

Let's obtain the magnetic field at point A, which is at a distance  $r$  from an infinitely long wire carrying current  $I$ , using Ampère's law.



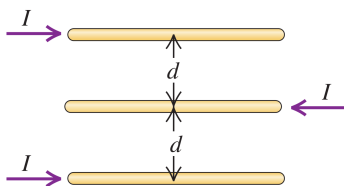
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Let's discuss the results of Ampère's law through examples.



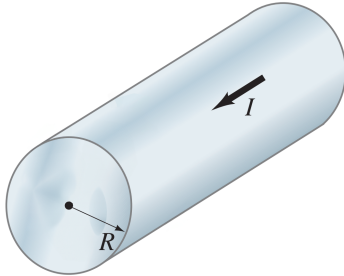
## Your turn

4. Find the magnetic forces exerted on each other by the sufficiently long, straight wires carrying current  $I$  in the same plane as shown in the figure.

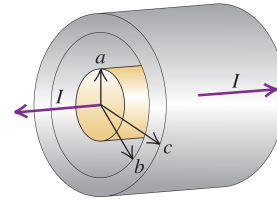


**EXAMPLE 5:** An infinitely long cylindrical straight wire carrying a current  $I$  is shown in the figure.

- Find the magnetic field intensities for the regions inside and outside the wire.
- Plot the magnetic field intensity as a function of the distance from the center of the wire.



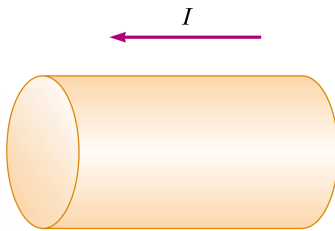
**EXAMPLE 6:** Two nested coaxial cables carry currents in opposite directions as shown in the figure. The dimensions of the cables are shown in the figure. Find the magnetic field intensities in all regions.



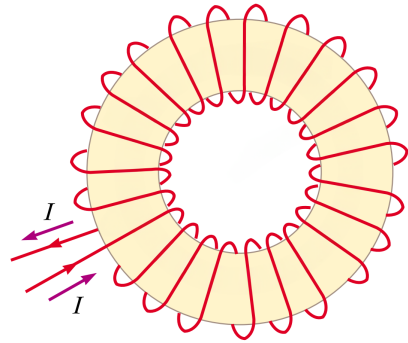
**Your turn**

5. An  $I$  current passes through the conducting cylindrical wire of radius  $R$  shown in the figure. Given that  $\alpha$  is a constant, the current density in the wire is given as  $J(r) = \alpha r$  depending on the distance from the center.

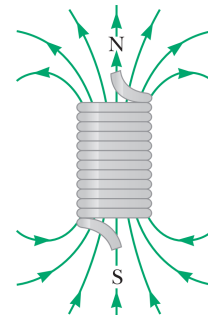
- Find the constant  $\alpha$ .
- Find the magnetic field intensity inside the wire as a function of the distance from the center.
- Find the magnetic field intensity outside the wire as a function of the distance from the center.

**Magnetic Field of a Toroid**

Let's obtain the magnetic field inside a toroid carrying a current  $I$ .

**Magnetic Field of a Solenoid**

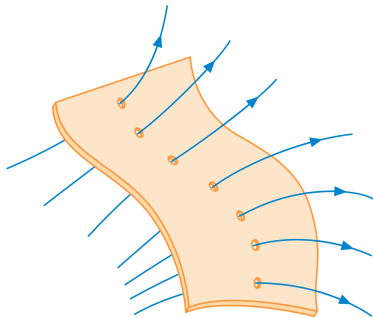
Let us examine the magnetic field created by a solenoid. Let us discuss the similarity of the magnetic field between a solenoid and a bar magnet. Let us obtain the magnetic field intensity inside an infinitely long solenoid.



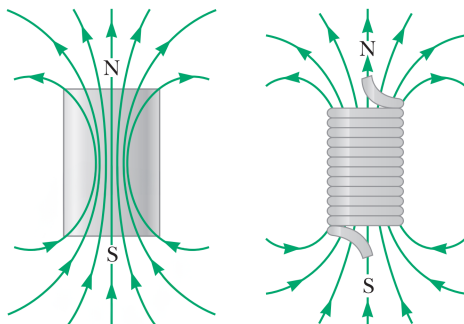
**EXAMPLE 7:** A thin solenoid with a length of 10 cm has a total of 800 turns. Given that the magnetic field intensity near the center of the solenoid is  $2.0 \times 10^{-2}$  T, find the current flowing through the solenoid.

### Gauss's Law in Magnetism

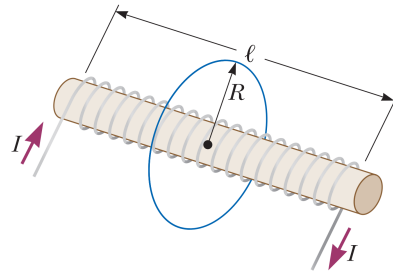
Let us define the concept of magnetic flux. Let us write the relevant mathematical expression for magnetic flux.



Let us discuss Gauss's law for magnetism and write the relevant mathematical expression.

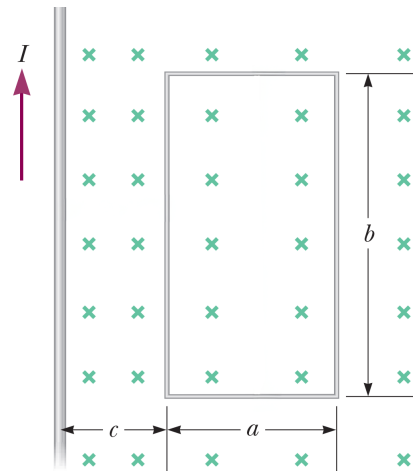


**EXAMPLE 8:** A solenoid shown in the figure with length  $\ell$  and radius  $r$  has  $N$  turns. While a current  $I$  is flowing through the solenoid, find the magnetic flux in the circular region of radius  $R$  shown in the figure.



