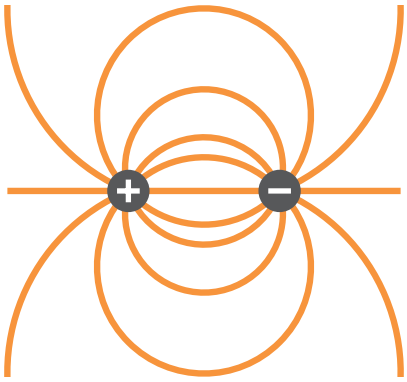
ELECTROMAGNETIC THEORY 1

PROBLEM BOOK

SAMPLE PAGES

11 chapters · 178 problems · step-by-step solutions



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How to use this book

- Each chapter opens with a reminder box summarizing the definitions and formulas the problems rely on.
- Difficulty: ★☆☆ basic ★★☆ intermediate ★★★ advanced. Each chapter has a few multiple-choice concept questions; each chapter ends with one or more exam-style mixed problems.
- The dotted area below each problem is for your own work. Solve first; if stuck, go back to the reminder box and the lecture notes, and look at the solution last.
- All solutions are at the end of the book. The Solution → link next to a problem takes you to its solution, and ← Problem takes you back. Short answers are in the **ANSWER** box.
- **Notation:** SI units are used; $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m. Vectors are bold ($\mathbf{E}$) and unit vectors carry a hat ($\hat{\mathbf{x}}$). Cylindrical coordinates are (r, ϕ, z) and spherical coordinates (R, θ, ϕ). In the figures the electric field is orange and the magnetic field turquoise; positive charge is red and negative charge blue. Current is shown by a black arrow, velocity by a blue arrow and force by a red arrow; a charge whose sign is asked is drawn in black.
- **Midterm and final:** Electromagnetic Theory 1 is usually taught in the second year. The midterm usually covers vector analysis and electrostatics (Chapters 1–6); the final also covers Poisson's and Laplace's equations and the method of images, steady current, magnetostatics and Faraday's law (Chapters 7–11). Chapter 11 (Faraday and Maxwell) also appears unchanged in *Electromagnetic Theory 2*, for schools that cover it in the second course. Choose the chapters to study according to your own course syllabus.

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CHAPTER 3

GAUSS'S LAW

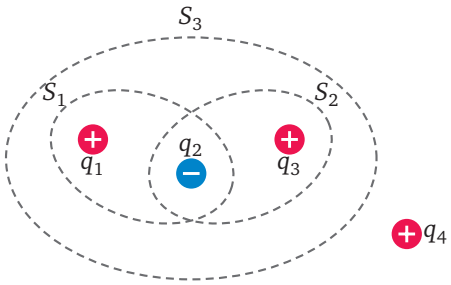
electric flux density	$\mathbf{D} = \varepsilon_0 \mathbf{E}$ (free space)	spherical symmetry	$D_R\, 4\pi R^2 = Q_{\text{enc}}$
Gauss's law	$\oint_S \mathbf{D} \cdot d\mathbf{S} = Q_{\text{enc}}$	cylindrical symmetry	$D_r\, 2\pi r L = Q_{\text{enc}}$
differential form	$\nabla \cdot \mathbf{D} = \rho_v$	infinite sheet	$\mathbf{E} = \pm \frac{\rho_s}{2\varepsilon_0} \hat{\mathbf{n}}$
divergence theorem	$\oint_S \mathbf{A} \cdot d\mathbf{S} = \int_V \nabla \cdot \mathbf{A}\, dv$	divergence	$\frac{1}{r}\frac{\partial (r D_r)}{\partial r}$, $\frac{1}{R^2}\frac{\partial (R^2 D_R)}{\partial R}$

In this chapter all charges are in free space: $\mathbf{D} = \varepsilon_0 \mathbf{E}$, $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m. Cylindrical coordinates are written (r, ϕ, z) and spherical coordinates (R, θ, ϕ). On a closed surface $d\mathbf{S}$ points outward. Gauss's law holds for every closed surface, but it yields the field only when, by symmetry, D is constant on the surface or tangent to it. The divergence formulas apply to fields that have only the indicated component.

