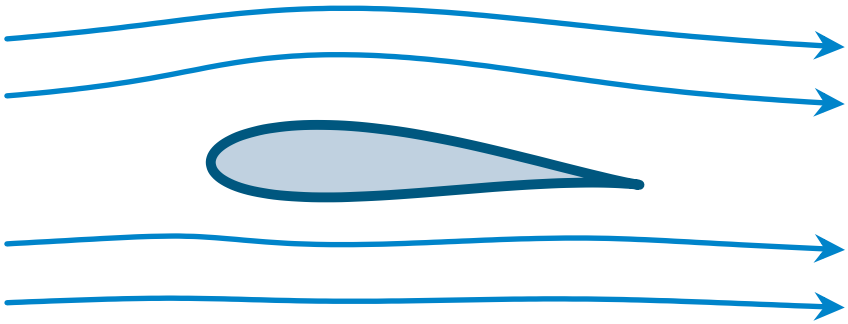
FLUID MECHANICS 2

PROBLEM BOOK

SAMPLE PAGES

7 chapters · 140 problems · step-by-step solutions



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How to use this book

- Each chapter opens with a reminder box summarizing the definitions and formulas the problems rely on.
- Difficulty: ★☆☆ basic ★★☆ intermediate ★★★ advanced. Each chapter has a few multiple-choice concept questions; its last problem is an exam-style mixed problem combining several topics.
- The dotted area below each problem is for your own work. Solve first; if stuck, go back to the reminder box and the lecture notes, and look at the solution last.
- All solutions are at the end of the book. The Solution → link next to a problem takes you to its solution, and ← Problem takes you back. Short answers are in the **ANSWER** box.
- **Shared chapters:** Chapters 6–7 (dimensional analysis, internal flow) are the same as in *Fluid Mechanics 1*. They are included here for schools whose second course begins with these topics; for others they serve as review.
- **Midterm and final:** Fluid Mechanics II is mostly taught in mechanical, energy, and automotive engineering. The midterm usually covers pipe flow and differential analysis (Chapters 6–8); the final covers external flow, compressible flow, open-channel flow, and turbomachinery (Chapters 9–12), with the selection of topics varying from school to school. Choose the chapters to study according to your own course syllabus.

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CHAPTER 9

EXTERNAL FLOW: DRAG AND LIFT

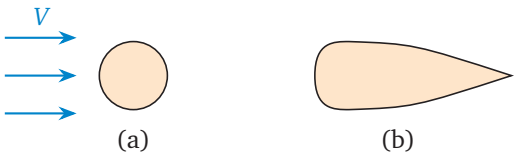
drag force	$F_D = C_D \frac{1}{2}\rho V^2 A$	terminal velocity (falling body)	$W = F_D + F_B$
lift force	$F_L = C_L \frac{1}{2}\rho V^2 A$	Stokes' law ($Re < 1$)	$F_D = 3\pi\mu V D$
Reynolds number	$Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$	stall speed	$V_{\min} = \sqrt{\frac{2W}{\rho\, C_{L,\max} A}}$
friction drag	$F_f = C_f \frac{1}{2}\rho V^2 A_{\text{wet}}$		
transition on a flat plate	$Re_x \approx 5 \times 10^5$	Strouhal number	$St = \frac{f D}{V}$

Unless stated otherwise, the gravitational acceleration is taken as $g = 9.81$ m/s^2. The density and viscosity of air and water, the drag and lift coefficients, and the flat-plate correlations are given in each problem; each correlation is used only within its stated Re range, and Re is checked first. The flow is steady and incompressible. Unless stated otherwise, C_D is based on the projected (frontal) area normal to the flow; for wings, C_L and C_D are based on the planform area. A_{wet} is the wetted surface area. In the figures, velocity and flow direction are blue, force is red, rotation is green, solid bodies are peach, and water is light cyan.

Drag and lift forces

9.1 ★☆☆ Solution →

The two bodies in the figure have equal thickness normal to the flow and equal length (normal to the page): (a) a circular cylinder, (b) a strut given a streamlined shape by extending its tail. Both are in an air stream with the same velocity and a high Reynolds number. Which of the following is correct?