### Your turn

6. A rectangular loop of width  $a$  and length  $b$  is placed next to a straight wire carrying current as shown in the figure. The straight wire is positioned parallel to the long sides of the rectangular loop. Given that the current passing through the wire is  $I$ , find the magnetic flux through the loop.

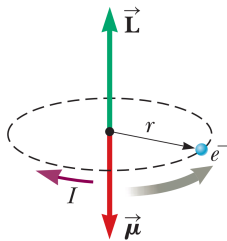


## Magnetism in Matter

Let us discuss how the material medium can affect the magnetic field.

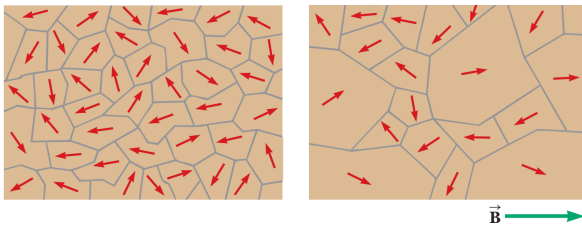
### Magnetic Field of Atoms

Let us discuss how the magnetic field of an atom is formed. Let us examine the particles and interactions that can contribute to the magnetic field of the atom.



### Ferromagnetism

Let us examine the concept of ferromagnetism. Let us discuss how materials with this property behave in a magnetic field.



### Paramagnetism and Diamagnetism

Let us examine the concepts of paramagnetism and diamagnetism. Let us discuss how materials with these properties behave in a magnetic field.

## Answers

### Example questions

- 1 Magnitudes:  $B_E > B_C > B_D$   
Refer to the video for directions.
- 2  $\vec{B} = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}} \hat{i}$   
When point P is moved to the origin:  $\vec{B} = \frac{\mu_0 I}{2R} \hat{i}$
- 3  $\vec{B} = \frac{\mu_0 I}{2R} \left(1 + \frac{1}{\pi}\right) \otimes$
- 4  $\vec{F}_{net} = \frac{\mu_0 I_1 I_2 \ell a}{2.0 \pi c(c+a)}$ , to the left
- 5 a.  $B = \frac{\mu_0 I r}{2\pi R^2}$  and  $B = \frac{\mu_0 I}{2\pi r}$   
b. Go to video.
- 6 For  $r < a$ :  $B = \frac{\mu_0 I r}{2\pi a^2}$   
For  $a \leq r \leq b$ :  $B = \frac{\mu_0 I}{2\pi r}$   
For  $b < r < c$ :  $B = \frac{\mu_0 I}{2\pi r} \left(\frac{c^2 - r^2}{c^2 - b^2}\right)$   
For  $r \geq c$ :  $B = 0$
- 7  $I = 2.0 \text{ A}$
- 8  $\Phi = \frac{\mu_0 N I \pi r^2}{\ell}$

### Your turn questions

- 1  $\vec{B} = \frac{\mu_0 I}{4\pi x} \otimes$
- 2  $\vec{B}_p = \frac{\mu_0 I}{8r} \left(1 + \frac{4}{\pi}\right) \otimes$
- 3  $\vec{B} = \frac{\mu_0 I}{12} \left(\frac{1}{a} - \frac{1}{b}\right) \odot$
- 4 Top wire:  $\vec{F}_1 = \frac{\mu_0 I^2}{4\pi d} \hat{j}$   
Middle wire:  $\vec{F}_2 = 0$   
Bottom wire:  $\vec{F}_3 = -\frac{\mu_0 I^2}{4\pi d} \hat{j}$
- 5 a.  $\alpha = \frac{3I}{2\pi R^3}$   
b.  $B = \frac{\mu_0 I r^2}{2\pi R^3}$   
c.  $B = \frac{\mu_0 I}{2\pi r}$
- 6  $\Phi_B = \frac{\mu_0 I b}{2\pi} \ln\left(\frac{c+a}{c}\right)$

# CHAPTER 9

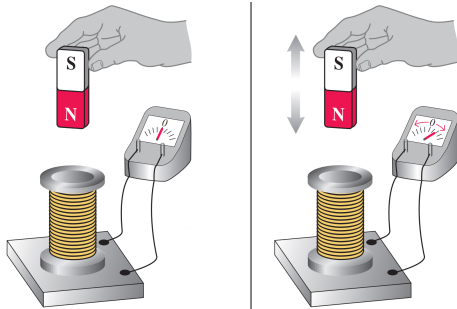
## ELECTROMAGNETIC INDUCTION



watch video

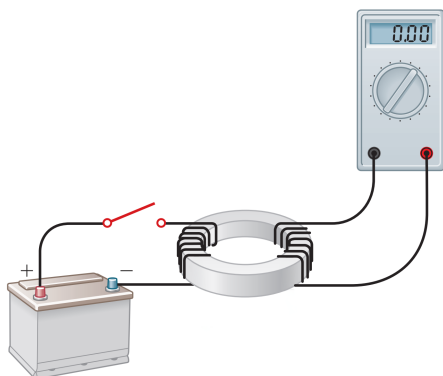
### Faraday's Law of Induction

Let us examine the concept of induced current. Let us discuss the relationship between the magnetic field and the electric field, and write Faraday's law of induction mathematically.

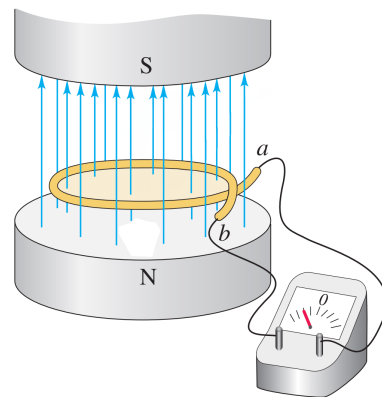


Let us examine under which time-varying parameters an induced emf is obtained.

