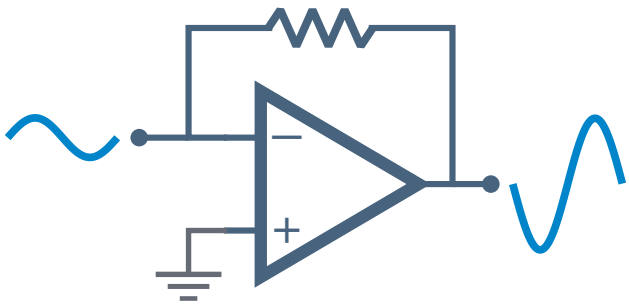
CIRCUIT ANALYSIS I

WORKBOOK

SAMPLE PAGES

9 chapters · 160 problems · step-by-step solutions



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How to use this book

- Each chapter opens with a reminder box summarizing the definitions and formulas the problems rely on.
- Difficulty: ★☆☆ basic ★★☆ intermediate ★★★ advanced. Each chapter has a few multiple-choice concept questions; each chapter ends with exam-style mixed problems.
- The dotted area below each problem is for your own work. Solve first; if stuck, go back to the reminder box and the lecture notes, and look at the solution last.
- All solutions are at the end of the book. The Solution → p. link next to a problem takes you to its solution, and ← Problem p. takes you back. Short answers are in the **ANSWER** box.
- **Units and signs:** About half of the problems use Ω, A and W, the other half kΩ, mA and mW; both appear in exams. The passive sign convention is used throughout: if current enters an element at its + terminal, $p = vi$ is the power absorbed by that element. In the figures, currents are red and voltage polarities are blue.
- **Midterm and final:** At most universities Circuits I (usually the fall of the second year) covers this whole book. The first midterm usually covers resistive circuits and Thévenin–Norton (Chapters 1–5); the second midterm or the final covers op amps, inductors and capacitors, and transients (Chapters 6–9). Where basic circuit topics are taught in the first year, Chapters 1–4 serve as review; where Circuits II starts with inductors and capacitors, Chapters 7–9 belong to that course. Choose the chapters to study according to your own course syllabus.

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CHAPTER 3

THE NODE-VOLTAGE METHOD

essential node	three or more elements meet	conductance matrix	$\mathbf{G}\mathbf{v} = \mathbf{i}$
number of equations	$n_e - 1$ KCL, $b_b - (n_e - 1)$ KVL	dependent source	write the controlling variable in node voltages
node equation	$\sum$(currents leaving the node) $= 0$	supernode	one KCL + $v_a - v_b = v_s$

In this chapter n_e is the number of essential nodes. An essential branch is a path between two essential nodes that does not pass through another essential node; b_e is the number of essential branches, and b_b is the number of those whose current is unknown. A loop (closed path) is a path that ends at the node where it starts; a mesh is a loop that contains no other loop. A node voltage is the voltage of a node with respect to the reference node; unless stated otherwise, the reference is the bottom node. The passive sign convention is used: if the current enters an element at its + terminal, $p = vi$ is the power *absorbed* by the element. With resistances in kΩ and currents in mA, power comes out directly in mW; with Ω and A, it comes out in W.

The node-voltage method

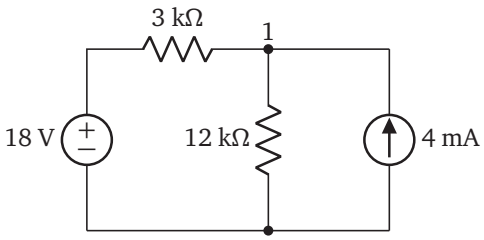
3.3

★☆☆ Solution → p. 10

For the circuit shown,

a. find the voltage of node 1 by the node-voltage method;

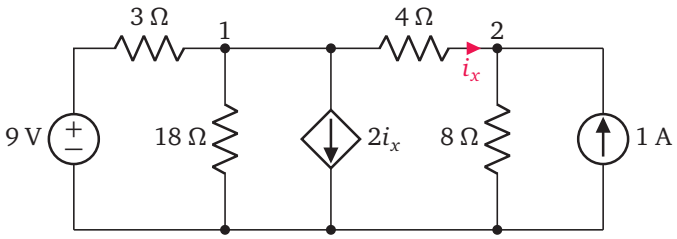
b. does the 18 V source deliver or absorb power? How much?