Electric flux and Gauss's law

3.2 ★☆☆ Solution →

The figure shows four point charges, $q_1 = 4$ nC, $q_2 = -7$ nC, $q_3 = 2$ nC and $q_4 = 5$ nC, and the cross sections of three closed surfaces. S_1 encloses q_1 and q_2, S_2 encloses q_2 and q_3, and S_3 encloses the first three charges; q_4 is outside all of them.



a. Find the net outward flux of $\mathbf{D}$ through each surface.

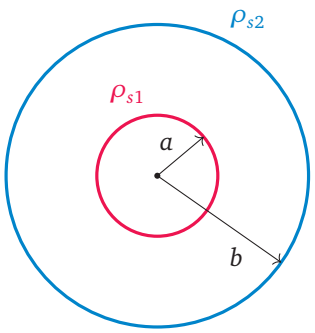
b. Find the net outward flux of $\mathbf{E}$ through S_3.

c. If q_4 is moved slightly closer to S_3 (still outside it), do the flux through S_3 and the $\mathbf{E}$ on S_3 change?

Spherical symmetry

3.5 ★★☆ Solution →

Of two thin concentric spherical shells, the inner one has radius $a = 2$ cm and surface charge density $\rho_{s1} = 50\ \text{nC/m}^2$; the outer one has radius $b = 5$ cm.



a. What must the density ρ_{s2} of the outer shell be for the field to vanish in the outer region ($R > b$)?

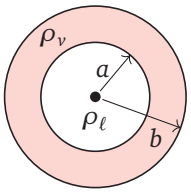
b. In that case, find $\mathbf{E}$ at $R = 1$ cm, 3 cm and 6 cm.

c. Sketch $E(R)$. Where is the field discontinuous, and what is the size of each discontinuity?

Cylindrical symmetry

3.7 ★★☆ Solution →

An infinitely long structure has a uniform line charge ρ_ℓ on the z axis and a uniform volume charge $\rho_v = 5\ \mu\text{C/m}^3$ in the cylindrical shell $a = 1.5\ \text{cm} < r < b = 2.5\ \text{cm}$.



a. What must ρ_ℓ be for the field to vanish for $r > b$?

b. In that case, find $\mathbf{E}$ in the regions $r < a$ and $a < r < b$.

c. Compute the field at $r = 0.75$ cm and $r = 2$ cm, and state its direction.

CHAPTER 6

CAPACITANCE

capacitance	$C = \dfrac{Q}{V}$	series, parallel	$\dfrac{1}{C} = \sum \dfrac{1}{C_i}, \quad C = \sum C_i$
parallel plate	$C = \dfrac{\varepsilon A}{d}$	energy	$W_e = \frac{1}{2}CV^2 = \dfrac{Q^2}{2C} = \displaystyle\int_V \frac{1}{2}\varepsilon E^2 \, \mathrm{d}v$
coaxial	$C = \dfrac{2\pi\varepsilon L}{\ln(b/a)}$	force, constant Q	$F_x = -\dfrac{\partial W_e}{\partial x} = \dfrac{Q^2}{2C^2}\dfrac{\partial C}{\partial x}$
spherical	$C = \dfrac{4\pi\varepsilon ab}{b-a}$	force, constant V	$F_x = +\dfrac{\partial W_e}{\partial x} = \dfrac{V^2}{2}\dfrac{\partial C}{\partial x}$

The permittivity of free space is $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, and that of a dielectric is $\varepsilon = \varepsilon_r \varepsilon_0$. In this chapter a capacitor consists of two conductors separated by a dielectric; one conductor carries charge $+Q$, the other $-Q$, and the voltage between them is denoted by V. Unless stated otherwise, edge effects (fringing) are neglected. The dielectric strength of a material is the largest field intensity it can sustain without breakdown; for air it is taken as 3 MV/m. In the force formulas, x is the direction of the virtual displacement of the body.

Capacitance and basic capacitors

6.1 ★☆☆ Solution →

An air-filled parallel-plate capacitor has capacitance C. Which of the following is true?

A) If the voltage between the plates is doubled, the charge on the plates also doubles; C does not change.

B) C increases as the charge on the plates increases.

C) If the space between the plates is completely filled with a dielectric of $\varepsilon_r = 3$, C drops to one third.

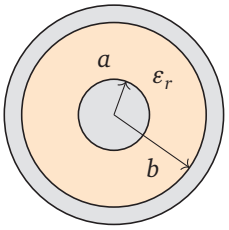
D) If both the plate area and the plate separation are doubled, C becomes four times larger.

