ELECTROMAGNETIC THEORY 2

PROBLEM BOOK

SAMPLE PAGES

9 chapters · 140 problems · step-by-step solutions



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How to use this book

- Each chapter opens with a reminder box summarizing the definitions and formulas the problems rely on.
- Difficulty: ★☆☆ basic ★★☆ intermediate ★★★ advanced. Each chapter has a few multiple-choice concept questions; each chapter ends with one or more exam-style mixed problems.
- The dotted area below each problem is for your own work. Solve first; if stuck, go back to the reminder box and the lecture notes, and look at the solution last.
- All solutions are at the end of the book. The Solution → link next to a problem takes you to its solution, and ← Problem takes you back. Short answers are in the **ANSWER** box.
- **Notation:** SI units are used; $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m. Vectors are bold ($\mathbf{E}$) and unit vectors carry a hat ($\hat{\mathbf{x}}$). Time-harmonic fields use $e^{j\omega t}$ with a cosine reference: $\mathbf{E}(z,t) = \mathrm{Re}[\mathbf{E}(z)\,e^{j\omega t}]$; a phasor's magnitude is its peak value, and the time-average power density is $\frac{1}{2}\mathrm{Re}(\mathbf{E}\times\mathbf{H}^*)$. The handedness of polarization follows the IEEE convention. In the figures the electric field is orange and the magnetic field turquoise.
- **Shared chapter:** Chapter 11 (Faraday's law and Maxwell's equations) is the same as in *Electromagnetic Theory 1*. It is included here for schools that cover it in the second course or review it before waves. Results needed from Chapters 1–10 (boundary conditions, capacitance and inductance formulas) are given briefly in the problems and reminder boxes.
- **Midterm and final:** Electromagnetic Theory 2 (electromagnetic waves) is usually taught in the fall of the third year. The midterm usually covers Maxwell's equations, plane waves and lossy media (Chapters 11–14); the final also covers reflection, transmission lines, the Smith chart and waveguides (Chapters 15–19). Chapters 17–18 rely only on circuit theory, so schools that teach transmission lines in a separate course or before waves can use them directly. Choose the chapters to study according to your own course syllabus.

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CHAPTER 13

PLANE WAVES IN LOSSLESS MEDIA AND POLARIZATION

phase constant
$\beta = \omega\sqrt{\mu\varepsilon} = \dfrac{2\pi}{\lambda}, \quad u_p = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{\mu\varepsilon}}$

intrinsic impedance $\eta = \sqrt{\dfrac{\mu}{\varepsilon}} = \eta_0\sqrt{\dfrac{\mu_r}{\varepsilon_r}}$

plane wave $\mathbf{E} = \mathbf{E}_0\, e^{-j\mathbf{k}\cdot\mathbf{R}}, \quad \mathbf{k} = \beta\,\hat{\mathbf{k}}, \quad \hat{\mathbf{k}}\cdot\mathbf{E}_0 = 0$

TEM relation $\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{k}}\times\mathbf{E}, \quad \mathbf{E} = -\eta\,\hat{\mathbf{k}}\times\mathbf{H}$

power density $\mathbf{S} = \mathbf{E}\times\mathbf{H}, \quad \mathbf{S}_{\text{avg}} = \dfrac{|\mathbf{E}_0|^2}{2\eta}\hat{\mathbf{k}}$

polarization $\mathbf{E} = \left(a_x\hat{\mathbf{x}} + a_y e^{j\delta}\hat{\mathbf{y}}\right)e^{-j\beta z}$

circular ($a_x = a_y$)
$\delta = -90°$: right-hand, $\delta = +90°$: left-hand

ellipse $\tan\psi_0 = \dfrac{a_y}{a_x}, \quad \tan 2\zeta = \tan 2\psi_0 \cos\delta$

$\sin 2\chi = \sin 2\psi_0 \sin\delta, \quad$ axial ratio $\dfrac{1}{|\tan\chi|}$