Let us examine Faraday's experiment and discuss its results.

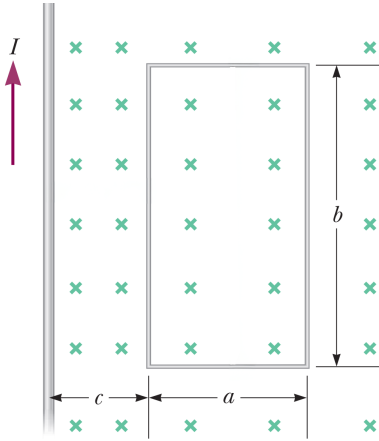


**EXAMPLE 1:** An insulated conducting wire is bent into a loop and placed in a magnetic field region, as shown, with the plane of the loop perpendicular to the magnetic field. The area enclosed by the wire is  $0.2 \text{ m}^2$ . Find the potential difference between the ends of the wire while the magnitude of the magnetic field is increased from  $0.1 \text{ T}$  to  $0.3 \text{ T}$  in  $10$  seconds.

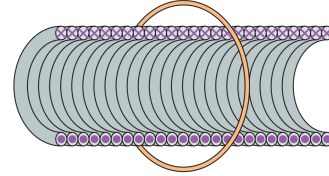


**Your turn**

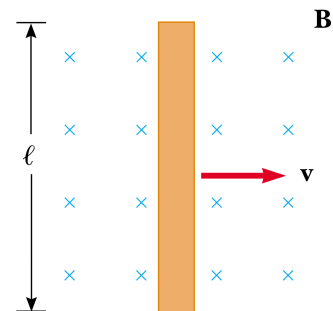
1. A rectangular loop of width  $a$  and length  $b$  is placed next to a long straight wire carrying current as shown in the figure. The straight wire is oriented parallel to the longer sides of the rectangular loop. The time-dependent current through the wire is given by  $I(t) = I_0 \sin \omega t$ . Find the induced emf in the loop in terms of the given quantities.



2. A solenoid of radius  $r$ , number of turns  $N$ , and length  $L$  carries a time-dependent current  $I(t) = \alpha t$  ( $\alpha > 0$ ), as shown in the figure. A conducting loop of radius  $R$  surrounds the solenoid. The central axes of the solenoid and the loop coincide. Find the induced emf in the loop in terms of the given quantities.

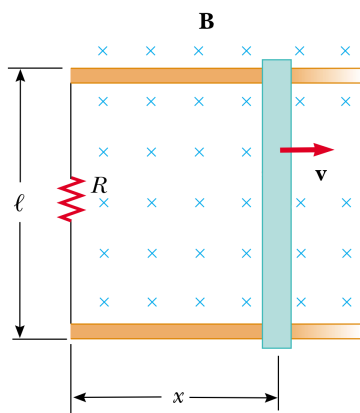
**Motional emf**

Let us derive the potential difference that appears between the ends of a conductor moving in a uniform magnetic field.

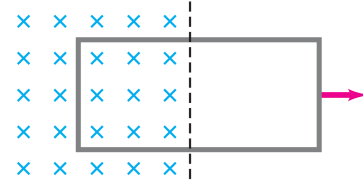


Let us examine the circuit setup shown in the figure and complete the following steps.

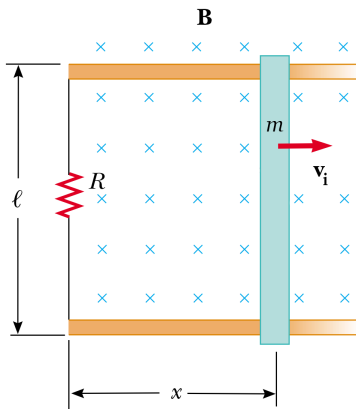
- Let us derive the induced emf between the ends of the moving conductor.
- Let us obtain the same emf from the change in area as well.
- Let us determine the current through the circuit.
- Let us find the magnetic force acting on the moving conductor.
- Let us compare the power dissipated in the resistor with the mechanical power required to pull the conductor.



**EXAMPLE 2:** A rectangular wire loop with resistance  $R$  has a short side of length  $a$  and a long side of length  $b$ . The loop is pulled with speed  $v$  from a region where there is a uniform magnetic field of magnitude  $B$  into a region where there is no magnetic field. Find the current through the loop and its direction.



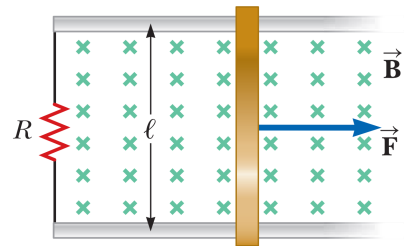
**EXAMPLE 3:** In a frictionless rail system, a conducting bar of mass  $m$  is launched as shown with speed  $v_i$  at time  $t = 0$ . Derive the equation for the velocity of the conducting bar as a function of time.



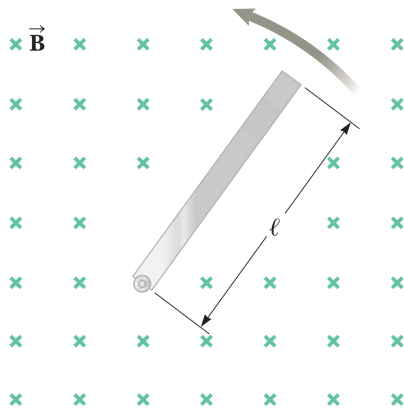
### Your turn

3. In a frictionless rail system whose resistance is negligible, a conducting bar is pulled as shown with a constant speed of  $v = 120 \text{ cm/s}$  by a force of magnitude  $F$  in a region where there is a uniform magnetic field of magnitude  $B = 600 \text{ mT}$ . The magnetic field is directed into the page, and the conducting bar is pulled along the plane of the page. Given that  $\ell = 150 \text{ cm}$  and  $R = 100 \Omega$ .