E) C depends on the conductivity of the plate metal; better-conducting plates give a larger capacitance.

6.3

★★☆ Solution →

The inner conductor of a coaxial cable has radius $a = 0.6$ mm; the space between the conductors is filled with polyethylene ($\varepsilon_r = 2.25$). The cable is to have a capacitance per unit length of $C' = 100$ pF/m.



a. What must the inner radius b of the outer conductor be?

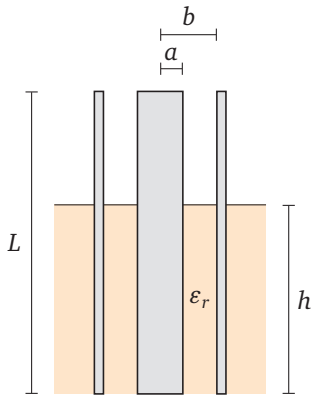
b. With 2 kV applied between the conductors, find the charge per unit length and the field at $r = a$ and $r = b$. Why is the field largest at the surface of the inner conductor?

c. The field in the polyethylene must not exceed 9 MV/m (half its dielectric strength). What is the highest voltage at which the cable can be used?

Mixed problem

6.14 ★★★ Solution →

The liquid level in a fuel tank is measured with a vertical coaxial probe: the inner rod has radius $a = 4$ mm, the outer tube has inner radius $b = 10$ mm, and the probe is $L = 40$ cm long. The liquid ($\varepsilon_r = 2.2$, nonconducting) fills the tube up to the same height h as the level outside. Fringing is neglected.



a. Find $C(h)$; compute the capacitance when the tank is empty and when the probe is fully immersed.

b. If the measured capacitance is 42.5 pF, what is the liquid level?

c. If 500 V is applied to the probe, what electric force pulls the liquid upward?

d. For $h = 25$ cm and 500 V, find the field and the surface charge density at the surface of the inner rod, separately for the parts in air and in the liquid.

CHAPTER 11

FARADAY'S LAW AND MAXWELL'S EQUATIONS

Faraday	$\mathscr{V} = \oint_C \mathbf{E}\cdot d\mathbf{l} = -\dfrac{d\Phi}{dt}$	continuity	$\nabla\cdot\mathbf{J} = -\dfrac{\partial \rho_v}{\partial t}$
moving circuit	$\mathscr{V} = \oint_C (\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l} - \int_S \dfrac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{S}$	charge relaxation	$\rho_v = \rho_{v0}\, e^{-t/\tau}, \quad \tau = \dfrac{\varepsilon}{\sigma}$
curl	$\nabla\times\mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}, \quad \nabla\times\mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t}$	boundary cond.	$E_{1t} = E_{2t}, \quad \hat{\mathbf{n}}_{21}\cdot(\mathbf{D}_1 - \mathbf{D}_2) = \rho_s$
			$B_{1n} = B_{2n}, \quad \hat{\mathbf{n}}_{21}\times(\mathbf{H}_1 - \mathbf{H}_2) = \mathbf{J}_s$
divergence equations	$\nabla\cdot\mathbf{D} = \rho_v, \quad \nabla\cdot\mathbf{B} = 0$	potentials	$\mathbf{B} = \nabla\times\mathbf{A}, \quad \mathbf{E} = -\nabla V - \dfrac{\partial \mathbf{A}}{\partial t}$

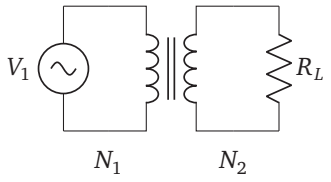
In this chapter $\varepsilon_0 = 8.854\times10^{-12}$ F/m and $\mu_0 = 4\pi\times10^{-7}$ H/m. In the flux $\Phi = \int_S \mathbf{B}\cdot d\mathbf{S}$ the direction of $d\mathbf{S}$ is related to the direction of travel around the contour C by the right-hand rule; the emf is positive in this direction of travel. For an N-turn coil the flux linkage is $N\Phi$. The displacement current density is $\mathbf{J}_d = \partial\mathbf{D}/\partial t$ and the conduction current density is $\mathbf{J} = \sigma\mathbf{E}$. $\hat{\mathbf{n}}_{21}$ points from medium 2 to medium 1. Cylindrical coordinates are (r, ϕ, z) and spherical coordinates are (R, θ, ϕ). Results needed from static topics, such as the field of a solenoid or the capacitance of a capacitor, are given in the problem.