Doppler $f' \approx f\left(1 + \dfrac{u}{c}\cos\theta\right)$

The fields are time-harmonic ($e^{j\omega t}$, cosine reference); the magnitude of a phasor is the peak value. The medium is lossless, linear, isotropic and homogeneous; unless stated otherwise it is nonmagnetic. Constants: $c = 3\times10^8$ m/s, $\eta_0 = 120\pi \approx 377\ \Omega$. $\hat{\mathbf{k}}$ is the unit vector in the direction of propagation; $\mathbf{R} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$. Polarization follows the IEEE convention: if, with the thumb of the right hand pointing in the direction of propagation, the fingers point in the direction in which $\mathbf{E}$ rotates with time at a fixed point, the polarization is right-hand; otherwise it is left-hand. For a wave propagating in the $+z$ direction, δ is the phase difference of E_y relative to E_x; $\sin\delta < 0$ means right-hand and $\sin\delta > 0$ means left-hand. For the ellipse, ζ is the angle between the major axis and the x-axis (same sign as $\cos\delta$) and χ is the ellipticity angle (same sign as $\sin\delta$).

Uniform plane waves and wave parameters

13.1 ★☆☆ Solution →

For a uniform plane wave traveling in the $+z$ direction in a lossless, nonmagnetic medium, which of the following is true?

A) If the frequency is doubled, the phase velocity also doubles, while the wavelength stays the same.

B) In a medium with $\varepsilon_r = 4$ the wavelength is one quarter of the free-space wavelength at the same frequency.

C) $\mathbf{E}$ and $\mathbf{H}$ are in phase; the ratio of their amplitudes is η and does not depend on frequency.

D) $\mathbf{E}$ may have a component in the direction of propagation; only $\mathbf{H}$ is perpendicular to it.

E) Since the intrinsic impedance is $\sqrt{\mu/\varepsilon}$, it increases as ε_r increases.

Polarization

13.8 ★★☆ Solution →

In a lossless medium, which of the following waves is right-hand circularly polarized according to the IEEE convention?

A) $\mathbf{E} = \hat{\mathbf{x}}\cos(\omega t - \beta z) - \hat{\mathbf{y}}\sin(\omega t - \beta z)$

B) $\mathbf{E} = \hat{\mathbf{x}}\cos(\omega t + \beta z) + \hat{\mathbf{y}}\sin(\omega t + \beta z)$

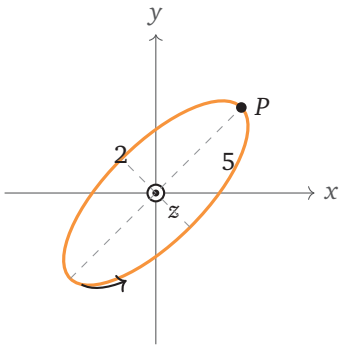
C) $\mathbf{E} = 2\,\hat{\mathbf{x}}\cos(\omega t - \beta z) + \hat{\mathbf{y}}\sin(\omega t - \beta z)$

D) $\mathbf{E} = \hat{\mathbf{y}}\sin(\omega t - \beta x) + \hat{\mathbf{z}}\cos(\omega t - \beta x)$

E) $\mathbf{E} = \hat{\mathbf{x}}\sin(\omega t + \beta z) + \hat{\mathbf{y}}\cos(\omega t + \beta z)$

13.10 ★★★ Solution →

For a plane wave traveling in air in the $+z$ direction (out of the page), the tip of the vector $\mathbf{E}$ at $z = 0$ traces the ellipse shown. The semi-major axis is 5 V/m and makes 45° with the x-axis; the semi-minor axis is 2 V/m. The arrow shows the sense of rotation, and point P marks the tip of $\mathbf{E}$ at $t = 0$.



a. Determine the sense of rotation of the polarization (IEEE) and the axial ratio.

b. Write the phasor $\mathbf{E}$ in terms of its $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ components; find a_x, a_y and δ.

c. Decompose $\mathbf{E}$ into right-hand and left-hand circular components.

d. Find the time-average power density.

