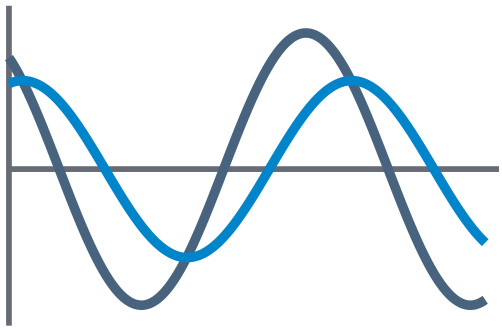
CIRCUIT ANALYSIS II

WORKBOOK

SAMPLE PAGES

12 chapters · 188 problems · step-by-step solutions



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How to use this book

- Each chapter opens with a reminder box summarizing the definitions and formulas the problems rely on.
- Difficulty: ★☆☆ basic ★★☆ intermediate ★★★ advanced. Each chapter has a few multiple-choice concept questions; each chapter ends with exam-style mixed problems.
- The dotted area below each problem is for your own work. Solve first; if stuck, go back to the reminder box and the lecture notes, and look at the solution last.
- All solutions are at the end of the book. The Solution → p. link next to a problem takes you to its solution, and ← Problem p. takes you back. Short answers are in the **ANSWER** box.
- **Notation and signs:** Phasors are set in bold (**V**, **I**); a phasor's magnitude is its peak value, and rms values are written as V_{rms}. In ac power (Chapter 12) each problem states whether peak or rms values are used; in three-phase circuits (Chapter 14) phasors are rms phasors. Sinusoids are referenced to the cosine, and angles are in degrees. The passive sign convention is used throughout: if current enters an element at its + terminal, $p = vi$ is the power absorbed by that element. In the figures, currents are red and voltage polarities are blue; the polarity of a sinusoidal source is marked by a black + and − beside it.
- **Shared chapters:** Chapters 7–9 (inductors and capacitors, first- and second-order transients) are the same as in *Circuit Analysis I*. They are included here for schools whose Circuits II course starts with these topics or reviews them before the Laplace transform.
- **Midterm and final:** Circuits II is usually taught in the spring of the second year. The midterm usually covers phasors, ac power, mutual inductance and three-phase circuits (Chapters 10–14); the final also covers the Laplace transform, frequency response and two-port circuits (Chapters 15–18). For schools that teach the Laplace transform before phasors, Chapters 15–16 are written without relying on phasors; in that order you can follow 7–9 → 15–16 → 10–14 → 17–18. Choose the chapters to study according to your own course syllabus.

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CHAPTER 13

MUTUAL INDUCTANCE AND TRANSFORMERS

coupled coils	$v_1 = L_1\frac{di_1}{dt} \pm M\frac{di_2}{dt}$	frequency domain	$\mathbf{V}_1 = j\omega L_1\mathbf{I}_1 \pm j\omega M\mathbf{I}_2$
energy	$w = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2$	reflected impedance	$Z_r = \frac{(\omega M)^2}{Z_{22}} = \left(\frac{\omega M}{\lvert Z_{22}\rvert}\right)^2 Z_{22}^*$
coefficient of coupling	$k = \frac{M}{\sqrt{L_1 L_2}}, \quad 0 \le k \le 1$	ideal, $1:n$	$\mathbf{V}_2 = n\mathbf{V}_1, \quad \mathbf{I}_1 = -n\mathbf{I}_2$
in series	$L_{\text{eq}} = L_1 + L_2 \pm 2M$	impedance reflection	$Z_{\text{in}} = Z_L/n^2$

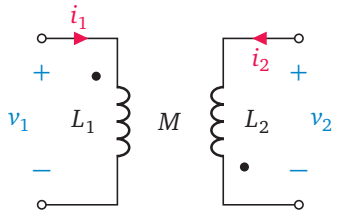
Dot convention: if current enters a coil at its dotted terminal, the + terminal of the mutual voltage induced in the other coil is that coil's dotted terminal. If both currents enter at the dotted terminals (or both leave), the mutual term has the same sign as the self term. If one enters and the other leaves, it has the opposite sign. In an ideal transformer $1:n$ is the turns ratio $N_1:N_2$. The formulas are written with the dots on the same side, the + terminals of the voltages at the dotted terminals and the currents entering the dotted terminals; if a dot is reversed, the sign changes. Z_{22} is the total impedance of the secondary mesh, and Z_{in} is the impedance seen from the primary terminals. The magnitude of a phasor is the amplitude: $S = \frac{1}{2}\mathbf{V}\mathbf{I}^*$. The first two sections (13.1–13.5) are in the time domain and need no phasors; coupled coils in the s domain are treated in 16.11–16.12.