The node method with dependent sources

3.7 ★☆☆ Solution → p. 10

In the circuit shown, the current-controlled current source draws a current $2i_x$ from node 1; i_x is the current in the 4 Ω resistor from 1 to 2. Which of the following is the KCL equation at node 1?



A) $(9-v_1)/3+v_1/18+(v_1-v_2)/4+2i_x=0$

B) $(v_1-9)/3+v_1/18+(v_1-v_2)/4=0$

C) $(v_1-9)/3+v_1/18+(v_1-v_2)/4-2(v_1-v_2)/4=0$

D) $(v_1-9)/3+v_1/18+(v_1-v_2)/4+2v_1/4=0$

E) $(v_1-9)/3+v_1/18+(v_1-v_2)/4+2(v_1-v_2)/4=0$

Special cases and the supernode

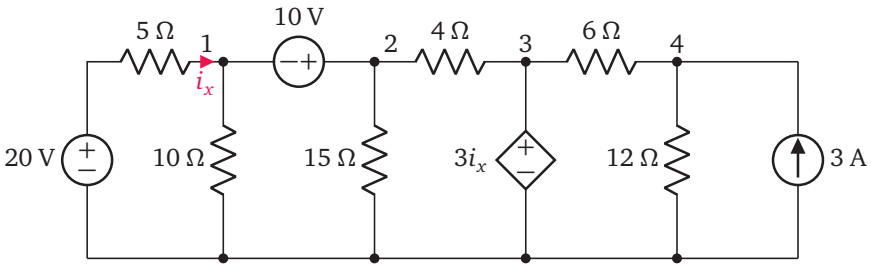
3.15 ★★★ Solution → p. 10

In the circuit shown, the 10 V source is between nodes 1 and 2, and the current-controlled voltage source ($3i_x$; volts, i_x in amperes; i_x is the current in the 5 Ω resistor) is between node 3 and the reference.

a. Which node voltage is tied directly to the controlling variable? Taking v_1, v_2, v_4, i_x as the unknowns, write the four equations in matrix form.

b. Find the node voltages.

c. Find the power of every source and show that power is conserved.



CHAPTER 5

SOURCE TRANSFORMATIONS, THÉVENIN–NORTON, MAXIMUM POWER, SUPERPOSITION

source transformation $v_s = i_s R$ (R series $\leftrightarrow$ R parallel)

Thévenin $V_{\text{Th}} = v_{\text{oc}}, \quad R_{\text{Th}} = \dfrac{v_{\text{oc}}}{i_{\text{sc}}}$

Norton $I_{\text{N}} = i_{\text{sc}} = \dfrac{V_{\text{Th}}}{R_{\text{Th}}}$

test source $R_{\text{Th}} = \dfrac{v_t}{i_t}$ (independent sources off)

maximum power $R_L = R_{\text{Th}}, \quad P_{\max} = \dfrac{V_{\text{Th}}^2}{4R_{\text{Th}}}$

superposition

independent sources singly; dependent ones stay

In this chapter, v_{oc} is the open-circuit voltage and i_{sc} the short-circuit current. The passive sign convention is used: if current enters an element at its + terminal, $p = vi$ is the power *absorbed* by the element. To deactivate a source, replace a voltage source with a short circuit and a current source with an open circuit. With resistances in kΩ and currents in mA, power comes out directly in mW; with Ω and A, in W.

Thévenin and Norton equivalents

5.6

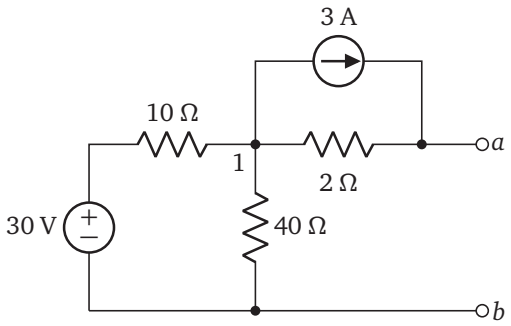
★★☆ Solution → p. 11

In the circuit shown, the 3 A current source is connected in parallel with the 2 Ω resistor.

a. Find the Thévenin equivalent at terminals a–b.

b. If a 14 Ω resistor is connected to the terminals, what is its current?

c. In this case, does the 3 A source deliver or absorb power? How much?