CHAPTER 15

REFLECTION AND TRANSMISSION AT NORMAL INCIDENCE

plane wave $\beta = \omega\sqrt{\mu\varepsilon}, \quad \eta = \sqrt{\mu/\varepsilon}, \quad \mathbf{H} = \frac{1}{\eta}\hat{\mathbf{k}} \times \mathbf{E}$

lossy medium

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}, \quad \eta_c = \frac{j\omega\mu}{\gamma}$$

good conductor

$$\alpha = \beta = \frac{1}{\delta} = \sqrt{\pi f \mu\sigma}, \quad \eta_c = (1+j)R_s, \quad R_s = \frac{1}{\sigma\delta}$$

boundary conditions

$E_{1t} = E_{2t}, \quad H_{1t} = H_{2t} \quad (\mathbf{J}_s = 0)$

perfect conductor $E_{1t} = 0, \quad \mathbf{J}_s = \hat{\mathbf{n}} \times \mathbf{H}_1$

average power $\mathbf{S}_{\text{avg}} = \frac{1}{2}\text{Re}(\mathbf{E} \times \mathbf{H}^*)$

reflection, transmission

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}, \quad \tau = 1 + \Gamma = \frac{2\eta_2}{\eta_2 + \eta_1}$$

medium 1 $E_1(z) = E_0\left(e^{-j\beta_1 z} + \Gamma e^{j\beta_1 z}\right)$

standing-wave ratio $S = \dfrac{|E_1|_{\max}}{|E_1|_{\min}} = \dfrac{1+|\Gamma|}{1-|\Gamma|}$

power (lossless) $|\Gamma|^2 + \dfrac{\eta_1}{\eta_2}|\tau|^2 = 1$

wave impedance

$$Z(-l) = \eta_1 \frac{\eta_2 \cos\beta_1 l + j\eta_1 \sin\beta_1 l}{\eta_1 \cos\beta_1 l + j\eta_2 \sin\beta_1 l}$$

input of the layer $\Gamma_0 = \dfrac{Z - \eta_1}{Z + \eta_1}$

The fields are time-harmonic: $\mathbf{E}(z,t) = \text{Re}[\mathbf{E}(z)\,e^{j\omega t}]$; the magnitude of a phasor is the peak value. The boundary is the plane $z = 0$; medium 1 ($z < 0$) carries the incident and reflected waves, medium 2 ($z > 0$) the transmitted wave. The incident wave propagates in the $+z$ direction with $\mathbf{E} = E\,\hat{\mathbf{x}}$ and $\mathbf{H} = H\,\hat{\mathbf{y}}$. Γ and τ are the ratios of the reflected and transmitted electric fields to the incident electric field at the boundary. The media are linear, isotropic and homogeneous; unless stated otherwise they are nonmagnetic ($\mu = \mu_0$) and the index of refraction is $n = \sqrt{\varepsilon_r}$. Constants: $\varepsilon_0 = 8.854 \times 10^{-12}$ F/m, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $c = 3 \times 10^8$ m/s, $\eta_0 = 120\pi \approx 377\ \Omega$. The loss tangent $\sigma/(\omega\varepsilon)$ is the ratio of the conduction current to the displacement current. If the ratio is $\ll 1$, the medium is a low-loss dielectric (for a ratio below 0.1 the relative error of the approximation is of the order of $(\sigma/\omega\varepsilon)^2/8$, i.e., about one part in a thousand); if the ratio is $\gg 1$, the medium is a good conductor (in practice above 100). In between, the medium is a quasi-conductor. Unless stated otherwise, the second boundary behind a thick layer is ignored. The wave impedance $Z(z) = E_x(z)/H_y(z)$ is the ratio of the total fields; the formula for $Z(-l)$ holds at a plane a distance l from the boundary and is applied step by step in a layered medium.

Reflection and transmission coefficients at normal incidence

15.2

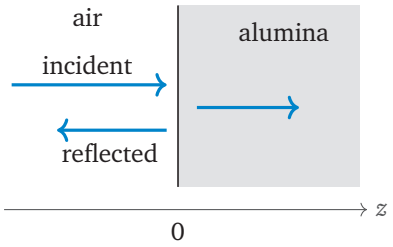
★☆☆ Solution →

Alumina ceramic ($\varepsilon_r = 9.8$, $\mu_r = 1$, lossless) is being studied for a microwave lens antenna. A plane wave from air is normally incident on the flat face of a thick alumina block.

a. Find Γ and τ.

b. What percentage of the incident power is reflected, and what percentage enters the block?

c. Find the standing-wave ratio in air and express it in dB. Compare the result with ε_r.