- Find the induced emf between the ends of the conductor.
- Find the current through the resistor.
- Find the magnitude of the force  $F$ .
- Find the power dissipated by the resistor.

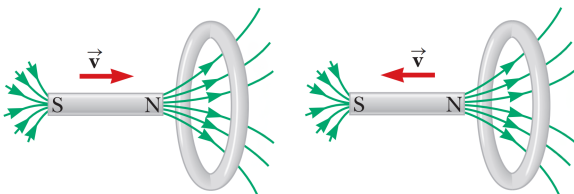
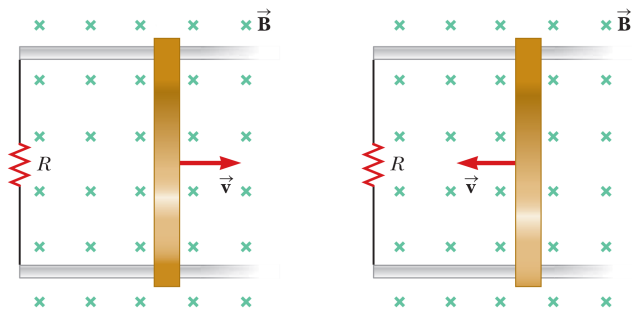


Let us derive the potential difference that appears between the ends of a conductor of length  $\ell$  when it is rotated with angular speed  $\omega$  as shown in a uniform magnetic field.

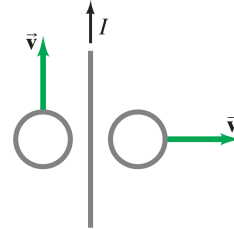


### Lenz's Law

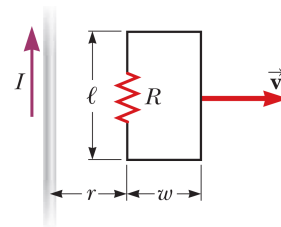
Let us discuss Lenz's law and determine the directions of the induced currents for the following examples.



**EXAMPLE 4:** Two conducting loops are moved around a sufficiently long current-carrying wire as shown in the figure. Discuss whether induced currents are formed in the loops, and determine the direction of the induced currents if they are formed.

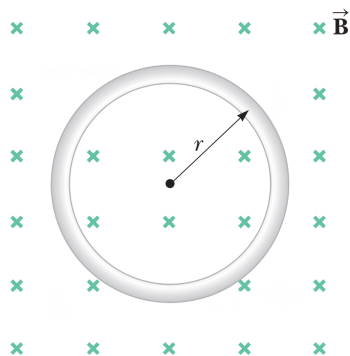


**EXAMPLE 5:** A rectangular loop of width  $w$  and length  $l$  is placed next to an infinitely long straight current-carrying wire as shown in the figure. The straight wire is positioned parallel to the longer sides of the rectangular loop. The current through the wire is constant and equal to  $I$ . As the loop is pulled with speed  $v$ , find the current through the resistor  $R$  for the position shown in the figure.



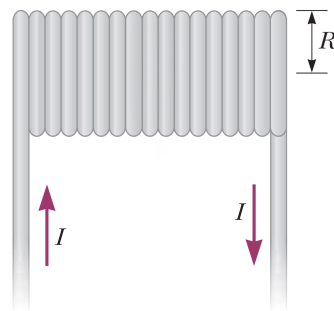
## General Form of Faraday's Law

Let us discuss the relationship between the change in magnetic flux and the electric field. Let us write the general form of Faraday's law for the case in which no conductor is present.

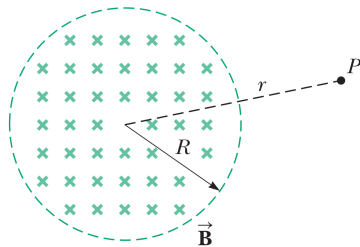


**EXAMPLE 6:** A solenoid with turn density  $n$  and radius  $R$  is shown in the figure. The time-dependent current through the coil is given by  $I(t) = I_0 \sin \omega t$ .

- Find the magnitude of the induced electric field at a point outside the solenoid, at a distance  $r > R$  from the central axis.
- Find the magnitude of the induced electric field at a point inside the solenoid, at a distance  $r < R$  from the central axis.

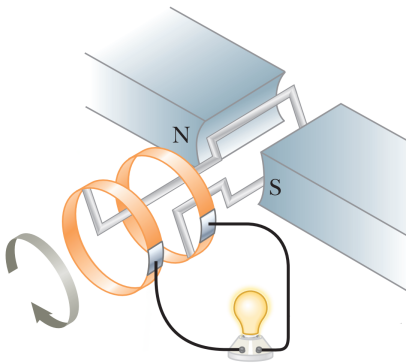


**EXAMPLE 7:** There is a time-varying magnetic field region in the area shown by dashed lines in the figure. In this region, the magnetic field magnitude is given by  $B(t) = 2t^2 - t + 3$ . Find the magnitude of the induced electric field at point P outside the region as a function of time and indicate its direction.