Faraday's law and transformer emf

11.4

★☆☆ Solution →

An ideal transformer has $N_1 = 600$ turns on the primary winding and $N_2 = 90$ turns on the secondary. A voltage $V_1 = 120$ V (rms) is applied to the primary, and $R_L = 12\ \Omega$ is connected to the secondary. Find V_2, I_2, I_1, the resistance seen from the primary, and the load power. What are the assumptions of an ideal transformer?

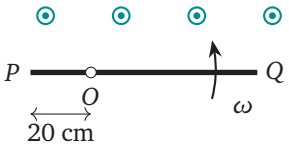


Motional emf and generators

11.5

★★☆ Solution →

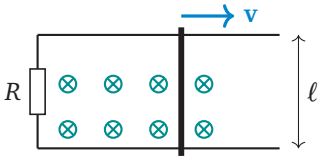
A conducting rod PQ has length $L = 75$ cm. The rod rotates in a horizontal plane at 120 revolutions per minute (counterclockwise viewed from above) about a vertical axis through the point O, which is 20 cm from end P. There is a vertical field $\mathbf{B} = 50\ \mu\text{T}\,\hat{\mathbf{z}}$ in the region ($+z$ points up). Find the voltages of ends P and Q relative to V_O, and $V_Q - V_P$. Where must the axis pass for there to be no voltage between the ends?



11.6

★★★ Solution →

Two horizontal, frictionless conducting rails are $\ell = 25$ cm apart; one end of the rails is closed by a resistor $R = 0.3\ \Omega$. A conducting rod of mass $m = 50$ g rests perpendicular to the rails. There is a vertical field $B = 0.6$ T in the region. At $t = 0$ the rod is given a speed $v_0 = 4$ m/s away from the resistor and released (the resistance of the rails and rod is negligible).



a. Find the emf, the current and the force on the rod at $t = 0$.

b. Find $v(t)$.

c. How far does the rod travel before it stops?

d. Find the total heat dissipated in the resistor and verify energy conservation.

SOLUTIONS

3 GAUSS'S LAW

3.2

← Problem

a. By Gauss's law, the net flux of **D** equals the total charge enclosed:

$$\Psi_1 = q_1 + q_2 = -3.00\ \text{nC}, \qquad \Psi_2 = q_2 + q_3 = -5.00\ \text{nC},$$

$$\Psi_3 = q_1 + q_2 + q_3 = -1.00\ \text{nC}.$$

The minus sign means that the net flux *enters* the surface.

b. Since $\mathbf{E} = \mathbf{D}/\varepsilon_0$ in free space,

$$\oint_{S_3} \mathbf{E}\cdot d\mathbf{S} = \frac{-1.00\times10^{-9}}{8.854\times10^{-12}} = -113\ \text{V·m}.$$

c. The net flux does not change; as long as q_4 stays outside, Q_{enc} is the same. But **E** on S_3 does change: the field on the surface is the sum of the fields of all charges, q_4 included. The field of q_4 enters on one side of the surface and leaves on the other; its contribution to the net flux is zero.

ANSWER a. −3.00 nC, −5.00 nC, −1.00 nC b. −113 V·m c. net flux unchanged; **E** changes

3.5

← Problem

a. A zero field for $R > b$ means that the total enclosed charge is zero:

$$4\pi a^2\rho_{s1} + 4\pi b^2\rho_{s2} = 0,$$

$$\rho_{s2} = -\rho_{s1}\frac{a^2}{b^2} = -50\cdot\frac{4}{25} = -8.00\ \text{nC/m}^2.$$

b. Spherical Gaussian surface. $R < a$: no charge inside, $\mathbf{E} = 0$. $a < R < b$: $Q_{\text{enc}} = 4\pi a^2\rho_{s1}$,

$$\mathbf{E} = \frac{\rho_{s1}a^2}{\varepsilon_0 R^2}\hat{\mathbf{R}}.$$

At $R = 3$ cm:

$$E = \frac{(50\times10^{-9})(0.02)^2}{(8.854\times10^{-12})(0.03)^2} = 2.51\ \text{kV/m}.$$