Multilayer media

15.11

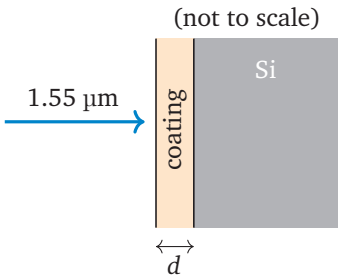
★☆☆ Solution →

A silicon photodetector detects light of $\lambda_0 = 1.55$ μm; at this wavelength the index of refraction of silicon is 3.48. A single-layer lossless antireflection coating is to be applied to the surface.

a. What percentage of normally incident light is reflected by the uncoated surface?

b. Find the index of refraction and the minimum thickness of the coating that eliminates the reflection.

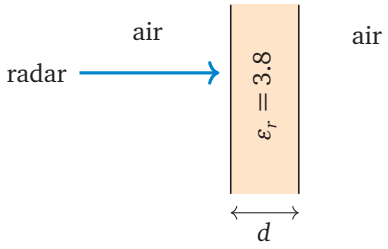
c. What is the next suitable thickness?



15.14

★★★ Solution →

The radome of a 5.6 GHz weather radar is to be made of a lossless composite ($\varepsilon_r = 3.8$, $\mu_r = 1$). The radome is treated as a flat slab with air on both sides, and the wave is normally incident on the slab. For strength the thickness must be at least 2 cm.



a. Find the reflectionless thicknesses at 5.6 GHz and choose the suitable one.

b. The radar is also used at 5.3 GHz. What is the reflected power fraction for the chosen thickness?

c. Without the strength requirement, how much would the thinnest reflectionless slab reflect at 5.3 GHz? Interpret the result.

d. How much does a slab mistakenly made a quarter wavelength thick reflect at 5.6 GHz?

CHAPTER 18

THE SMITH CHART AND IMPEDANCE MATCHING

normalized impedance
$z = \dfrac{Z}{Z_0} = r + jx, \quad \Gamma = \dfrac{z-1}{z+1}, \quad z = \dfrac{1+\Gamma}{1-\Gamma}$

constant-r circle center $\left(\dfrac{r}{1+r}, 0\right)$, radius $\dfrac{1}{1+r}$

constant-x arc center $\left(1, \dfrac{1}{x}\right)$, radius $\dfrac{1}{|x|}$

along the line $\Gamma(d) = \Gamma_L e^{-j2\beta d}$: clockwise $720^\circ\, d/\lambda$

admittance
$y = \dfrac{1}{z} = g + jb$: point symmetric about the center

standing-wave ratio
$S = r$ (where the circle crosses the positive real axis)

voltage maximum, minimum
$\Gamma(d)$ angle $0^\circ : z = S; \quad 180^\circ : z = 1/S$

stubs $y_{sc} = -j \cot \beta l, \quad y_{oc} = j \tan \beta l$

quarter-wave transformer $Z_T = \sqrt{R_a R_b}$

lumped elements $Z_L = j\omega L, \quad Z_C = \dfrac{1}{j\omega C}$

The lines are lossless and Z_0 is real. The position d is measured from the load toward the generator; lengths are mostly given in wavelengths. The Smith chart is the unit circle in the complex Γ-plane: the distance of a point from the center is $|\Gamma|$ and its angle is the angle of Γ. Moving from the load toward the generator means rotating clockwise on the chart; one full revolution is $\lambda/2$. The normalized admittance $y = 1/z$ can also be read on the same chart. The chart is then an admittance chart; its curves are g circles and b arcs. Unless stated otherwise, stubs are short-circuited line sections with the same Z_0 as the main line, connected in shunt with it. Lumped elements are ideal. Abbreviations: sc: short circuit, oc: open circuit. Every chart reading is also verified with the formulas; results are given to 3 significant figures.

Structure of the chart: impedance and admittance

18.3 ★★☆ Solution →

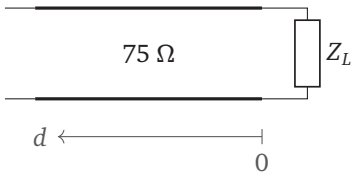
A lossless 75 Ω line is terminated at 200 MHz in a load $Z_L = 27 + j54\ \Omega$.

a. Locate z_L on the chart and find Γ.

b. Find y_L from the point symmetric about the center on the chart and verify it with the formula; write Y_L in millisiemens (mS).

c. If the load is modeled as a resistor in parallel with a reactive element, what are the element values?