Mutual inductance and the dot convention

13.1

★☆☆ Solution → p. 9

In the coupled coils shown, i_1 enters the first coil at its upper terminal and i_2 enters the second coil at its upper terminal; the + terminals of v_1 and v_2 are at the top. Which are the terminal equations?



A) $v_1 = L_1\, di_1/dt + M\, di_2/dt, \quad v_2 = L_2\, di_2/dt + M\, di_1/dt$

B) $v_1 = L_1\, di_1/dt - M\, di_2/dt, \quad v_2 = L_2\, di_2/dt + M\, di_1/dt$

C) $v_1 = L_1\, di_1/dt + M\, di_2/dt, \quad v_2 = L_2\, di_2/dt - M\, di_1/dt$

D) $v_1 = L_1\, di_1/dt - M\, di_2/dt, \quad v_2 = L_2\, di_2/dt - M\, di_1/dt$

E) $v_1 = -L_1\, di_1/dt + M\, di_2/dt, \quad v_2 = -L_2\, di_2/dt + M\, di_1/dt$

The linear transformer

13.8

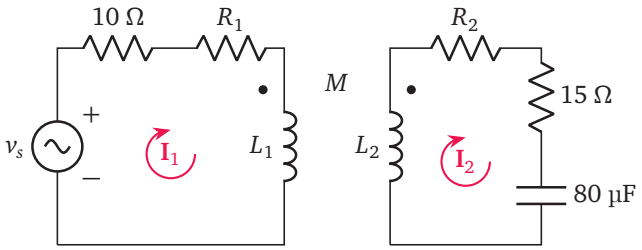
★★☆ Solution → p. 9

The parameters of the linear transformer shown are $R_1 = 2\ \Omega$, $R_2 = 3\ \Omega$, $L_1 = 50$ mH, $L_2 = 72$ mH, $M = 42$ mH. The source is $v_s = 100\cos 500t$ V with an internal resistance of 10 Ω; the load is 15 Ω in series with 80 μF. The dots are at the top.

a. Find the self-impedances of the primary and secondary meshes (Z_{11}, Z_{22}) and the coefficient of coupling.

b. Find the impedance reflected into the primary and the impedance seen from the source.

c. Find $\mathbf{I}_1$, $\mathbf{I}_2$ and the average power delivered to the load.



CHAPTER 16

CIRCUIT ANALYSIS WITH THE LAPLACE TRANSFORM; TRANSFER FUNCTIONS

resistor	$V = RI$
inductor, series form	$V = sL\,I - LI_0$
inductor, parallel form	$I = \dfrac{V}{sL} + \dfrac{I_0}{s}$
capacitor, series form	$V = \dfrac{I}{sC} + \dfrac{V_0}{s}$
capacitor, parallel form	$I = sC\,V - CV_0$
coupled coils	$V_1 = sL_1I_1 - L_1i_1(0) \pm \left[sMI_2 - Mi_2(0)\right]$

transfer function	$H(s) = \dfrac{Y(s)}{X(s)}$ (no initial energy)
impulse response	$h(t) = \mathscr{L}^{-1}\{H(s)\}$
step response	$y_u(t) = \mathscr{L}^{-1}\{H(s)/s\}, \quad h = \dfrac{dy_u}{dt}$
convolution	$y(t) = \displaystyle\int_0^t h(\lambda)\,x(t-\lambda)\,d\lambda$