A) Most of the drag of cylinder (a) comes from the shear stresses on its surface (friction drag).

B) Streamlining greatly reduces pressure drag, but because the wetted surface area grows, friction drag may increase.

C) Since the friction coefficient decreases as flow speed increases, the friction force on strut (b) also decreases.

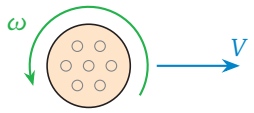
D) Pressure drag is independent of viscosity; pressure drag acts on cylinder (a) even in a frictionless (ideal) flow.

E) The closer flow separation is to the rear of the body, the wider the wake and the larger the drag.

Spheres and cylinders; terminal velocity

9.11

★★☆ Solution →



A dimpled golf ball has a diameter of 42.7 mm and a mass of 45.9 g. The ball is struck with backspin (the bottom surface rotating forward) at 70 m/s. For air, $\rho = 1.2\ \text{kg/m}^3$ and $\mu = 1.8 \times 10^{-5}\ \text{Pa}\cdot\text{s}$. At this speed and spin, $C_D = 0.25$ and $C_L = 0.21$.

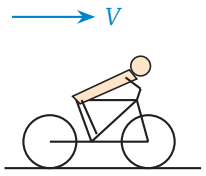
a. Find the Reynolds number and the drag and lift forces. How many times the weight is the lift force? What does this mean for the ball's motion?

b. For a smooth sphere the drag crisis begins at $Re \approx 3.5 \times 10^5$, for a dimpled golf ball at $Re \approx 4 \times 10^4$. Find the speeds corresponding to these Reynolds numbers and explain why golf balls are made with dimples.

Mixed problems

9.20

★★★ Solution →



The total mass of a road cyclist and the bicycle is 84 kg. In racing position $C_DA = 0.28\ \text{m}^2$ and the rolling resistance coefficient is 0.0045 (rolling resistance = coefficient × force normal to the road). For air, $\rho = 1.2\ \text{kg/m}^3$. Neglect drivetrain losses.

a. Find the power required for a constant speed of 38 km/h on a level road in still air. What percentage of this power is spent on aerodynamic drag?

b. Find the power required to climb a 7% grade (rise/run = 0.07) at 14 km/h in still air, and the share of drag.

c. Find the power required for a ground speed of 38 km/h on a level road against a 6 m/s headwind.

CHAPTER 10
COMPRESSIBLE FLOW

speed of sound, Mach number
$$c = \sqrt{kRT},\quad Ma = \frac{V}{c}$$

stagnation temperature
$$\frac{T_0}{T} = 1 + \frac{k-1}{2} Ma^2$$

isentropic
$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{k}{k-1}},\quad \frac{\rho_0}{\rho} = \left(\frac{T_0}{T}\right)^{\frac{1}{k-1}}$$

area–Mach
$$\frac{A}{A^*} = \frac{1}{Ma}\left[\frac{2}{k+1}\frac{T_0}{T}\right]^{\frac{k+1}{2(k-1)}}$$

critical ($k = 1.4$)
$$\frac{T^*}{T_0} = 0.8333,\quad \frac{P^*}{P_0} = 0.5283$$

choked flow rate
$$\dot{m} = A^* P_0 \sqrt{\frac{k}{RT_0}}\left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}}$$

normal shock
$$Ma_2^2 = \frac{(k-1)Ma_1^2 + 2}{2kMa_1^2 - (k-1)}$$
$$\frac{P_2}{P_1} = \frac{1 + kMa_1^2}{1 + kMa_2^2}$$
$$\frac{T_2}{T_1} = \frac{2 + (k-1)Ma_1^2}{2 + (k-1)Ma_2^2}$$

across shock
$$T_{02} = T_{01},\quad P_{01}A_1^* = P_{02}A_2^*$$

ideal gas
$$P = \rho RT,\quad c_p = \frac{kR}{k-1}$$

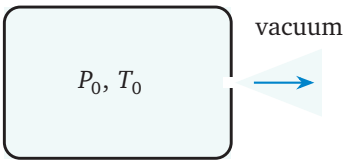
thrust (nozzle)
$$F = \dot{m}V_e + (P_e - P_a)A_e$$