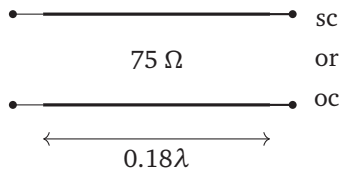
d. Find the normalized admittance of the line at the point 0.1λ from the load.



Moving along the line on the chart

18.6 ★☆☆ Solution →

A section of lossless 75 Ω line is 0.18λ long.



a. Find the input impedance with the end short-circuited using the Smith chart, and verify it with the formula.

b. Find the input impedance with the end open-circuited.

c. Which one is inductive?

Single-stub matching

18.8 ★★☆ Solution →

Which of the following is true for impedance matching with a single short-circuited shunt stub?

A) If the stub is placed right at the load, any load can be matched with a suitable stub length.

B) Since the stub is connected in shunt, the calculation is done on the impedance chart; switching to admittance offers no advantage.

C) The stub is placed at the point where the real part of the normalized line admittance is 1.

D) If the stub is shorter than $\lambda/4$, it provides a capacitive susceptance.

E) For every load there is a single stub location; there is no second solution closer than $\lambda/2$ to the load.

SOLUTIONS

13 PLANE WAVES IN LOSSLESS MEDIA AND POLARIZATION

13.1

← Problem

In a lossless medium $\eta = \sqrt{\mu/\varepsilon}$ is real and independent of frequency; since $\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{z}} \times \mathbf{E}$, $\mathbf{E}$ and $\mathbf{H}$ are in phase. (C) is correct.

- (A): $u_p = 1/\sqrt{\mu\varepsilon}$ is independent of frequency; when the frequency doubles, the wavelength is halved.
- (B): $\lambda = \lambda_0/\sqrt{\varepsilon_r} = \lambda_0/2$, not one quarter.
- (D): $\nabla \cdot \mathbf{E} = 0$ requires that $\mathbf{E}$ also have no component in the direction of propagation: the wave is TEM.
- (E): $\eta = \eta_0/\sqrt{\varepsilon_r}$; it decreases as ε_r increases.

ANSWER C

13.8

← Problem

Method: at a fixed point, the cross product of $\mathbf{E}(\omega t = 0)$ and $\mathbf{E}(\omega t = 90°)$ gives the axis of rotation. If this axis is in the direction of propagation, the polarization is right-hand.

- (A): $+z$; at $z = 0$ it rotates from $\hat{\mathbf{x}}$ to $-\hat{\mathbf{y}}$, $\hat{\mathbf{x}} \times (-\hat{\mathbf{y}}) = -\hat{\mathbf{z}}$: left-hand.
- (B): $-z$; it rotates from $\hat{\mathbf{x}}$ to $\hat{\mathbf{y}}$, $\hat{\mathbf{x}} \times \hat{\mathbf{y}} = +\hat{\mathbf{z}}$, opposite to propagation: left-hand.
- (C): The component amplitudes differ (2 and 1): right-hand but elliptical, not circular.
- (D): $+x$; at $x = 0$ it rotates from $\hat{\mathbf{z}}$ to $\hat{\mathbf{y}}$, $\hat{\mathbf{z}} \times \hat{\mathbf{y}} = -\hat{\mathbf{x}}$: left-hand.

(E): $-z$; at $z = 0$, $\mathbf{E}$ is along $\hat{\mathbf{y}}$ at $\omega t = 0$ and along $\hat{\mathbf{x}}$ at $\omega t = 90°$: $\hat{\mathbf{y}} \times \hat{\mathbf{x}} = -\hat{\mathbf{z}}$, the same as the direction of propagation. The amplitudes are equal: right-hand circular. (E) is correct.