components of the response

poles of H: transient; of X: steady state

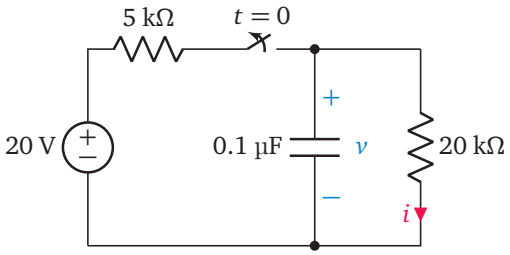
In this chapter I_0 and V_0 are the values of the inductor current and the capacitor voltage at $t = 0^-$; by the passive sign convention, I enters the element at the + terminal of V. In the series form LI_0 and V_0/s are voltage sources; in the parallel form I_0/s and CV_0 are current sources. The series form suits the mesh-current method and the parallel form the node-voltage method. In the s domain voltage is in V·s and current in A·s; impedance is still in ohms. For coupled coils the mutual term also brings an initial value; its sign follows the dot convention of Chapter 13. The transfer function is defined with all initial conditions zero. Time functions found by the inverse transform are valid for $t \geq 0$.

Natural and step responses in the s domain

16.4

★☆☆ Solution → p. 9

In the circuit shown, the switch has been closed for a long time and opens at $t = 0$, disconnecting the source.



a. Find $v(0^-)$.

b. Model the capacitor in series form and find the current in the 20 kΩ resistor ($I(s)$, downward). Find $i(0^+)$ with the initial-value theorem and check it from the circuit.

c. Model the capacitor in parallel form and find $V(s)$; show that the model of part b gives the same $V(s)$. Write $v(t)$.

Mesh, node and Thévenin; switched circuits

16.9

★★★ Solution → p. 10

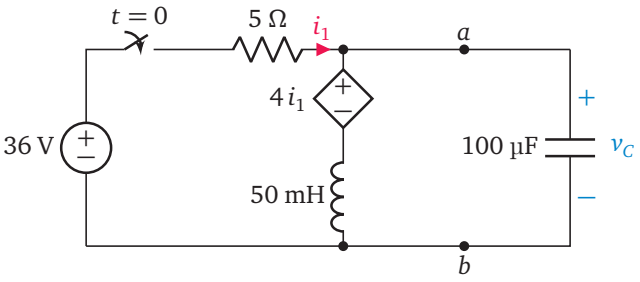
In the circuit shown, the switch closes at $t = 0$; there is no initial energy in the circuit. The dependent source (a current-controlled voltage source) is in series with the inductor, and its voltage is $4\,i_1$ (i_1 in amperes, voltage in volts).

a. Find the s-domain Thévenin voltage (V_{Th}) of the circuit to the left of terminals a–b.

b. Find the Thévenin impedance (Z_{Th}) using a test current.

c. Find $V_C(s)$ and $v_C(t)$ for $t \geq 0$. Check $v_C(\infty)$ with the dc circuit.

d. Find $dv_C/dt(0^+)$. Does v_C rise above its final value? Find its maximum value and the instant at which it occurs.



CHAPTER 17

FREQUENCY RESPONSE: RESONANCE, PASSIVE FILTERS, BODE PLOTS

steady state	$y_{ss} = A\lvert H(j\omega)\rvert \cos(\omega t + \phi + \theta)$	resonance	$\omega_0 = \dfrac{1}{\sqrt{LC}}$
cutoff frequency	$\lvert H(j\omega_c)\rvert = H_{max}/\sqrt{2}$	bandwidth	$\beta_{series} = \dfrac{R}{L}, \quad \beta_{parallel} = \dfrac{1}{RC}$
RC and RL	$\omega_c = \dfrac{1}{RC}, \quad \omega_c = \dfrac{R}{L}$	quality factor	$Q = \dfrac{\omega_0}{\beta}, \quad \omega_{c1}\,\omega_{c2} = \omega_0^2$
Bode magnitude	$A_{dB} = 20\log_{10}\lvert H(j\omega)\rvert$	cutoff frequencies	$\omega_{c1,c2} = \mp\dfrac{\beta}{2} + \sqrt{\left(\dfrac{\beta}{2}\right)^2 + \omega_0^2}$

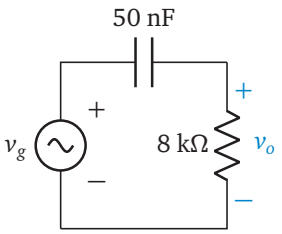
In this chapter sinusoids are written as cosines and amplitudes are peak values. Do not confuse the angular frequency in radians per second with the frequency in hertz; they are related by $\omega = 2\pi f$. The transfer function $H(s)$ is the ratio of the Laplace transforms of the output and the input when the circuit initially stores no energy. Substituting $s = j\omega$ gives the magnitude $|H(j\omega)|$ and the phase angle $\theta(\omega)$. H_{max} denotes the largest magnitude; at the cutoff (half-power) frequency the magnitude falls to $1/\sqrt{2}$ times H_{max}. Logarithms in Bode plots are base 10; a decade is an interval over which the frequency increases tenfold. Sources are ideal unless stated otherwise.