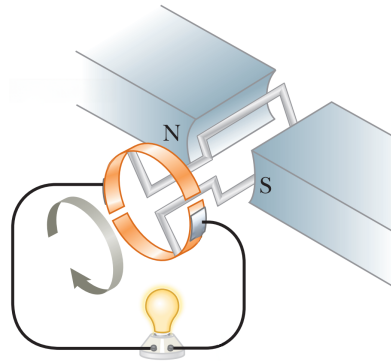


## Generators and Motors

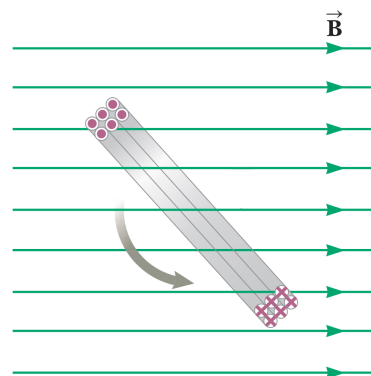
Let us examine how an alternating current generator works.



Let us examine how a direct current generator works.



Let us obtain a mathematical expression for the induced emf generated by an alternating current source.

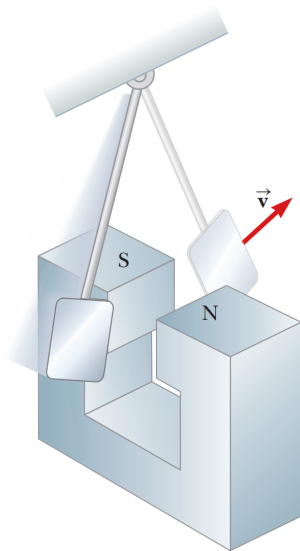


**EXAMPLE 8:** An alternating voltage is obtained by rotating a conducting square loop with a side length of 40 cm in a region of uniform magnetic field. The magnetic field strength is 400 mT, and the loop is rotated at a frequency of 50 Hz.

- Find the time-dependent equation for the magnetic flux through the loop.
- Find the emf induced in the loop.

## Eddy Currents

Let us examine the concept of eddy currents. Let us discuss why a conductor loses energy faster than usual while oscillating near a magnet.



## Answers

### Example questions

1  $\mathcal{E} = 4.0 \times 10^{-3} \text{ V}$

2  $I = \frac{Bav}{R}$

Current direction: Clockwise

3  $v(t) = v_i e^{-\frac{B^2 \ell^2}{mK} t}$

Left loop: No current is induced.

4 Right loop: A current is induced, and its direction is clockwise.

5  $i = \frac{\mu_0 I l v w}{2\pi R r(r+w)}$

6 a.  $E = \frac{\mu_0 n I_0 R^2 \omega}{2r} \cos \omega t$

b.  $E = \frac{\mu_0 n I_0 r \omega}{2} \cos \omega t$

7  $E = \frac{(4.0t-1.0)R^2}{2r} \text{ V/m}$ , its direction is counterclockwise.

8 a.  $\Phi(t) = 0.064 \cos(100\pi t) \text{ Wb}$

b.  $\mathcal{E}(t) = 6.4\pi \sin(100\pi t) \text{ V}$

### Your turn questions

1  $\mathcal{E} = -\frac{\mu_0 I_0 b \omega}{2\pi} \ln\left(\frac{c+d}{c}\right) \cos \omega t$

2  $\mathcal{E} = -\frac{\mu_0 N \pi r^2 \alpha}{L}$

3 a.  $\mathcal{E} = 1.08 \text{ V}$

b.  $I = 1.08 \times 10^{-2} \text{ A}$

c.  $F = 9.7 \times 10^{-3} \text{ N}$

d.  $P = 1.2 \times 10^{-2} \text{ W}$

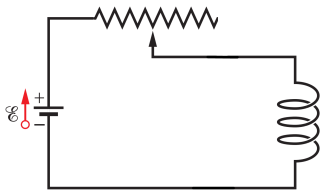
# CHAPTER 10

## INDUCTANCE



### Self-Induction and Inductance

Let's define the concept of self-induction using a simple circuit arrangement. Let's mathematically express the self-induced emf for the following circuit.

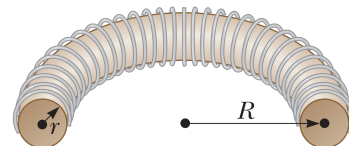


- Let's define the concept of inductance and write the corresponding mathematical expression.
- Let's examine the unit of inductance.
- Let's examine the inductor as a circuit element.

**EXAMPLE 1:** A solenoid whose length is sufficiently greater than its radius has  $N$  turns, length  $\ell$ , and radius  $r$ .

- Find the inductance of the solenoid.
- If the current through the solenoid is  $I(t) = I_0 \sin \omega t$ , find the self-induced emf across the solenoid.

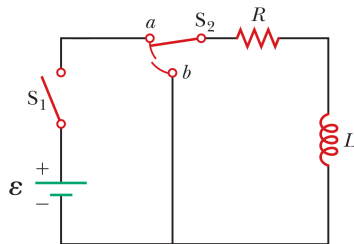
**EXAMPLE 2:** A toroid with major radius  $R$  and minor radius  $r$  has  $N$  turns wound around it. For  $R \gg r$ , find the inductance of this toroid. (Assume that the magnetic field magnitude is uniform over any cross section of the toroid.)



## RL Circuits

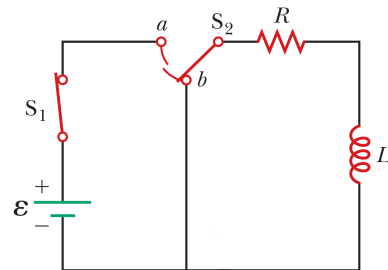
Let us discuss what happens after closing switch  $S_1$  in the RL circuit shown. Then let us carry out the following steps.

- Obtain the time-dependent equation for the current in the circuit.
- Obtain the time-dependent equation for the potential difference across the inductor.
- Let us define the concept of the **time constant** and examine its unit.
- Draw the graphs of the current and the time derivative of the current as functions of time.