Unless stated otherwise, the gas is air treated as an ideal gas: $k = 1.4$, $R = 287\ \text{J/(kg·K)}$, $c_p = 1004.5\ \text{J/(kg·K)}$. The flow is one-dimensional, steady, and adiabatic; in nozzles without a shock it is taken as isentropic. Pressures and temperatures are absolute; atmospheric pressure is 101.3 kPa. Isentropic relations are not used across a normal shock: across a shock the stagnation temperature is conserved and the stagnation pressure drops. The Mach number is solved from the area–Mach relation by trial and error. In the figures the gas is shown with a light fill, the flow direction is blue, and a normal shock is a thick black line.

Converging nozzles and choking

10.8 ★☆☆ Solution →

A micrometeorite has made a round hole 6 mm in diameter in the wall of a space station module. The air in the module is at 101.3 kPa and 295 K; the outside is vacuum. The hole behaves like a short, frictionless converging nozzle.



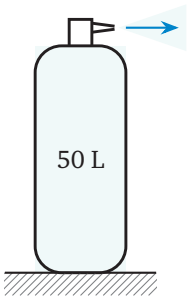
a. Show that the flow is choked and find the air loss (g/s).

b. Find the temperature, pressure, and velocity of the jet leaving the hole.

c. The volume of the module is 100 m^3. At this loss rate, how long does it take to lose 1% of the module's air?

10.10

★★★ Solution →



The valve of a 50 L compressed-air cylinder breaks, and the cylinder begins to discharge to the atmosphere through a frictionless converging opening 4 mm in diameter. Initially the cylinder pressure is 800 kPa. The cylinder is thin-walled and metallic; heat exchange is assumed adequate, so the air temperature in the cylinder is taken as constant at 295 K.

a. Find the initial mass flow rate.

b. Show that, as long as the flow remains choked, the cylinder pressure decreases as $P_0(t) = P_{0i}\, e^{-t/\tau}$; find the time constant.

c. When does the flow stop being choked? What percentage of the initial air remains in the cylinder at that moment?

CHAPTER 12
TURBOMACHINERY

water power, pump efficiency $\dot{W}_{\text{w}} = \rho g \dot{V} H, \;\; \eta_p = \dfrac{\dot{W}_{\text{w}}}{\dot{W}_{\text{shaft}}}$

operating point $H_{\text{pump}}(\dot{V}) = H_{\text{system}}(\dot{V})$

NPSH $\text{NPSH} = \left(\dfrac{P}{\rho g} + \dfrac{V^2}{2g} \right)_{\text{in}} - \dfrac{P_v}{\rho g}$

available NPSH $\text{NPSH}_m = \dfrac{P_{\text{atm}} - P_v}{\rho g} + z_s - h_{L,\text{suc}}$

Euler equation $\dot{W}_{\text{shaft}} = \rho \dot{V} \left(r_2 V_{2,t} - r_1 V_{1,t} \right)$

affinity (flow rate) $\dfrac{\dot{V}_B}{\dot{V}_A} = \dfrac{n_B}{n_A} \left(\dfrac{D_B}{D_A} \right)^3$

affinity (head, power) $H \propto n^2 D^2, \;\; \dot{W} \propto \rho n^3 D^5$

pump specific speed $N_{Sp} = \dfrac{\omega \dot{V}^{1/2}}{(gH)^{3/4}}$

turbine specific speed $N_{St} = \dfrac{\omega \dot{W}_{\text{shaft}}^{1/2}}{\rho^{1/2} (gH)^{5/4}}$

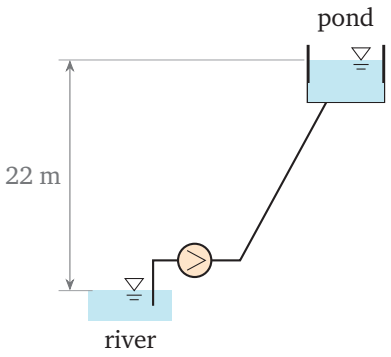
wind turbine $\dot{W} = C_P \, \frac{1}{2} \rho A V^3, \;\; C_{P,\max} = \frac{16}{27}$