Low-pass and high-pass filters

17.3

★☆☆ Solution → p. 11

In the series RC circuit shown, the output (v_o) is the voltage across the 8 kΩ resistor.



a. Find the cutoff frequency in radians per second and in hertz.

b. Find v_o for $v_g = 10\cos(200\pi t - 15°)$ V.

Bode plots

17.11

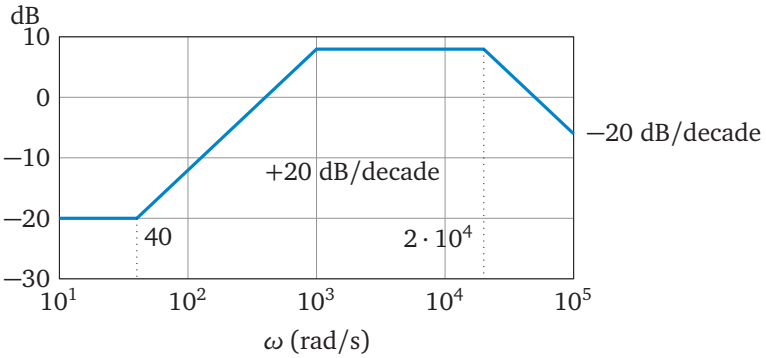
★★☆ Solution → p. 11

The figure shows the straight-line (asymptotic) Bode magnitude plot of a transfer function whose zero and poles are real and negative and for which $H(0) > 0$. The corner frequencies are 40, 10^3 and $2 \cdot 10^4$ rad/s.

a. Find $H(s)$. What is the horizontal asymptote in the middle region, in decibels?

b. Find the actual magnitude in decibels at $\omega = 80$ rad/s and $\omega = 10\,000$ rad/s; compare it with the asymptote. How are these frequencies related to the corners?

c. Find the steady-state output for $v_g = 3\cos(4000t)$ V. How far off are the amplitude read from the asymptote and the straight-line phase approximation?



SOLUTIONS

13 MUTUAL INDUCTANCE AND TRANSFORMERS

13.1

← Problem p. 3

Self terms: since both currents enter their own coil at the + terminal (passive sign convention), these are positive: $L_1\,di_1/dt$ and $L_2\,di_2/dt$.

Mutual terms: i_1 enters the first coil at the dotted terminal; the + terminal of the voltage it induces in the second coil is the dotted (lower) terminal. Since the + terminal of v_2 is at the top, this term is $-M\,di_1/dt$. i_2 enters the second coil at the undotted terminal; the + terminal of the voltage it induces in the first coil is the undotted (lower) terminal, so it enters v_1 as $-M\,di_2/dt$. In short: one current enters at a dot and the other at an undotted terminal, so the mutual terms are negative.

- (A): This is the case in which both currents enter at the dotted terminals.
- (B) and (C): Mutual inductance is symmetric ($M_{12} = M_{21}$); a term that is negative in one equation is negative in the other as well.
- (E): The sign of the self terms comes from the passive sign convention, not from the dots.