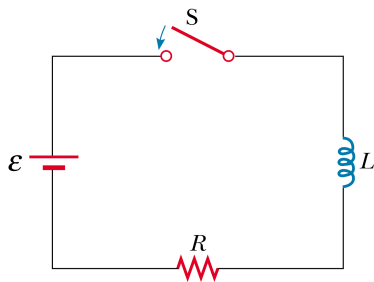
Let us discuss what happens in the RL circuit shown when switch  $S_1$  is first closed, the circuit is allowed to reach steady state, and then  $S_1$  is moved from  $a$  to  $b$ .

- Obtain the time-dependent equation for the current through the resistor.
- Obtain the time-dependent equation for the potential difference across the inductor.
- Draw the current-versus-time graph for the resistor.



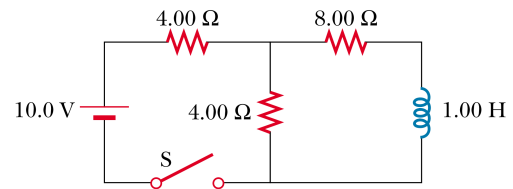
**EXAMPLE 3:** In the circuit shown,  $\mathcal{E} = 12 \text{ V}$ ,  $L = 10 \text{ mH}$ , and  $R = 5 \Omega$  are given. The switch is closed at  $t = 0$ .

- Find the time constant of the circuit.
- Obtain the time-dependent equation for the current in the circuit.
- Find the current in the circuit after waiting for a long enough time.
- Find the time required for the current to reach half of its maximum value.



**EXAMPLE 4:** A circuit consisting of resistors, a source, and an inductor is shown in the figure.

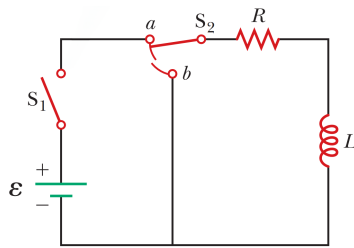
- Find the currents through all branches immediately after the switch is closed.
- Find the currents in all branches after waiting for a long enough time.



## Energy in a Magnetic Field

Let us examine the energy stored in the inductor after switch  $S_1$  is closed in the RL circuit shown.

- Find the rate at which the inductor stores energy.
- Find the energy that will be stored in the inductor after waiting for a long enough time.
- Treating the inductor as an ideal solenoid, find the magnetic energy density per unit volume inside the coil.



**EXAMPLE 5:** When a current  $I$  passes through a coil with  $N$  turns, a magnetic flux  $\Phi_B$  is produced through each loop. Find the magnetic field energy of this coil in terms of the given quantities.

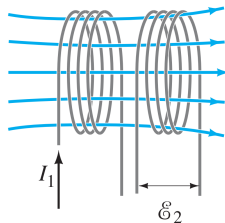
**EXAMPLE 6:** Find the magnetic energy density at the center of a circular wire of radius  $R$  carrying a current  $I$  in terms of the given quantities.

**EXAMPLE 7:** A circuit consisting of a 12 V battery, a switch, a resistor, and an inductor is constructed. It is given that  $R = 6 \, \Omega$  and  $L = 2 \, \text{H}$ .

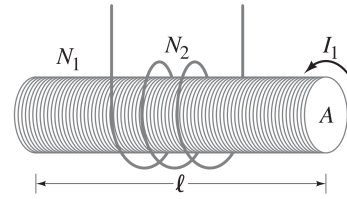
- Find the energy stored in the inductor after the switch is closed and a sufficiently long time has passed.
- Find the energy stored in the inductor after one time constant has elapsed since the switch was closed.

## Mutual Inductance

Let us examine the concept of mutual inductance for two side-by-side coils. Let us derive the relevant mathematical expressions.



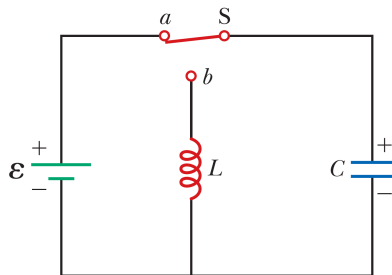
**EXAMPLE 8:** Find the mutual inductance of the two-coil arrangement whose properties are given in the figure.



**EXAMPLE 9:** Two sufficiently long concentric solenoids of the same length have radii  $r_1 > r_2$ . The solenoids have  $n_1$  and  $n_2$  turns per unit length. Accordingly, find the mutual inductance per unit length of this arrangement.

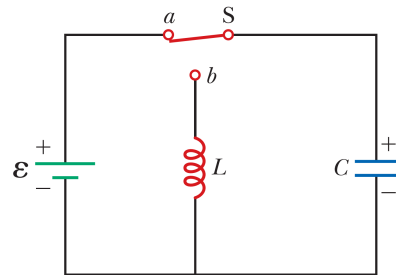
## Oscillations in LC Circuits

Let us discuss the oscillations in LC circuits and derive the relevant mathematical results.



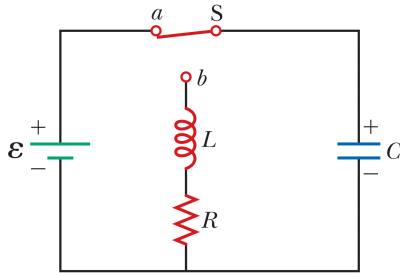
**EXAMPLE 10:** In the circuit shown,  $\mathcal{E} = 10 \text{ V}$  and while the switch is in position  $a$ , the capacitor becomes fully charged; then the switch is moved to position  $b$ . The circuit parameters are given as  $L = 2 \text{ mH}$  and  $C = 8 \text{ mF}$ .

- Find the frequency of oscillation in the LC circuit.
- Find the maximum value of the charge stored in the capacitor.
- Find the maximum value of the current through the circuit.



## RLC Circuits

Let us examine RLC circuits and obtain the related mathematical results.