Unless stated otherwise, take the density of water as $\rho = 1000$ kg/m^3, that of air as 1.2 kg/m^3, the gravitational acceleration as $g = 9.81$ m/s^2, and the atmospheric pressure as $P_{\text{atm}} = 101.3$ kPa. H is the net head (meters of liquid column). The pump and system curves are given as equations in the problem; the unit of the flow rate in these equations (L/s or m^3/s) is stated in the problem. z_s is the height of the free surface in the suction tank above the pump axis (negative if the surface is below the axis). The affinity laws apply to similar operating points of geometrically similar machines; efficiency is assumed to be the same. Specific speed is evaluated at the best efficiency point with ω in rad/s: $N_{Sp} < 1.5$ suits a centrifugal pump, 1.5–3.5 a mixed-flow pump, and above 3.5 an axial pump; $N_{St} < 0.3$ suits an impulse turbine, 0.3–2 a Francis turbine, and above 2 a Kaplan turbine. The blade angles are measured between the relative velocity and the tangent direction opposite to the direction of rotation.

Pump performance, system curve, and efficiency

12.3 ★★☆ Solution →

An irrigation cooperative pumps water taken from a river into a balancing pond on a hilltop. The pump curve is $H_p = 38 - 0.0055 \dot{V}^2$ and the system curve is $H_s = 22 + 0.0036 \dot{V}^2$ (H in meters, $\dot{V}$ in L/s). The pump efficiency curve is $\eta = 0.78 \left[2(\dot{V}/48) - (\dot{V}/48)^2 \right]$.



a. Find the flow rate and head at the operating point.

b. Find the efficiency and shaft power at this point.

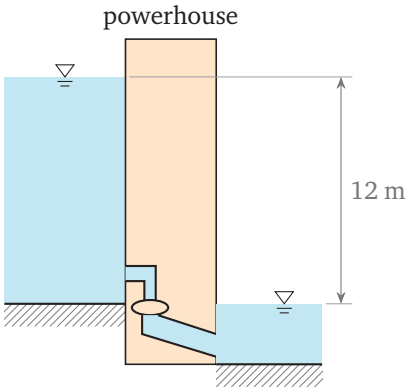
c. If the water level in the pond rises by 3 m (static head 25 m), by what percentage does the flow rate decrease?

Euler equation, velocity triangles, and turbines

12.16

★☆☆ Solution →

At a run-of-river plant the gross head between the diversion weir and the tailwater is 12 m; the losses in the intake structure and the canal are 0.6 m. The flow rate of one unit is 140 m^3/s, the turbine efficiency is 0.91, and the generator efficiency is 0.97. The synchronous speed of the generator is 107.1 rpm.



a. Find the shaft power of the turbine and the electric power of the unit.

b. Find the specific speed of the turbine. Which turbine type is suitable?

c. If a Francis turbine were used at the same site, what speed would be needed for the optimum specific speed ($N_{s_t} \approx 1$)? Comment.

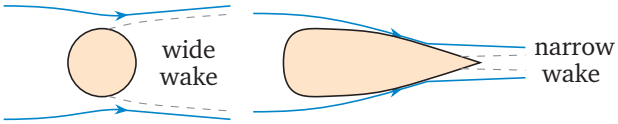
SOLUTIONS

9 EXTERNAL FLOW: DRAG AND LIFT

9.1

← Problem

Total drag is the sum of the flow-direction components of the pressure forces normal to the surface (pressure drag) and the shear stresses tangent to the surface (friction drag). On a bluff body the flow separates early and a wide, low-pressure wake forms behind it; streamlining delays separation and narrows the wake. The price is a longer body and therefore a larger wetted surface. **B** is correct.