ANSWER D

13.8

← Problem p. 4

a. $\omega = 500$ rad/s: $j\omega L_1 = j25$, $j\omega L_2 = j36$, $j\omega M = j21\ \Omega$; the capacitor is $-j/(500 \cdot 80 \times 10^{-6}) = -j25\ \Omega$.

$$Z_{11} = 10 + 2 + j25 = 12 + j25\ \Omega,$$

$$Z_{22} = 3 + j36 + 15 - j25 = 18 + j11\ \Omega,$$

$$k = \frac{42}{\sqrt{50(72)}} = \frac{42}{60} = 0.700.$$

b. $|Z_{22}|^2 = 445$; the scale factor is $(\omega M/|Z_{22}|)^2 = 441/445 = 0.991$:

$$Z_r = 0.991(18 - j11) = 17.8 - j10.9\ \Omega.$$

The reflected impedance is proportional to the conjugate of Z_{22}: an inductive secondary appears capacitive to the primary. The impedance seen from the source (including its internal resistance):

$$Z_{\text{in}} = Z_{11} + Z_r = 29.8 + j14.1\ \Omega = 33.0\angle 25.3^\circ\ \Omega.$$

c. Primary current:

$$\mathbf{I}_1 = \frac{100}{33.0\angle 25.3^\circ} = 3.03\angle -25.3^\circ\ \text{A}.$$

In the secondary mesh (clockwise), $\mathbf{I}_2$ leaves the dotted terminal: $-j21\mathbf{I}_1 + Z_{22}\mathbf{I}_2 = 0$:

$$\mathbf{I}_2 = \frac{j21\,\mathbf{I}_1}{18 + j11} = 3.02\angle 33.3^\circ\ \text{A},$$

$$P_L = \tfrac{1}{2}(3.02^2)(15) = 68.2\ \text{W}.$$

Check: The power in the reflected resistance, $\tfrac{1}{2}(3.03^2)(17.84) = 81.9$ W, equals the total power in the secondary: $P_{R_2} + P_L = \tfrac{1}{2}(3.02^2)(3) + 68.2 = 13.6 + 68.2 = 81.9$ W.

Common mistake: Writing the reflected impedance as $(\omega M)^2/Z_{22}^*$. The correct form is $(\omega M)^2/Z_{22}$, which turns out to be proportional to Z_{22}^*.

ANSWER a. $Z_{11} = 12 + j25\ \Omega$, $Z_{22} = 18 + j11\ \Omega$, $k = 0.700$ b. $Z_r = 17.8 - j10.9\ \Omega$; $Z_{\text{in}} = 29.8 + j14.1\ \Omega$ c. $\mathbf{I}_1 = 3.03\angle -25.3^\circ$ A, $\mathbf{I}_2 = 3.02\angle 33.3^\circ$ A, 68.2 W

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16 CIRCUIT ANALYSIS WITH THE LAPLACE TRANSFORM; TRANSFER FUNCTIONS

16.4

← Problem p. 5

a. For $t < 0$ the capacitor is an open circuit; by voltage division, $v(0^-) = 20 \cdot 20/(5 + 20) = 16.0$ V.

b. Series form: $1/(sC) = 10^7/s$ in series with a $16/s$ source (+ on top). A single mesh remains:

$$I(s) = \frac{16/s}{10^7/s + 20\,000} = \frac{8 \times 10^{-4}}{s + 500}.$$

$$i(0^+) = \lim_{s\to\infty} sI(s) = 8 \times 10^{-4}\ \text{A} = 0.800\ \text{mA}.$$

From the circuit: at $t = 0^+$ the capacitor voltage remains at 16 V, and this voltage is directly across the 20 kΩ resistor: $16/20 = 0.800$ mA.

c. Parallel form: $10^7/s$ in parallel with $CV_0 = 1.6\times10^{-6}$ A·s (toward the top node). KCL at the top node:

$$10^{-7}s\,V + \frac{V}{20\,000} = 1.6\times10^{-6} \;\Rightarrow\; V(s) = \frac{16}{s+500}.$$

In the series form $V = 20\,000\,I(s) = 16/(s+500)$, the same expression: the two forms are source transformations of each other. Inverse transform:

$$v(t) = 16.0\,e^{-500t}\text{ V}, \quad t \geq 0.$$

Check: The time constant is $\tau = 20\text{ k}\Omega\times0.1\,\mu\text{F} = 2$ ms; the pole of $V(s)$ is also at $-1/\tau = -500\text{ s}^{-1}$.

Common mistake: Forgetting the $16/s$ source in the series form and getting $I(s) = 0$, or taking the source as 16 (without the s). The initial-condition source is also a Laplace transform; a constant voltage enters as $16/s$.

