APPENDIX

MATHEMATICAL SUPPLEMENT FOR GENERAL PHYSICS

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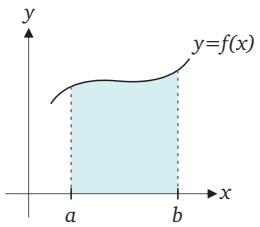
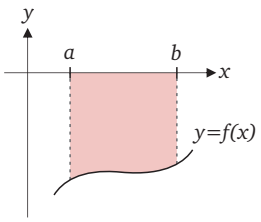
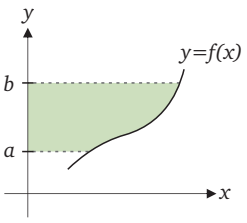
Calculations with Integrals

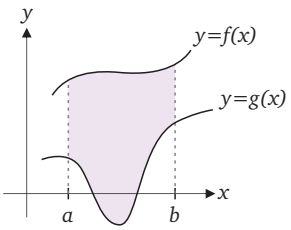
EXAMPLE 1: Let us find the length of the rod shown in the figure by using an integral.

EXAMPLE 2: Let us write the integrals that give the areas shown in the figures.

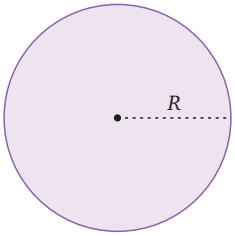




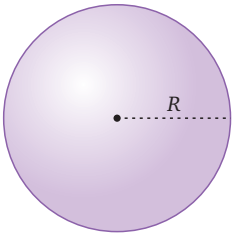




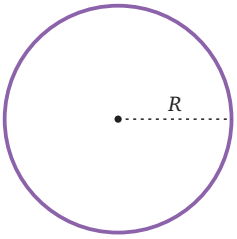
EXAMPLE 3: Let us find the area of a circle of radius R by integrating infinitesimal rings.



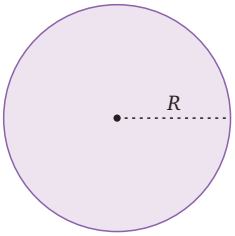
EXAMPLE 4: Let us find the volume of a sphere of radius R using integration.



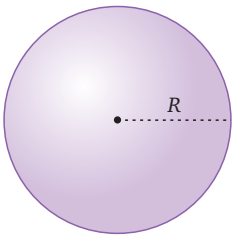
EXAMPLE 5: Let us find the circumference of a circle of radius R using integration.



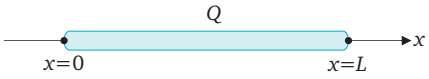
EXAMPLE 6: Let us find the area of a circle of radius R by integrating infinitesimal sectors.



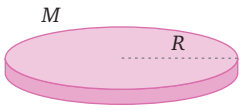
EXAMPLE 7: Let us find the surface area of a sphere of radius R using integration.



EXAMPLE 8: The linear charge density on the rod of length L and total charge Q shown in the figure is given by $\lambda(x) = Ax^2$. Accordingly, let us find the constant A in terms of the given quantities.



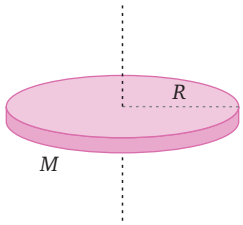
EXAMPLE 9: The thickness of the disk shown is negligible. The surface mass density of this disk, which has radius R and mass M, depends on the distance from the center as $\sigma(r) = \dfrac{A}{r}$. Accordingly, let us find the constant A in terms of the given quantities.



EXAMPLE 10: Let us find the gravitational force exerted by the uniform rod of mass M and length L shown in the figure on the object of mass m.

EXAMPLE 11: Let us find the moments of inertia of the uniform rod shown in the figure, which has mass M and length L, about one end and about its center.

EXAMPLE 12: The axis passing through the center of the uniform disk of mass M and radius R is shown in the figure. Let us find the moment of inertia of the disk about this axis.



Answers

1	$L = \int_a^b dx = b - a$
2	a. $A = \int_a^b f(x)\,dx$ b. $A = -\int_a^b f(x)\,dx$ c. $A = \int_a^b f^{-1}(y)\,dy$ d. $A = \int_a^b [f(x) - g(x)]\,dx$
3	$A = \int_0^R 2\pi r\,dr = \pi R^2$
4	$V = \frac{4}{3}\pi R^3$
5	Circumference $= 2\pi R$
6	$A = \int_0^R 2\pi r\,dr = \pi R^2$
7	$A = 4\pi R^2$
8	$A = \frac{3Q}{L^3}$
9	$A = \frac{M}{2.0\pi R}$
10	$F = \frac{GMm}{d(d+L)}$
11	a. $I_{\text{end}} = \frac{1}{3}ML^2$ b. $I_{\text{center}} = \frac{1}{12}ML^2$
12	$I = \frac{1}{2}MR^{2.0}$