- (A): At high Re, most of the cylinder's drag is pressure drag; it comes from the pressure difference between the front and rear faces.
- (C): C_f decreases but the force is proportional to $C_f V^2$; for a turbulent plate $C_f \propto V^{-0.2}$, so the force increases approximately as $V^{1.8}$.
- (D): In frictionless flow the pressure distribution is symmetric and the drag is zero (d'Alembert's paradox). The flow separation that causes pressure drag is a consequence of viscosity.
- (E): As separation is delayed the wake narrows and pressure drag decreases; the direction is the opposite.

ANSWER B

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9.11

← Problem

Frontal area $A = \pi(0.0427)^2/4 = 1.432 \times 10^{-3}$ m^2, dynamic pressure $\frac{1}{2}(1.2)(70)^2 = 2940$ Pa.

a. Reynolds number and forces:

$$Re = \frac{(1.2)(70)(0.0427)}{1.8 \times 10^{-5}} = 1.993 \times 10^5 \approx 1.99 \times 10^5,$$

$$F_D = (0.25)(2940)(1.432 \times 10^{-3}) = 1.053 \approx 1.05 \text{ N},$$

$$F_L = (0.21)(2940)(1.432 \times 10^{-3}) = 0.884 \text{ N}.$$

Weight $W = (0.0459)(9.81) = 0.4503$ N:

$$\frac{F_L}{W} = \frac{0.884}{0.4503} = 1.96.$$

The lift force (Magnus effect) is about twice the weight: after the shot the ball follows a path that rises for a while, curving upward, and travels much farther than a parabolic trajectory. The drag is $1.053/0.4503 = 2.34$ times the weight; the ball decelerates quickly.

b. $V = Re\,\mu/(\rho D)$; here $\rho D = 0.05124$ kg/m^2:

$$V_{\text{smooth}} = \frac{(3.5 \times 10^5)(1.8 \times 10^{-5})}{0.05124} = 123 \text{ m/s},$$

$$V_{\text{dimpled}} = \frac{(4 \times 10^4)(1.8 \times 10^{-5})}{0.05124} = 14.05 \approx 14.1 \text{ m/s}.$$

Golf ball speeds are at most 70–80 m/s; a smooth ball never reaches the drag crisis and flies with a high C_D (≈ 0.5). Dimples make the boundary layer turbulent early; separation is delayed, the wake narrows, and C_D is roughly halved over nearly the entire flight.

ANSWER a. $Re = 1.99 \times 10^5$, $F_D = 1.05$ N, $F_L = 0.884$ N, $F_L/W = 1.96$ b. smooth 123 m/s (unattainable), dimpled 14.1 m/s

9.20

← Problem

At constant speed the cyclist's propulsive force equals the sum of rolling resistance, drag, and (on a hill) the component of weight along the road; power is this force times ground speed. Drag is computed with the velocity relative to the air. $W = (84)(9.81) = 824.0$ N.

a. $V = 38/3.6 = 10.556$ m/s:

$$F_D = \tfrac{1}{2}(1.2)(0.28)(10.556)^2 = 18.72 \text{ N},$$

$$F_r = (0.0045)(824.0) = 3.708 \text{ N},$$

$$\dot{W} = (18.72 + 3.708)(10.556) \approx 237 \text{ W}.$$

$18.72/22.43 = 83.5\%$ of the power goes to drag.

b. $\theta = \arctan 0.07 = 4.00°$, $\sin\theta = 0.06983$, $\cos\theta = 0.9976$; $V = 14/3.6 = 3.889$ m/s:

$$W \sin\theta = (824.0)(0.06983) = 57.54 \text{ N},$$

$$F_r = (0.0045)(824.0)(0.9976) = 3.699 \text{ N},$$

$$F_D = \tfrac{1}{2}(1.2)(0.28)(3.889)^2 = 2.541 \text{ N},$$

$$\dot{W} = (57.54 + 3.699 + 2.541)(3.889) = 248.0 \approx 248 \text{ W}.$$

The share of drag is $2.541/63.78 = 3.98\%$. On a hill most of the power goes to climbing; the benefit of an aerodynamic position is small, and mass matters.

c. Velocity relative to the air $10.556 + 6 = 16.556$ m/s:

$$F_D = \tfrac{1}{2}(1.2)(0.28)(16.556)^2 = 46.05 \text{ N},$$

$$\dot{W} = (46.05 + 3.708)(10.556) = 525.2 \approx 525 \text{ W}.$$

Power is the force times *ground speed* (the cyclist travels relative to the ground). The headwind more than doubles the required power.