ANSWER E

13.10

← Problem

a. Viewed from $+z$ (with the wave approaching the observer), the arrow goes from the major axis toward the minor axis, counterclockwise. With the thumb along $+z$, the fingers of the right hand curl in this direction: right-hand. Axial ratio $5/2 = 2.5$.

b. The major axis is along $\hat{\mathbf{u}}_\xi = (\hat{\mathbf{x}} + \hat{\mathbf{y}})/\sqrt{2}$, the minor axis along $\hat{\mathbf{u}}_\eta = (-\hat{\mathbf{x}} + \hat{\mathbf{y}})/\sqrt{2}$. At $t = 0$, $\mathbf{E}$ is at P (the end of the major axis) and then rotates toward $\hat{\mathbf{u}}_\eta$:

$$\mathbf{E}(0, t) = 5\cos\omega t\,\hat{\mathbf{u}}_\xi + 2\sin\omega t\,\hat{\mathbf{u}}_\eta.$$

The phasor of $\sin\omega t$ is $-j$; $\mathbf{E} = 5\,\hat{\mathbf{u}}_\xi - j2\,\hat{\mathbf{u}}_\eta$:

$$\mathbf{E} = \tfrac{1}{\sqrt{2}}\big[(5 + j2)\hat{\mathbf{x}} + (5 - j2)\hat{\mathbf{y}}\big]e^{-j\beta z},$$

i.e., $3.81\,e^{j21.8°}\hat{\mathbf{x}} + 3.81\,e^{-j21.8°}\hat{\mathbf{y}}$ V/m:
$a_x = a_y = 3.81$ V/m, $\delta = -43.6°$.

Check: $\psi_0 = 45°$; $\tan 2\zeta = \tan 90° \cos\delta$ is infinite and $\cos\delta > 0$: $\zeta = 45°$. $\sin 2\chi = \sin(-43.6°)$, $\chi = -21.8°$, $\tan\chi = -0.4 = -2/5$: right-hand, ratio 2.5.

c. $\mathbf{E} = a_R(\hat{\mathbf{x}} - j\hat{\mathbf{y}}) + a_L(\hat{\mathbf{x}} + j\hat{\mathbf{y}})$; $a_R = (E_x + jE_y)/2$, $a_L = (E_x - jE_y)/2$:

$$a_R = \frac{(5 + j2) + j(5 - j2)}{2\sqrt{2}} = \frac{7(1 + j)}{2\sqrt{2}},$$

so $|a_R| = 3.50$ V/m. Similarly $|a_L| = |(5 + j2) - j(5 - j2)|/(2\sqrt{2})$, so $|a_L| = 1.50$ V/m. The ratio $(3.50 + 1.50)/(3.50 - 1.50) = 2.5$ equals the axial ratio.

d. The semi-axes of the ellipse are $a_\xi = 5$ V/m and $a_\eta = 2$ V/m; since $a_x^2 + a_y^2 = a_\xi^2 + a_\eta^2$,

$$S_{\text{avg}} = \frac{a_\xi^2 + a_\eta^2}{2\eta_0} = \frac{29}{754} = 38.5\ \text{mW/m}^2.$$

ANSWER a. right-hand; 2.5
b. $\frac{1}{\sqrt{2}}[(5 + j2)\hat{\mathbf{x}} + (5 - j2)\hat{\mathbf{y}}]e^{-j\beta z}$; 3.81 V/m, 3.81 V/m, $\delta = -43.6°$ c. $|a_R| = 3.50$ V/m, $|a_L| = 1.50$ V/m d. 38.5 mW/m^2

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15 REFLECTION AND TRANSMISSION AT NORMAL INCIDENCE

15.2

← Problem

a. In nonmagnetic media $\eta = \eta_0/\sqrt{\varepsilon_r}$; $\sqrt{9.8} = 3.13$:

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{1 - \sqrt{\varepsilon_r}}{1 + \sqrt{\varepsilon_r}} = \frac{1 - 3.130}{1 + 3.130} = -0.516,$$

$\tau = 1 + \Gamma = 0.484$.

b. The reflected power fraction is $|\Gamma|^2 = 0.266$, i.e., 26.6%. The transmitted power fraction is $(\eta_1/\eta_2)|\tau|^2 = 3.130(0.484)^2 = 0.734$, i.e., 73.4%. The sum is 1.

c. Standing-wave ratio:

$$S = \frac{1 + 0.516}{1 - 0.516} = 3.13, \qquad 20\log_{10} S = 9.91 \text{ dB}.$$

The result is $S = \sqrt{\varepsilon_r}$: if $\Gamma = (1-n)/(1+n)$, then $(1+|\Gamma|)/(1-|\Gamma|) = n$. For normal incidence from air on a lossless nonmagnetic medium, S equals the index of refraction of the medium.