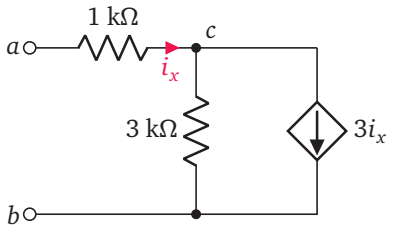


Thévenin equivalents of circuits with dependent sources

5.11

★★★ Solution → p. 11

The circuit shown has no independent source; the current of the current-controlled current source is $3i_x$.



a. Find the Thévenin equivalent at terminals a–b.

b. A 6 V source is connected to the terminals: its + terminal to a through 2 kΩ, its − terminal to b. Find the current from the source into a and the power of the source.

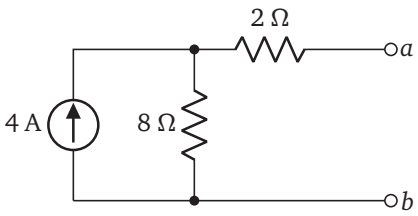
c. Where does the power absorbed by the source in (b) come from?

Maximum power transfer

5.13

★★☆ Solution → p. 12

An adjustable load R_L will be connected to terminals a–b of the circuit shown.



a. What should R_L be for maximum load power? What is this power in W?

b. Find the power delivered by the current source in this case. What percentage of the source power is transferred to the load?

CHAPTER 8

FIRST-ORDER RL AND RC CIRCUITS

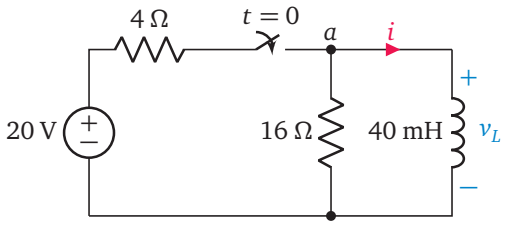
time constant	$\tau = R_{\text{Th}}C, \quad \tau = \dfrac{L}{R_{\text{Th}}}$	continuity	$v_C(0^+) = v_C(0^-), \quad i_L(0^+) = i_L(0^-)$
natural response	$x(t) = x(0^+)\,e^{-t/\tau}$	dc steady state	C open circuit, L short circuit
general solution	$x(t) = x(\infty) + \left[x(t_0^+) - x(\infty)\right]e^{-(t-t_0)/\tau}$	energy	$w_C = \frac{1}{2}Cv^2, \quad w_L = \frac{1}{2}Li^2$

In this chapter R_{Th} is the Thévenin resistance seen from the terminals of the capacitor or the inductor. The phrase "for a long time" means that the circuit is in dc steady state: the capacitor behaves like an open circuit and the inductor like a short circuit. $t = 0^-$ is the instant just before switching, and $t = 0^+$ the instant just after. The passive sign convention is used: if current enters an element at its + terminal, $p = vi$ is the power *absorbed* by the element. In time expressions t is in milliseconds; when Ω is used with mH or mF, or kΩ with μF, τ comes out directly in ms. The energy is in mJ with Ω–A–mH (or mF) and in μJ with kΩ–mA–μF.

Step response

8.9 ★☆☆ Solution → p. 12

The switch shown has been open for a long time and closes at $t = 0$.



a. Find the Thévenin equivalent seen from the inductor and the time constant.

b. Find $i(t)$ for $t \geq 0$ and the inductor voltage ($v_L(t)$) for $t \geq 0^+$.

c. At what instant does the current reach 4 A?

Sequential switching

8.19 ★★☆ Solution → p. 12

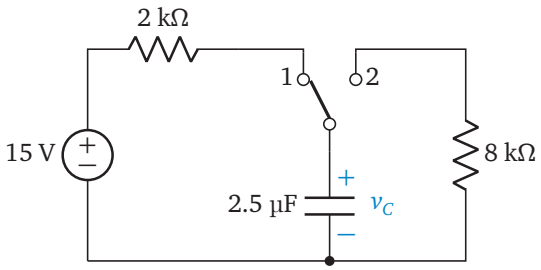
The switch shown has been in position 1 for a long time. It moves to position 2 at $t = 0$ and returns to position 1 at $t = 20$ ms.

a. Find $v_C(t)$ for $0 \le t \le 20$ ms and $t \ge 20$ ms.

b. Find the current in 8 kΩ at $t = 0^+$ and the current in 2 kΩ at $t = 20^+$ ms.

c. At what instant does v_C reach 14 V again?

d. Sketch $v_C(t)$; mark the time constants.