ANSWER a. about 237 W, 83.5% of it drag b. 248 W, drag 3.98% c. 525 W

10 COMPRESSIBLE FLOW

10.8

← Problem

a. The outside pressure $0 < 0.5283\,P_0$; $Ma = 1$ occurs at the hole and the flow is choked. $A = \pi(0.006)^2/4 = 2.827 \times 10^{-5}$ m^2:

$$\dot{m} = AP_0\sqrt{\frac{k}{RT_0}}\left(\frac{2}{2.4}\right)^3,$$

$$\dot{m} = (2.827 \times 10^{-5})(101\,300)\frac{0.06984}{\sqrt{295}}(0.5787),$$

$$\dot{m} = 6.74 \times 10^{-3} \text{ kg/s} = 6.74 \text{ g/s}.$$

b. The exit is at the critical state:

$$T^* = 0.8333\,(295) = 245.8 \approx 246 \text{ K},$$

$$P^* = 0.5283\,(101.3) = 53.51 \approx 53.5 \text{ kPa},$$

$$V^* = \sqrt{(1.4)(287)(245.8)} = 314.3 \approx 314 \text{ m/s}.$$

The jet leaves at a much higher pressure than the vacuum and continues to expand outside.

c. Air mass in the module:

$$m = \frac{P_0 V}{RT_0} = \frac{(101\,300)(100)}{(287)(295)} = 119.6 \text{ kg},$$

$$t = \frac{0.01\,(119.6)}{6.74 \times 10^{-3}} = 177.5 \approx 178 \text{ s},$$

about 3 min. Even a small hole leaves the crew only a few minutes of warning for evacuation; the flow rate increases with the hole area, that is, with the square of the diameter.

ANSWER a. choked, 6.74 g/s b. 246 K, 53.5 kPa, 314 m/s c. 178 s

10.10

← Problem

At the opening the critical pressure $0.5283\,P_0$ is larger than atmospheric pressure as long as $P_0 > 191.8$ kPa; during this period the flow is choked. $A = \pi(0.004)^2/4 = 1.257 \times 10^{-5}$ m^2 and $C = \sqrt{k/R}\,(2/2.4)^3 = 0.04042\ \sqrt{\text{K}}\cdot\text{s/m}$:

$$\dot{m} = \frac{CAP_0}{\sqrt{T_0}}.$$

a. Initially:

$$\dot{m}_i = \frac{(0.04042)(1.257 \times 10^{-5})(800\,000)}{\sqrt{295}},$$

$$\dot{m}_i = 0.02366 \approx 0.0237 \text{ kg/s}.$$

b. Mass in the cylinder $m = P_0 V/(RT_0)$, with T_0 constant:

$$\frac{V}{RT_0}\frac{dP_0}{dt} = -\frac{CAP_0}{\sqrt{T_0}},$$

$$\frac{dP_0}{P_0} = -\frac{dt}{\tau},$$

$$\tau = \frac{V}{R\sqrt{T_0}\,CA},$$

$$\tau = \frac{0.05}{(287)(17.18)(0.04042)(1.257 \times 10^{-5})},$$

$$\tau = 19.97 \approx 20.0 \text{ s}.$$

Check: τ is the ratio of the initial mass (0.4725 kg) to the initial flow rate.

c. Choking lasts until $P_0 = 191.8$ kPa:

$$t = \tau \ln\frac{800}{191.8} = 19.97\,(1.428) = 28.5 \text{ s}.$$

Since the temperature is constant, the remaining mass fraction equals the pressure ratio: $191.8/800 = 0.240$,

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that is, 24.0% (0.113 kg). In reality the cylinder cools during rapid discharge; the pressure falls faster, and the constant-temperature assumption gives a somewhat longer choking time (about 22.6 s in the adiabatic limit).