ANSWER a. 16.0 V b. $I = \frac{8\times10^{-4}}{s+500}$; $i(0^+) = 0.800$ mA c. $V = \frac{16}{s+500}$ (same in both forms); $v = 16.0e^{-500t}$ V, $t \geq 0$

16.9

← Problem p. 6

a. With the terminals open, i_1 flows through the branch containing the dependent source. KVL around the loop:

$$\frac{36}{s} = 5I_1 + 4I_1 + 0.05s\,I_1 \;\Rightarrow\; I_1 = \frac{720}{s\,(s+180)},$$

$$V_{\text{Th}} = (4+0.05s)\,I_1 = \frac{36\,(s+80)}{s\,(s+180)}.$$

b. The source is set to zero and a current I_t is injected at a. The 5 Ω resistor is now between a and the bottom node: $I_1 = -V_a/5$. The inductor carries $I_t + I_1$:

$$V_a = 4I_1 + 0.05s\,(I_t + I_1) = -0.8V_a + 0.05s\,I_t - 0.01s\,V_a,$$

$$Z_{\text{Th}} = \frac{V_a}{I_t} = \frac{0.05s}{1.8+0.01s} = \frac{5s}{s+180}\ \Omega.$$

The dependent source is not set to zero; compared with $5\ \Omega \parallel sL$, it shifts the pole from −100 to −180.

c. $1/(sC) = 10^4/s$; voltage divider:

$$V_C = V_{\text{Th}}\,\frac{10^4/s}{Z_{\text{Th}} + 10^4/s} = \frac{72\,000\,(s+80)}{s\,(s+200)(s+1800)}.$$

The coefficients are, for $s = 0$: $72\,000(80)/360\,000 = 16$; for $s = -200$: $72\,000(-120)/[(-200)(1600)] = 27$; for $s = -1800$: $72\,000(-1720)/[(-1800)(-1600)] = -43$:

$$v_C(t) = 16 + 27\,e^{-200t} - 43\,e^{-1800t}\text{ V}, \quad t \geq 0.$$

Check: At dc the inductor is a short circuit: $v_a = 4i_1$ and $36 = 5i_1 + 4i_1$, so $i_1 = 4$ A and $v_C = 16.0$ V. The final-value theorem also gives 16 V; $v_C(0) = 16 + 27 - 43 = 0$.

d. At $t = 0^+$, $v_C = 0$ and the inductor current is zero; all of $i_1 = 36/5 = 7.2$ A goes to the capacitor:

$$\frac{dv_C}{dt}(0^+) = \frac{7.2}{10^{-4}} = 72\,000\text{ V/s} = \lim_{s\to\infty} s^2V_C.$$

At the maximum $dv_C/dt = 0$: $5400\,e^{-200t} = 77\,400\,e^{-1800t}$, that is, $e^{1600t} = 43/3$:

$$t = \frac{\ln(43/3)}{1600} = 1.66\text{ ms}, \qquad v_{C,\max} = 33.2\text{ V}.$$

Yes, it rises to more than twice the final value (16 V). Initially the inductor acts like an open circuit; the capacitor charges rapidly toward 36 V through the 5 Ω resistor alone. Once the inductor current builds up, the branch with the dependent source pulls terminal a to 16 V. **Common mistake:** Also setting the dependent source to zero when finding Z_{Th}. This gives $5\ \Omega \parallel 0.05s = 5s/(s+100)$; the pole comes out at −100 instead of −180, and v_C is wrong.

ANSWER a. $V_{\text{Th}} = \frac{36(s+80)}{s(s+180)}$ b. $Z_{\text{Th}} = \frac{5s}{s+180}\ \Omega$ c. $v_C = 16 + 27e^{-200t} - 43e^{-1800t}$ V, $t \geq 0$; 16.0 V d. 72 000 V/s; yes: 33.2 V, 1.66 ms

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17 FREQUENCY RESPONSE: RESONANCE, PASSIVE FILTERS, BODE PLOTS

17.3

← Problem p. 7

a. Voltage divider: $H(s) = R/(R+1/sC) = s/(s+1/RC)$. $H(0) = 0$, $H(\infty) = 1$; the circuit is high-pass. The condition $|H(j\omega_c)| = 1/\sqrt{2}$ gives $\omega_c = 1/RC$:

$$\omega_c = \frac{1}{8\cdot 10^3 \cdot 50 \cdot 10^{-9}} = 2500 \text{ rad/s},$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{2500}{2\pi} = 398 \text{ Hz}.$$

b. The input frequency is $f = 100$ Hz, so its angular frequency is $\omega = 2\pi(100) = 628.3$ rad/s.