SOLUTIONS

3 THE NODE-VOLTAGE METHOD

3.3

← Problem p. 3

a. The essential nodes are 1 and the bottom node; with the bottom node as the reference, one equation suffices. The sum of the currents leaving 1 is zero (the current source drives 4 mA into 1):

$$\frac{v_1 - 18}{3} + \frac{v_1}{12} - 4 = 0.$$

Multiplying by 12 gives $4v_1 - 72 + v_1 - 48 = 0$, $5v_1 = 120$:

$$v_1 = 24.0\ \text{V}.$$

b. A current of $(24-18)/3 = 2$ mA flows in the 3 kΩ resistor from 1 toward the source. This current enters the source at its + terminal, so the source absorbs power:

$$P = 18(2) = 36.0\ \text{mW}.$$

Check: The current source delivers $24(4) = 96$ mW. Absorbed: 3 kΩ $2^2(3) = 12$ mW, 12 kΩ $24^2/12 = 48$ mW, source 36 mW; total 96 mW.

Common mistake: Assuming that a voltage source always delivers power. Here the 18 V source takes energy from the current source, like a battery being charged.

ANSWER a. $v_1 = 24.0$ V b. absorbs 36.0 mW

3.7

← Problem p. 4

The four currents leaving 1: $(v_1 - 9)/3$ through the 3 Ω resistor toward the source, $v_1/18$ through the 18 Ω, $i_x = (v_1 - v_2)/4$ through the 4 Ω, and $2i_x$ through the dependent source. The dependent source is written like an independent source, and then its controlling variable is expressed in terms of the node voltages (constraint equation):

$$i_x = \frac{v_1 - v_2}{4} \implies 2i_x = \frac{2(v_1 - v_2)}{4}.$$

- (A): The 3 Ω term is written as an entering current, the other terms as leaving currents. The signs are mixed.
- (B): The dependent source is forgotten.
- (C): The dependent source current is written as if it entered 1; the arrow points down, that is, out of 1.
- (D): Wrong controlling variable: i_x is the current in the 4 Ω resistor, not $v_1/4$.

ANSWER E

3.15

← Problem p. 5

a. The dependent source is between 3 and the reference: $v_3 = 3i_x$; no KCL is written for 3. Nodes 1 and 2 with the 10 V source form a supernode. Equations: the KCL equation of the supernode, its constraint, node 4, and the controlling variable $i_x = (20 - v_1)/5$:

$$\frac{v_1 - 20}{5} + \frac{v_1}{10} + \frac{v_2}{15} + \frac{v_2 - 3i_x}{4} = 0,$$
$$v_2 - v_1 = 10, \qquad v_1 + 5i_x = 20,$$
$$4: \ \frac{v_4 - 3i_x}{6} + \frac{v_4}{12} = 3.$$

Multiplying the supernode equation by 60 and that of 4 by 12:

$$\begin{pmatrix} 18 & 19 & 0 & -45 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 3 & -6 \\ 1 & 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_4 \\ i_x \end{pmatrix} = \begin{pmatrix} 240 \\ 10 \\ 36 \\ 20 \end{pmatrix}.$$

b. From the last two constraints, $v_1 = 20 - 5i_x$, $v_2 = 30 - 5i_x$; substituting into row 1 gives $930 - 230i_x = 240$:

$$i_x = 3.00\ \text{A}, \qquad v_1 = 5.00\ \text{V},$$
$$v_2 = 15.0\ \text{V}, \qquad v_3 = 9.00\ \text{V}.$$

From row 3, $3v_4 = 36 + 18$:

$$v_4 = 18.0\ \text{V}.$$

c. Sources:

- 20 V: delivers $20(3) = 60.0$ W.

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- 10 V: 3 A enters 1 through the 5 Ω and 0.5 A leaves through the 10 Ω; the remaining 2.5 A enters the source at its − terminal: delivers $10(2.5) = 25.0$ W.
- 3 A: its current goes into the 18 V node: delivers 54.0 W.
- $3i_x = 9$ V: $(15-9)/4 = 1.5$ A arrives at 3 through the 4 Ω and $(18-9)/6 = 1.5$ A through the 6 Ω; these 3 A enter the source at its + terminal: absorbs $9(3) = 27.0$ W.

Resistors: 5 Ω 45, 10 Ω 2.5, 15 Ω 15, 4 Ω 9, 6 Ω 13.5, 12 Ω 27 W; total 112 W.

Check: Absorbed $112 + 27 = 139$ W, delivered $60 + 25 + 54 = 139$ W.