ANSWER a. 0.0237 kg/s b. $\tau = 20.0$ s c. 28.5 s; 24.0%

12 TURBOMACHINERY

12.3

← Problem

At the operating point the pump head equals the head required by the system.

a. Equating:

$$38 - 0.0055\,\dot{V}^2 = 22 + 0.0036\,\dot{V}^2,$$

$$\dot{V}^2 = \frac{16}{0.0091} = 1758,$$

$$\dot{V} = 41.93 \approx 41.9 \text{ L/s},$$

$$H = 22 + 0.0036(1758) = 28.33 \approx 28.3 \text{ m}.$$

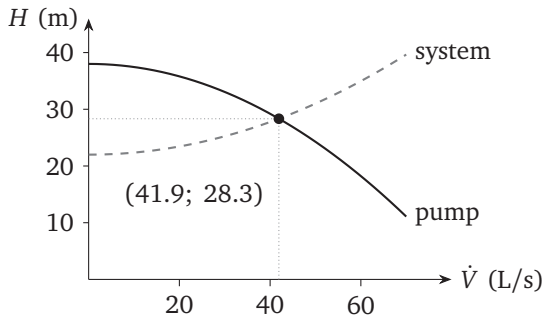
b. $\dot{V}/48 = 0.8736$:

$$\eta = 0.78\left[2(0.8736) - (0.8736)^2\right] = 0.7675 \approx 0.768,$$

$$\dot{W}_{\text{shaft}} = \frac{(1000)(9.81)(0.04193)(28.33)}{0.7675},$$

$$\dot{W}_{\text{shaft}} = \frac{11.65 \text{ kW}}{0.7675} = 15.18 \approx 15.2 \text{ kW}.$$

The pump operates close to, and slightly to the left of, its best efficiency point (48 L/s).



c. Static head 25 m:

$$\dot{V}^2 = \frac{13}{0.0091}, \quad \dot{V} = 37.80 \approx 37.8 \text{ L/s}.$$

Compared with part (a), the flow rate decreases by $(41.931 - 37.796)/41.931 = 9.86\%$. If the static head reaches 38 m (the pump's head at zero flow), the pump cannot deliver any water at all.

ANSWER a. 41.9 L/s, 28.3 m b. $\eta = 0.768$, 15.2 kW c. 37.8 L/s, decreases by 9.86%

12.16

← Problem

Net head $H = 12 - 0.6 = 11.4$ m.

a. Power chain:

$$\rho g \dot{V} H = (1000)(9.81)(140)(11.4) = 15.66 \text{ MW},$$

$$\dot{W}_{\text{shaft}} = (0.91)(15.66) = 14.25 \approx 14.2 \text{ MW},$$

$$\dot{W}_e = (0.97)(14.25) = 13.82 \approx 13.8 \text{ MW}.$$

b. $\omega = 2\pi(107.1)/60 = 11.22$ rad/s, $gH = 111.8$ m^2/s^2:

$$N_{St} = \frac{(11.22)(14.25 \times 10^6)^{1/2}}{(1000)^{1/2}(111.8)^{5/4}},$$

$$N_{St} = \frac{(11.22)(3775)}{(31.62)(363.7)} = 3.683 \approx 3.68.$$

$N_{St} > 2$: a Kaplan (axial-flow) turbine is suitable. Low head and large flow rate are the typical domain of the Kaplan turbine.

c. Because N_{St} is proportional to ω, the speed required for $N_{St} = 1$ is $\omega = 11.22/3.683 = 3.046$ rad/s, that is, $n = 29.09 \approx 29.1$ rpm. A generator rotating this slowly would require a very large number of poles and would be very large and expensive; the runner would also be gigantic. At low head, a turbine with a high specific speed is chosen.

ANSWER a. 14.2 MW (shaft), 13.8 MW (electric) b. $N_{St} = 3.68$; Kaplan c. 29.1 rpm; too slow to be practical

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- 7 chapters and 140 problems, all with step-by-step solutions.
- A reminder box at the start of each chapter, summarizing the relations the problems rely on.
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- One-click links from each problem to its solution and back.



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