$R > b$: $Q_{\text{enc}} = 0$, $\mathbf{E} = 0$.

c. E is zero for $R < a$; at $R = a$ it jumps to $\rho_{s1}/\varepsilon_0 = 5.65$ kV/m; for $a < R < b$ it decreases as $1/R^2$; at $R = b$ it drops from $\rho_{s1}a^2/(\varepsilon_0 b^2) = 0.904$ kV/m to zero. The discontinuities are where the surface charges are, and their size is ρ_s/ε_0: $50/8.854 = 5.65$ kV/m and $-8/8.854 = -0.904$ kV/m.

Check: The charge of the inner shell is $4\pi(0.02)^2(50) = 0.251$ nC and that of the outer shell $4\pi(0.05)^2(-8) = -0.251$ nC; the total is zero.

ANSWER a. −8.00 nC/m² b. 0; 2.51 kV/m $\hat{\mathbf{R}}$; 0 c. jumps of +5.65 kV/m at $R = a$, −0.904 kV/m at $R = b$ (ρ_s/ε_0)

3.7

← Problem

a. Use a cylindrical Gaussian surface of radius r and length L. If the field is zero for $r > b$, the total charge per unit length is zero:

$$\rho_\ell + \rho_v\pi(b^2 - a^2) = 0,$$

$$\rho_\ell = -\pi(5\times10^{-6})(6.25 - 2.25)\times10^{-4} = -6.28\ \text{nC/m}.$$

b. $D_r\,2\pi rL = Q_{\text{enc}}$:

$$r < a: \quad \mathbf{E} = \frac{\rho_\ell}{2\pi\varepsilon_0 r}\hat{\mathbf{r}},$$

$$a < r < b: \quad \mathbf{E} = \frac{\rho_\ell + \rho_v\pi(r^2 - a^2)}{2\pi\varepsilon_0 r}\hat{\mathbf{r}} = \frac{\rho_v(r^2 - b^2)}{2\varepsilon_0 r}\hat{\mathbf{r}}.$$

c. $r = 0.75$ cm:

$$\mathbf{E} = \frac{-6.283\times10^{-9}}{2\pi(8.854\times10^{-12})(0.0075)}\hat{\mathbf{r}} = -15.1\ \text{kV/m}\,\hat{\mathbf{r}},$$

$r = 2$ cm:

$$\mathbf{E} = \frac{(5\times10^{-6})(4 - 6.25)\times10^{-4}}{2(8.854\times10^{-12})(0.02)}\hat{\mathbf{r}} = -3.18\ \text{kV/m}\,\hat{\mathbf{r}}.$$

In both cases the field points toward the axis (inward): the net charge inside the cylinder of radius r is negative.

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Check: At $r = a$ both expressions give the same value (-7.53 kV/m); a volume charge leaves the field continuous. At $r = b$ the shell expression is zero.

ANSWER a. -6.28 nC/m b. $\frac{\rho_\ell}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}$; $\frac{\rho_v(r^2-b^2)}{2\varepsilon_0 r}\hat{\mathbf{r}}$
c. -15.1 kV/m $\hat{\mathbf{r}}$; -3.18 kV/m $\hat{\mathbf{r}}$ (inward)

6 CAPACITANCE

6.1

← Problem

Capacitance depends only on the shapes, sizes and relative positions of the conductors and on the permittivity of the medium between them: for parallel plates $C = \varepsilon A/d$. Q and V change in proportion; their ratio is constant. (A) is correct.

- (B): If Q increases, V increases in the same proportion; $C = Q/V$ does not change.
- (C): $C = \varepsilon_r\varepsilon_0 A/d$; the dielectric triples C.
- (D): When A and d are both doubled, A/d does not change; C stays the same.
- (E): In electrostatics the field inside a conductor is zero and the conductor is an equipotential; the value of the conductivity does not enter the capacitance.