Check: Solving directly from the boundary conditions $(E_0 + E_r = E_t,\ (E_0 - E_r)/\eta_1 = E_t/\eta_2)$ also gives $E_r/E_0 = -0.516$, $E_t/E_0 = 0.484$.

ANSWER a. $\Gamma = -0.516$, $\tau = 0.484$ b. 26.6% reflected, 73.4% transmitted c. $S = 3.13$ (9.91 dB) $= \sqrt{\varepsilon_r}$

15.11

← Problem

a. $\Gamma = (1 - 3.48)/(1 + 3.48) = -0.554$; the reflected power is $|\Gamma|^2 = 0.306$, i.e., 30.6%.

b. For a quarter-wave layer we need $\eta_2 = \sqrt{\eta_1 \eta_3}$; since $\eta = \eta_0/n$, $n_k = \sqrt{n_1 n_3} = \sqrt{3.48} = 1.87$. The thickness is a quarter of the wavelength in the coating:

$$d = \frac{\lambda_0}{4 n_k} = \frac{1.55}{4(1.865)} = 0.208 \text{ μm}.$$

At this thickness $\beta_k d = \pi/2$, and the wave-impedance formula gives $Z = \eta_k^2/\eta_3 = \eta_0$ (η_k: intrinsic impedance of the coating): from the air side the structure looks like unbounded air.

c. The thickness may be any odd multiple of $\lambda_k/4$: $d = 3\lambda_0/(4n_k) = 0.623$ μm. (A thicker coating has a narrower bandwidth.)

Check: Solving directly for the wave amplitudes in air, coating and silicon from the four boundary conditions gives zero reflected amplitude for $d = 0.208$ μm.

ANSWER a. 30.6% b. $n_k = 1.87$, $d = 0.208$ μm c. 0.623 μm

15.14

← Problem

a. When $\eta_1 = \eta_3 = \eta_0$, zero reflection requires $\beta_2 d = m\pi$, i.e., $d = m\lambda_2/2$ (half-wave window). $\lambda_0 = c/f = 5.36$ cm, $\lambda_2 = 5.357/\sqrt{3.8} = 2.75$ cm:

$$d = m(1.37 \text{ cm}):\ 1.37,\ 2.75,\ 4.12 \text{ cm},\ \ldots$$

The thinnest value exceeding 2 cm is $d = \lambda_2 = 2.75$ cm.

b. The phase thickness at 5.3 GHz is

$$\beta_2 d = \frac{2\pi(5.3 \times 10^9)\sqrt{3.8}\,(0.02748)}{3 \times 10^8} = 340.7^\circ,$$

$\eta_2 = 377/\sqrt{3.8} = 193.4\ \Omega$:

$$Z = \eta_2 \frac{\eta_0 \cos\beta_2 d + j\eta_2 \sin\beta_2 d}{\eta_2 \cos\beta_2 d + j\eta_0 \sin\beta_2 d} = 289 + j129\ \Omega,$$

$$|\Gamma|^2 = \left|\frac{Z - 377}{Z + 377}\right|^2 = 0.0533\ (5.33\%).$$

c. For $d = 1.37$ cm, $\beta_2 d = 170.4^\circ$; the same formula gives $|\Gamma|^2 = 0.0143$, i.e., 1.43%. The phase error is proportional to the thickness: for a slab of thickness λ_2 the deviation from 360° is 19.3°, for half of it 9.6°. A half-wave window is narrowband; the thinner one has a wider bandwidth.

d. At $d = \lambda_2/4$, $\beta_2 d = \pi/2$ and $Z = \eta_2^2/\eta_0 = \eta_0/3.8$:

$$\Gamma = \frac{1/3.8 - 1}{1/3.8 + 1} = -0.583, \qquad |\Gamma|^2 = 34.0\%.$$

A quarter-wave slab adds the reflections of the two surfaces; it is the worst thickness for an air-to-air window.

Check: Solving for the amplitudes in the three regions directly from the four boundary conditions gives exactly 5.33% and 1.43% at 5.3 GHz; at 5.6 GHz the reflection is zero for both thicknesses.