## Answers

### Example Problems

1

$$a. L = \frac{\mu_0 N^2 \pi r^2}{\ell}$$

$$b. \varepsilon = -\frac{\mu_0 N^2 \pi r^2 I_0 \omega \cos \omega t}{\ell}$$

2

$$L = \frac{\mu_0 N^2 r^2}{2R}$$

3

$$a. \tau = 2.0 \text{ ms}$$

$$b. i(t) = 2.4(1 - e^{-500t}) \text{ A}$$

$$c. I = 2.4 \text{ A}$$

$$d. t = 1.4 \text{ ms}$$

4

$$a. i_{\text{main}} = 1.25 \text{ A} \quad i_{\text{middle}} = 1.25 \text{ A} \quad i_{\text{right}} = 0.0 \text{ A}$$

$$b. i_{\text{main}} = 1.5 \text{ A} \quad i_{\text{middle}} = 1.0 \text{ A} \quad i_{\text{right}} = 0.50 \text{ A}$$

5

$$U_B = \frac{1}{2} N \Phi_B I$$

6

$$u_B = \frac{\mu_0 I^2}{8R^2}$$

7

$$a. U = 4.0 \text{ J}$$

$$b. U = 1.6 \text{ J}$$

8

$$M = \frac{\mu_0 N_1 N_2 A}{\ell}$$

9

$$m = \mu_0 n_1 n_2 \pi r_2^2$$

10

$$a. f = 40 \text{ Hz}$$

$$b. Q_{\text{max}} = 8.0 \times 10^{-2} \text{ C}$$

$$c. I_{\text{max}} = 20 \text{ A}$$

# CHAPTER 12

## ELECTROMAGNETIC WAVES



### Displacement Current

Let us examine a case in which the form of Ampère's law we wrote earlier is insufficient. Let us write the most general form of the law.

**EXAMPLE 1:** A parallel-plate capacitor with area  $A$  is being charged by a conduction current  $I$ . The magnitude of the uniform electric field formed between the capacitor plates is  $E$ , and the charge on a plate at any instant is  $q$ .

Show that the displacement current between the plates is equal to the conduction current  $I$  in the wires.

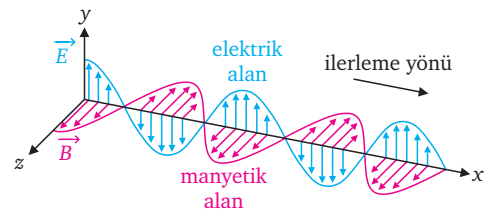
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### Maxwell's Equations

- Let us write Maxwell's equations of electromagnetism.
  - Gauss's law
  - Gauss's law for magnetism
  - Faraday's law
  - Ampère–Maxwell law
- Let us discuss what these equations express.
- Let us define the Lorentz force and discuss how the equations are used.
- Let us examine what form these equations take for a region of space containing no charge or current.

### Plane Electromagnetic Waves

Let us explain what plane electromagnetic waves are. Let us examine the following figure showing the state of an electromagnetic wave at a particular instant.



Let us write the wave equations expressing the electric and magnetic field components for electromagnetic waves, along with the solutions of these equations.

**EXAMPLE 2:** A sinusoidal electromagnetic wave with frequency 50 MHz propagates in the  $+x$  direction in vacuum. At a certain position and instant, the electric field has reached its maximum value of 600 N/C and is directed along the  $+y$  axis. Take the speed of light in vacuum as  $c = 3.0 \times 10^8$  m/s.

- Determine the wavelength and period of the wave.
- At this position and instant, calculate the magnetic field vector, including its magnitude and direction.

Let us define the concept of a linearly polarized wave. Let us express the ratio between the magnitudes of the electric and magnetic field components of electromagnetic waves.

### Energy Carried by Electromagnetic Waves

Let us define the Poynting vector. Let us obtain the unit of this quantity.

Let us obtain the intensity of an electromagnetic wave in terms of the magnitudes of the electric and magnetic field components of the wave.

**EXAMPLE 3:** A transmitting antenna at a military radar station, radiating equal power in all directions, emits electromagnetic waves with an average power of  $4.0 \times 10^1$  kW into the surroundings. It is assumed that the antenna behaves as a direction-independent point source and radiates all of its energy as spherical electromagnetic waves. Calculate the maximum electric and magnetic field magnitudes produced by these waves, which reach the surface of an unmanned aerial vehicle (UAV) flying a reconnaissance mission at a distance of  $2.0 \times 10^1$  km from this transmitting antenna and normally incident on its surface. Take the speed of light in vacuum as  $c = 3.0 \times 10^8$  m/s and the magnetic permeability of vacuum as  $\mu_0 = 4.0\pi \times 10^{-7}$  T · m/A.

- What is the magnitude of the maximum electric field on the UAV surface in V/m?
- What is the magnitude of the maximum magnetic field on the UAV surface in T?

Let us obtain the relationship between the intensity of an electromagnetic wave and its power through a given cross-sectional area.

## Momentum and Radiation Pressure

Let us write the momentum transferred to a surface by electromagnetic waves that are completely absorbed by the surface. Then let us obtain the pressure produced by these waves on the surface.

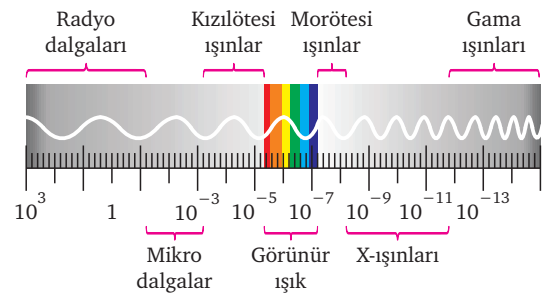
Let us discuss what happens when the waves are completely reflected from the surface.

**EXAMPLE 4:** A laser device used in a laboratory emits a beam with an average power of 6.0 mW. This laser beam forms a circular cross section with a diameter of 2.0 mm on the surface of a plate on which it is normally incident. The plate surface reflects %80 of the incident electromagnetic wave. Take the speed of light in vacuum as  $c = 3.0 \times 10^8$  m/s.