Common mistake: Writing a KCL equation also for the node of a dependent voltage source connected to the reference. The voltage of that node comes from the constraint; the source current is found by KCL only after the solution.

ANSWER a. $v_3 = 3i_x$; matrix above b. $v_1 = 5.00$, $v_2 = 15.0$, $v_3 = 9.00$, $v_4 = 18.0$ V c. 20 V 60.0 W, 10 V 25.0 W, 3 A 54.0 W delivered; $3i_x$ absorbs 27.0 W; 139 W = 139 W

5 SOURCE TRANSFORMATIONS, THÉVENIN–NORTON, MAXIMUM POWER, SUPERPOSITION

5.6

← Problem p. 6

a. With the terminals open, the 3 A current goes from 1 to a through the source and returns to 1 through 2 Ω; no net current enters node 1. Hence v_1 comes from the voltage divider alone:

$$v_1 = 30 \cdot \frac{40}{10+40} = 24\ \text{V}, \qquad V_{\text{Th}} = v_1 + 2(3) = 30.0\ \text{V}.$$

With the sources deactivated (30 V shorted, 3 A open):

$$R_{\text{Th}} = (10 \parallel 40) + 2 = 8 + 2 = 10.0\ \Omega.$$

b. $i_L = 30/(10+14) = 1.25$ A.

c. $v_a = 14(1.25) = 17.5$ V. A current of $3 - 1.25 = 1.75$ A flows through 2 Ω from a to 1; $v_1 = 17.5 - 2(1.75) = 14$ V. The source pushes its current from 1 to a, that is, from lower to higher potential:

$$P = (v_a - v_1)3 = 3.5(3) = 10.5\ \text{W (delivered)}.$$

ANSWER a. 30.0 V, 10.0 Ω b. 1.25 A c. 10.5 W delivered

5.11

← Problem p. 7

a. Since there is no independent source, $V_{\text{Th}} = 0$. For R_{Th}, apply a test current i_t to a; then $i_x = i_t$. At node c (currents leaving):

$$-i_x + 3i_x + \frac{v_c}{3} = 0 \implies v_c = -6i_x.$$

$$v_t = 1\,i_x + v_c = i_x - 6i_x = -5i_x,$$

$$R_{\text{Th}} = \frac{v_t}{i_t} = -5.00\ \text{k}\Omega.$$

b. The equivalent circuit is a single mesh of 6 V, 2 kΩ and −5 kΩ:

$$i = \frac{6}{2 + (-5)} = -2.00\ \text{mA}.$$

The current flows back into the + terminal of the source as 2 mA: the 6 V source *absorbs* $6(2) = 12.0$ mW. $v_a = 6 - 2(-2) = 10$ V, larger than the voltage of the source itself.

c. The 2 kΩ resistor also absorbs $2^2(2) = 8$ mW. The circuit delivers the total 20 mW; since resistors cannot deliver power, this power comes from the dependent current source. A negative resistance can appear only in a circuit containing an element that delivers power (an active element).

ANSWER a. $V_{\text{Th}} = 0$, $R_{\text{Th}} = -5.00$ kΩ b. 2.00 mA flows back into the source; the source absorbs 12.0 mW c. the dependent source

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5.13

← Problem p. 7

a. Thévenin equivalent: With the terminals open, no current flows through 2 Ω, and all of the 4 A flows through 8 Ω: $V_{\text{Th}} = 4(8) = 32$ V. With the current source open, $R_{\text{Th}} = 8 + 2 = 10\ \Omega$.

$$R_L = R_{\text{Th}} = 10.0\ \Omega, \qquad P_{\max} = \frac{32^2}{4(10)} = 25.6\ \text{W}.$$

b. With $R_L = 10\ \Omega$ the load branch is $2 + 10 = 12\ \Omega$. Current divider:

$$i_L = 4 \cdot \frac{8}{8+12} = 1.6\ \text{A}, \qquad i_8 = 2.4\ \text{A}.$$

The source voltage is that of the 8 Ω resistor: $v = 8(2.4) = 19.2$ V; the delivered power is $19.2(4) = 76.8$ W.

$$\frac{P_{\max}}{P_{\text{source}}} = \frac{25.6}{76.8} = \frac{1}{3} \approx 33.3\%.$$

In the equivalent circuit the ratio would be 50%. In the original circuit, however, the 8 Ω resistor in parallel with the source dissipates more power than is transferred to the load.

Check: The 8 Ω resistor absorbs $2.4^2(8) = 46.08$ W, the 2 Ω resistor $1.6^2(2) = 5.12$ W, and the load 25.6 W. Their sum is 76.8 W, equal to the power delivered by the source.