ANSWER A

6.3

← Problem

a. From $C' = 2\pi\varepsilon/\ln(b/a)$,

$$\ln\frac{b}{a} = \frac{2\pi\varepsilon_r\varepsilon_0}{C'} = \frac{2\pi(2.25)(8.854\times10^{-12})}{10^{-10}} = 1.252,$$

$$\frac{b}{a} = e^{1.252} = 3.50, \qquad b = 2.10 \text{ mm}.$$

b. $\rho_\ell = C'V = (10^{-10})(2000) = 200$ nC/m. From Gauss's law, $E = \rho_\ell/(2\pi\varepsilon r)$:

$$E(a) = \frac{2\times10^{-7}}{2\pi(2.25)(8.854\times10^{-12})(6\times10^{-4})} = 2.66 \text{ MV/m},$$

$$E(b) = E(a)\frac{a}{b} = 762 \text{ kV/m}.$$

On the surface of smaller radius the same charge is spread over a smaller area: $D = \rho_\ell/(2\pi r)$ grows as r decreases. Breakdown always starts at the surface of the inner conductor.

c. Since $E(a) = V/[a\ln(b/a)]$,

$$V_{\max} = E_{\text{allow}}\, a \ln\frac{b}{a},$$

$$V_{\max} = (9\times10^6)(6\times10^{-4})(1.252) = 6.76 \text{ kV}.$$

Check: $\int_a^b E\,\mathrm{d}r = \frac{\rho_\ell}{2\pi\varepsilon}\ln\frac{b}{a} = 2000$ V.

ANSWER a. $b = 2.10$ mm ($b/a = 3.50$)
b. 200 nC/m; 2.66 MV/m; 762 kV/m c. 6.76 kV

6.14

← Problem

a. The liquid-filled and air-filled parts lie between the same two conductors, so they are in parallel. The capacitance per unit length of an air-filled coaxial cable is

$$C_0' = \frac{2\pi\varepsilon_0}{\ln(b/a)} = \frac{2\pi(8.854\times10^{-12})}{\ln 2.5} = 60.7 \text{ pF/m},$$

$$C(h) = C_0'\big[(L-h) + \varepsilon_r h\big] = C_0'\big[L + (\varepsilon_r - 1)h\big].$$

Empty: $C_0'L = 24.3$ pF. Full: $\varepsilon_r C_0'L = 53.4$ pF.

b. C varies linearly with h:

$$h = \frac{C/C_0' - L}{\varepsilon_r - 1} = \frac{0.7000 - 0.40}{1.2} = 0.250 \text{ m} = 25.0 \text{ cm}.$$

c. At constant voltage the vertical force is

$$F = \frac{V^2}{2}\frac{\mathrm{d}C}{\mathrm{d}h} = \tfrac{1}{2}V^2 C_0'(\varepsilon_r - 1),$$

$$F = \tfrac{1}{2}(500)^2(60.71\times10^{-12})(1.2) = 9.11 \text{ μN}.$$

The force is upward (C increases with h) and does not depend on h. It is very small compared with the weight of the liquid column; the probe does not noticeably change the level.

d. The voltage is the same in both parts, and so is the field over the cross section: $E(r) = V/[r\ln(b/a)]$.

$$E(a) = \frac{500}{(0.004)\ln 2.5} = 136 \text{ kV/m}.$$

In the air-filled part $\rho_s = \varepsilon_0 E(a) = 1.21$ μC/m^2, and in the liquid-filled part $\rho_s = \varepsilon_r\varepsilon_0 E(a) = 2.66$ μC/m^2. The extra charge gathers on the liquid-filled part; this is why the capacitance increases with h.

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Check: The force in (c) is the integral over the cross section of the difference in energy density: $\int_a^b \tfrac{1}{2}(\varepsilon-\varepsilon_0)E^2\,2\pi r\,dr = \tfrac{1}{2}(\varepsilon_r-1)C_0'V^2$.

ANSWER a. $C_0'[L+(\varepsilon_r-1)h]$, $C_0' = 60.7$ pF/m; 24.3 pF; 53.4 pF b. 25.0 cm c. 9.11 μN, upward d. 136 kV/m (in both); 1.21 and 2.66 μC/m^2

11 FARADAY'S LAW AND MAXWELL'S EQUATIONS

11.4

← Problem

In an ideal transformer the core permeability is taken as infinite (zero magnetizing current), and there is no leakage flux and no loss. The same flux passes through every turn of both windings; Faraday's law gives $V_1/V_2 = N_1/N_2$. The ampere-turn balance is $N_1 I_1 = N_2 I_2$.

$$V_2 = V_1\frac{N_2}{N_1} = 120\cdot\frac{90}{600} = 18.0\text{ V}, \qquad I_2 = \frac{V_2}{R_L} = 1.50\text{ A},$$

$$I_1 = I_2\frac{N_2}{N_1} = 0.225\text{ A}, \qquad R_{\text{in}} = \left(\frac{N_1}{N_2}\right)^2 R_L = 533\ \Omega.$$

The load power is $P = V_2 I_2 = 27.0$ W; the power drawn by the primary is also $V_1 I_1 = 27.0$ W.