ANSWER a. $d = m(1.37$ cm$)$; $d = 2.75$ cm b. 5.33% c. 1.43%; the thin slab has a wider bandwidth d. $\Gamma = -0.583$, 34.0%

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18 THE SMITH CHART AND IMPEDANCE MATCHING

18.3

← Problem

a. $z_L = (27+j54)/75 = 0.36+j0.72$; the point is in the upper half (inductive):

$$\Gamma = \frac{-0.64+j0.72}{1.36+j0.72} = 0.626\angle 104^\circ.$$

b. The point symmetric to z about the center is $-\Gamma$; since $(1-\Gamma)/(1+\Gamma) = 1/z$, this point is read directly as y on the chart. The angle of the symmetric point is $103.7^\circ - 180^\circ = -76.3^\circ$:

$$y_L = \frac{1}{0.36+j0.72} = 0.556 - j1.11.$$

$Y_L = y_L/Z_0 = 7.41 - j14.8$ mS. The symmetric point corresponds to a rotation of $\lambda/4$ (180°) along the line: a $\lambda/4$ line inverts the impedance.

c. In the parallel model $Y_L = G + jB$: $R = 1/G = 135\ \Omega$. B is negative, so the element is an inductor: $B = -1/(\omega L)$,

$$L = \frac{1}{2\pi(2\times 10^8)(0.0148)} = 53.7\text{ nH}.$$

d. The admittance point also rotates clockwise along the line; for 0.1λ the rotation is 72°. The angle goes from −76.3° down to −148° and

$$y(0.1\lambda) = \frac{y_L + j\tan 36^\circ}{1 + j y_L \tan 36^\circ} = 0.248 - j0.268.$$

Check: If $z(0.1\lambda)$ is first found with the input-impedance formula and then inverted, the result is again $0.248 - j0.268$.

ANSWER a. $0.626\angle 104^\circ$ b. $y_L = 0.556 - j1.11$; $Y_L = 7.41 - j14.8$ mS c. 135 Ω in parallel with 53.7 nH d. $0.248 - j0.268$

18.6

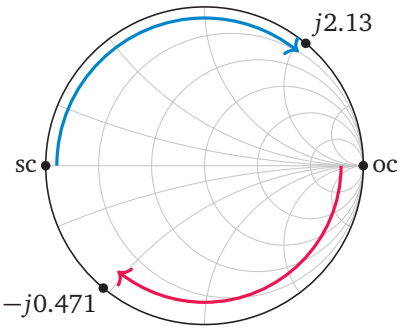
← Problem

For 0.18λ the point is rotated clockwise by $720^\circ(0.18) = 129.6^\circ$; $\beta l = 64.8^\circ$. For a purely reactive load $|\Gamma| = 1$; the point stays on the unit circle.

a. Short circuit: $\Gamma = -1$ (left end, angle 180°). After rotating 129.6° the angle is 50.4°; this point is in the upper half on the $x = 2.13$ arc: $z = j2.13$, $Z = j159\ \Omega$. With the formula, $jZ_0 \tan 64.8^\circ = j75(2.13) = j159\ \Omega$.

b. Open circuit: $\Gamma = +1$ (right end, angle 0). After the rotation the angle is −129.6°; in the lower half $x = -0.471$: $Z = -j35.3\ \Omega$. With the formula, $-jZ_0 \cot 64.8^\circ = -j35.3\ \Omega$.

c. The short-circuited section is inductive ($+j159\ \Omega$), because $l < \lambda/4$. The open-circuited one is capacitive.



ANSWER a. $j159\ \Omega$ b. $-j35.3\ \Omega$ c. the short-circuited section

18.8

← Problem

The admittances of shunt elements add. The stub can add only an imaginary admittance ($-jb$); therefore the normalized line admittance at the stub location must be $1 + jb$. The stub cancels the imaginary part with $-jb$, leaving $y = 1$. (C) is correct.

- (A): If $g_L \neq 1$ at the load, the stub cannot change the real part; the load cannot be matched.
- (B): In a shunt connection the admittances add, not the impedances; that is why the chart is used as an admittance chart.
- (D): $y_{sc} = -j\cot\beta l$; for $l < \lambda/4$, $\cot\beta l > 0$ and the susceptance is negative: inductive.
- (E): The constant-$|\Gamma|$ circle intersects the $g = 1$ circle at two points; there are two solutions closer than $\lambda/2$ to the load.

ANSWER C

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