ANSWER a. $R_L = 10.0\ \Omega$, $P_{\max} = 25.6$ W
b. 76.8 W; 33.3%

8 FIRST-ORDER RL AND RC CIRCUITS

8.9

← Problem p. 8

a. From terminals a–ground:

$$V_{\text{Th}} = 20 \cdot \frac{16}{4+16} = 16.0\ \text{V}, \quad R_{\text{Th}} = 4 \parallel 16 = 3.20\ \Omega,$$

$$\tau = \frac{40}{3.2} = 12.5\ \text{ms}.$$

b. $i(0^-) = 0$, continuous; $i(\infty) = 16/3.2 = 5$ A (with the inductor shorted, 16 Ω draws no current, $20/4 = 5$ A):

$$i(t) = 5.00\left(1 - e^{-t/12.5}\right)\text{A}, \qquad t \geq 0,$$

$$v_L(t) = L\frac{di}{dt} = 40 \cdot \frac{5}{12.5}\, e^{-t/12.5},$$

$$v_L(t) = 16.0\, e^{-t/12.5}\ \text{V}, \qquad t \geq 0^+.$$

$v_L(0^+) = V_{\text{Th}} = 16$ V: an inductor with zero current behaves like an open circuit at $t = 0^+$.

c. If $5(1 - e^{-t/12.5}) = 4$, then $e^{-t/12.5} = 0.2$, which gives $t = 12.5 \ln 5 = 20.1$ ms.

ANSWER a. 16.0 V, 3.20 Ω, $\tau = 12.5$ ms
b. $i = 5.00(1 - e^{-t/12.5})$ A $(t \geq 0)$,
$v_L = 16.0\, e^{-t/12.5}$ V $(t \geq 0^+)$ c. 20.1 ms

8.19

← Problem p. 9

a. $v_C(0^-) = 15$ V (steady state in position 1, no current in 2 kΩ). In position 2 the capacitor discharges through 8 kΩ: $\tau_b = 8(2.5) = 20.0$ ms.

$$v_C(t) = 15.0\, e^{-t/20}\ \text{V}, \qquad 0 \leq t \leq 20\ \text{ms},$$

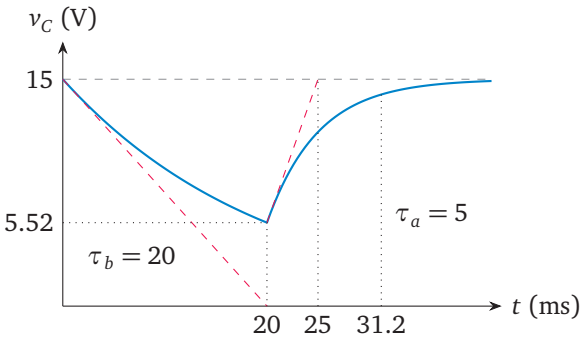
$v_C(20) = 15\, e^{-1} = 5.52$ V. In position 1 it charges toward 15 V through 2 kΩ: $\tau_a = 2(2.5) = 5.00$ ms.

$$v_C(t) = 15.0 - 9.48\, e^{-(t-20)/5}\ \text{V}, \qquad t \geq 20\ \text{ms}.$$

b. $i_8(0^+) = 15/8 = 1.88$ mA (downward). $i_2(20^+) = (15 - 5.52)/2 = 4.74$ mA (from the source to the capacitor).

c. If $15 - 9.482\, e^{-(t-20)/5} = 14$, then $t - 20 = 5 \ln 9.482 = 11.2$, that is, $t = 31.2$ ms.

d. The discharge is slow ($\tau_b = 20$ ms); the charging is four times faster ($\tau_a = 5$ ms). The dashed lines are the initial tangents of each interval; a tangent crosses the final-value line one time constant later.



ANSWER a. $15.0\, e^{-t/20}$ V ($\tau_b = 20.0$ ms);
$15.0 - 9.48\, e^{-(t-20)/5}$ V ($\tau_a = 5.00$ ms)
b. 1.88 mA; 4.74 mA c. 31.2 ms

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IN THE FULL BOOK

- 9 chapters and 160 problems, all with step-by-step solutions.
- A reminder box at the start of each chapter, summarizing the relations the problems rely on.
- Basic, intermediate and advanced problems, each with a dotted area for your own work.
- One-click links from each problem to its solution and back.



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