ANSWER $V_2 = 18.0$ V, $I_2 = 1.50$ A, $I_1 = 0.225$ A, $R_{\text{in}} = 533\ \Omega$, $P = 27.0$ W

11.5

← Problem

The point of the rod at distance r from O has speed $u = \omega r$ in the $\hat{\boldsymbol{\phi}}$ direction. $\mathbf{u}\times\mathbf{B} = \omega r B(\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}}) = \omega r B\,\hat{\mathbf{r}}$: positive charges are pushed outward from the axis, and both ends become positive relative to O. $\omega = 2\pi(120/60) = 12.57$ rad/s.

$$V_Q - V_O = \int_0^{\ell_Q} \omega r B\,dr = \frac{\omega B \ell_Q^2}{2},$$

$$V_Q - V_O = \frac{(12.57)(5\times10^{-5})(0.55)^2}{2} = 95.0\ \mu\text{V},$$

$$V_P - V_O = \frac{\omega B \ell_P^2}{2} = 12.6\ \mu\text{V}.$$

$$V_Q - V_P = \frac{\omega B}{2}(\ell_Q^2 - \ell_P^2) = 82.5\ \mu\text{V}.$$

For the ends to be at the same voltage we need $\ell_P = \ell_Q$: the axis must pass through the middle of the rod. For a rod rotating about one end the difference would be $\omega B L^2/2 = 177$ μV.

Check: From the area swept by the rod: the segment O–Q sweeps an area $\tfrac{1}{2}\omega\ell_Q^2$ per unit time; the emf is $B\cdot\tfrac{1}{2}\omega\ell_Q^2$.

ANSWER $V_P - V_O = 12.6$ μV, $V_Q - V_O = 95.0$ μV, $V_Q - V_P = 82.5$ μV; axis through the middle of the rod

11.6

← Problem

a. The motional emf is $\mathscr{V} = B\ell v$ and the current is $I = B\ell v/R$; the magnetic force on the current-carrying rod is $\mathbf{F} = I\ell\times\mathbf{B}$, of magnitude $I\ell B$, opposite to the motion (Lenz):

$$\mathscr{V}_0 = (0.6)(0.25)(4) = 0.600\text{ V}, \qquad I_0 = 2.00\text{ A},$$

$$F_0 = \frac{B^2\ell^2 v_0}{R} = 0.300\text{ N}.$$

b. Newton: $m\,dv/dt = -\frac{B^2\ell^2}{R}v$, so $v = v_0 e^{-t/\tau}$:

$$\tau = \frac{mR}{B^2\ell^2} = \frac{(0.05)(0.3)}{(0.36)(0.0625)} = 0.667\text{ s}.$$

c. Integrating the velocity over time:

$$x_\infty = \int_0^\infty v_0 e^{-t/\tau}\,dt = v_0\tau = 2.67\text{ m}.$$

The velocity never becomes zero at any finite time, yet the distance is finite.

d. The power in the resistor is $\mathscr{V}^2/R$:

$$W_R = \int_0^\infty \frac{(B\ell v)^2}{R}\,dt = \frac{B^2\ell^2 v_0^2}{R}\cdot\frac{\tau}{2} = 0.400\text{ J}.$$

This equals the initial kinetic energy ($\tfrac{1}{2}mv_0^2 = 0.400$ J): all the kinetic energy is converted into heat.

Common mistake: Taking the force as constant and assuming uniform deceleration. The force is proportional to the velocity; the decay is exponential.

ANSWER a. 0.600 V, 2.00 A, 0.300 N (opposing the motion) b. $v = v_0 e^{-t/\tau}$, $\tau = 0.667$ s c. 2.67 m d. 0.400 J $= \tfrac{1}{2}mv_0^2$

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