- Calculate the magnitude of the average Poynting vector on the plate surface ( $S_{\text{avg}}$ ).
- Calculate the radiation pressure exerted by the laser beam on the plate ( $P_{\text{avg}}$ ).

Let us examine the spectrum of electromagnetic waves using the table.

| Wave type     | Wavelength range |
|---------------|------------------|
| Radio waves   | $> 1$ m          |
| Microwaves    | 1 mm – 1 m       |
| Infrared      | 750 nm – 1 mm    |
| Visible light | 400 nm – 750 nm  |
| Ultraviolet   | 10 nm – 400 nm   |
| X-rays        | 0.01 nm – 10 nm  |
| Gamma rays    | $< 0.01$ nm      |



Let us examine the colors and wavelengths in the visible region.

| Light color | Wavelength range |
|-------------|------------------|
| Violet      | 400 nm – 450 nm  |
| Blue        | 450 nm – 495 nm  |
| Green       | 495 nm – 570 nm  |
| Yellow      | 570 nm – 590 nm  |
| Orange      | 590 nm – 620 nm  |
| Red         | 620 nm – 750 nm  |

## Production and Spectrum of Electromagnetic Waves

Let us discuss how electromagnetic waves are produced in an antenna.

## Exercises and Problems

1. A laser source emits a sinusoidal electromagnetic wave propagating in the  $-y$  direction in vacuum. The wavelength of this wave is  $\pi \times 10^{-6}$  m, the electric field vector is parallel to the  $x$ -axis, and its maximum amplitude is given as  $E_{\max} = 3.0 \times 10^6$  V/m. Take the speed of light in vacuum as  $c = 3.0 \times 10^8$  m/s.

- Find the wave number ( $k$ ) and angular frequency ( $\omega$ ) of this wave.
- Write the vector equations of the electric field ( $\vec{E}$ ) and magnetic field ( $\vec{B}$ ) as functions of position and time.

2. The electric field of an electromagnetic wave propagating in the  $+z$  direction in vacuum is of the form

$$\vec{E}(z, t) = E_0 \cos(kz - \omega t) \hat{i}. \text{ Here, take } E_0 = 6.0 \times 10^2 \text{ V/m, } \mu_0 = 4.0\pi \times 10^{-7} \text{ T} \cdot \text{m/A, and } c = 3.0 \times 10^8 \text{ m/s.}$$

- Obtain the time-dependent expression for the Poynting vector ( $\vec{S}$ ) and write its SI unit.
- Express the average intensity of the wave in terms of  $E_0$ . Calculate its numerical value.
- The wave is completely absorbed by a surface of area  $A = 2.0 \text{ m}^2$  perpendicular to the direction of propagation. Find the average power transferred to the surface.

3. The intensity of an electromagnetic wave beam propagating in vacuum is given as  $I = 1.2 \times 10^3 \text{ W/m}^2$ . This wave beam is normally incident on a plane surface of area  $A = 5.0 \text{ m}^2$  perpendicular to the direction of propagation. Assume that the wave reaches the surface for a duration of  $\Delta t = 10 \text{ s}$ . Take the speed of light in vacuum as  $c = 3.0 \times 10^8 \text{ m/s}$ .

- Calculate the total energy carried by the wave beam to the surface during this time interval.
- Assume that the surface completely absorbs the wave beam. In this case, calculate the magnitude of the total momentum transferred to the surface.
- Find the radiation pressure exerted on the surface in the case of complete absorption.
- Now assume that the wave beam is completely reflected from the surface. In this case, calculate the magnitude of the total momentum transferred to the surface.
- Find the radiation pressure exerted on the surface in the case of complete reflection.

## Cevaplar

### Sıra sende soruları

1 Go to the video.

2 a.  $\lambda = 6.0 \text{ m}$ ,  $T = 2.0 \times 10^{-8} \text{ s}$   
b.  $\vec{B} = (2.0 \times 10^{-6} \text{ T})\hat{k}$

3 a.  $E_{\text{max}} = 7.7 \times 10^{-2} \text{ V/m}$   
b.  $B_{\text{max}} = 2.6 \times 10^{-10} \text{ T}$

4 a.  $S_{\text{avg}} = 1.9 \times 10^3 \text{ W/m}^2$   
b.  $P_{\text{avg}} = 1.1 \times 10^{-5} \text{ Pa}$

### Alıştırmalar ve problemler

1 a.  $k = 2.0 \times 10^6 \text{ rad/m}$ ,  $\omega = 6.0 \times 10^{14} \text{ rad/s}$   
b.  $\vec{E}(y, t) = (3.0 \times 10^6 \text{ V/m})\cos(ky + \omega t)\hat{i}$   
 $\vec{B}(y, t) = (1.0 \times 10^{-2} \text{ T})\cos(ky + \omega t)\hat{k}$

2 a.  $\vec{S}(z, t) = \frac{E_0^2}{\mu_0 c} \cos^2(kz - \omega t)\hat{k}$ , Unit:  $\text{W/m}^2$   
b.  $I = \frac{E_0^2}{2\mu_0 c} = 4.8 \times 10^2 \text{ W/m}^2$   
c.  $P_{\text{avg}} = 9.5 \times 10^2 \text{ W}$

3 a.  $U = 6.0 \times 10^4 \text{ J}$   
b.  $p = 2.0 \times 10^{-4} \text{ kg} \cdot \text{m/s}$   
c.  $P = 4.0 \times 10^{-6} \text{ N/m}^2$   
d.  $p' = 4.0 \times 10^{-4} \text{ kg} \cdot \text{m/s}$   
e.  $P' = 8.0 \times 10^{-6} \text{ N/m}^2$