$$|H| = \frac{\omega}{\sqrt{\omega^2+\omega_c^2}} = \frac{628.3}{\sqrt{628.3^2+2500^2}} = 0.244,$$

$$\theta = 90^\circ - \tan^{-1}\frac{628.3}{2500} = 90^\circ - 14.1^\circ = 75.9^\circ,$$

$$v_o = 2.44\cos(200\pi t + 60.9^\circ)\text{ V}.$$

Common mistake: Substituting 100 in hertz directly for ω; this gives $|H| \approx 0.04$, a value about six times too small.

ANSWER a. $\omega_c = 2500$ rad/s, $f_c = 398$ Hz
b. $v_o = 2.44\cos(200\pi t + 60.9^\circ)$ V

17.11

← Problem p. 8

a. The plot is horizontal at −20 dB at low frequencies: $H(0) = 10^{-20/20} = 0.1$. The slope becomes +20 dB/decade at 40 rad/s (zero), levels off again at 1000 rad/s (pole), and falls at −20 dB/decade beyond $2\cdot 10^4$ rad/s (second pole):

$$H(s) = \frac{0.1\,(1+s/40)}{(1+s/1000)(1+s/20\,000)} = \frac{50\,000\,(s+40)}{(s+1000)(s+20\,000)}.$$

The middle asymptote rises over the $\log_{10} 25 = 1.398$ decades from 40 to 1000 rad/s:

$$-20 + 20(1.398) = 7.96 \text{ dB} \quad (|H| = 2.50).$$

b. 80 rad/s is twice the corner of the zero; 10 000 rad/s is half the corner of the second pole. The asymptote values are $-20 + 20\log_{10} 2 = -14.0$ dB and 7.96 dB. Substituting the numbers gives $|H(j80)| = 0.2229$ and $|H(j10^4)| = 2.225$:

$$20\log_{10} 0.2229 = -13.0 \text{ dB},$$

$$20\log_{10} 2.225 = 6.95 \text{ dB}.$$

The actual curve is about 1 dB above the asymptote at 80 rad/s (+0.9 dB, zero) and about 1 dB below it at 10 000 rad/s (−1.0 dB, pole). At twice or half a corner frequency the correction is $10\log_{10}(1.25) \approx 1$ dB; the small contribution of the neighboring corners changes the difference slightly.

c. Substitute $s = j4000$:

$$H(j4000) = \frac{50\,000\,(40+j4000)}{(1000+j4000)(20\,000+j4000)} = 2.38\angle 2.2^\circ,$$

$$v_o = 7.14\cos(4000t + 2.2^\circ)\text{ V}.$$

At 4000 rad/s the asymptote gives 7.96 dB, that is, $|H| = 2.50$; the amplitude read is 7.50 V, 5% larger than the actual value. In the straight-line phase approximation the zero contributes +90° (beyond 400 rad/s), the pole at 1000 rad/s contributes $-45^\circ(1+\log_{10} 4) = -72.1^\circ$, and the pole at $2\cdot 10^4$ rad/s contributes $-45^\circ \log_{10}(4000/2000) = -13.5^\circ$:

$$\theta \approx 90^\circ - 72.1^\circ - 13.5^\circ = 4.4^\circ \quad \text{(actual } 2.2^\circ\text{)}.$$

ANSWER
a. $H(s) = 50\,000(s+40)/[(s+1000)(s+20\,000)]$; 7.96 dB b. −13.0 dB (0.9 dB above the asymptote); 6.95 dB (1.0 dB below)
c. $v_o = 7.14\cos(4000t + 2.2^\circ)$ V; 7.50 V from the asymptote, phase approximation 4.4°

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IN THE FULL BOOK

- 12 chapters and 188 problems, all with step-by-step solutions.
- A reminder box at the start of each chapter, summarizing the relations the problems rely on.
- Basic, intermediate and advanced problems, each with a dotted area for your own work.
- One-click links from each problem to its solution and back.



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