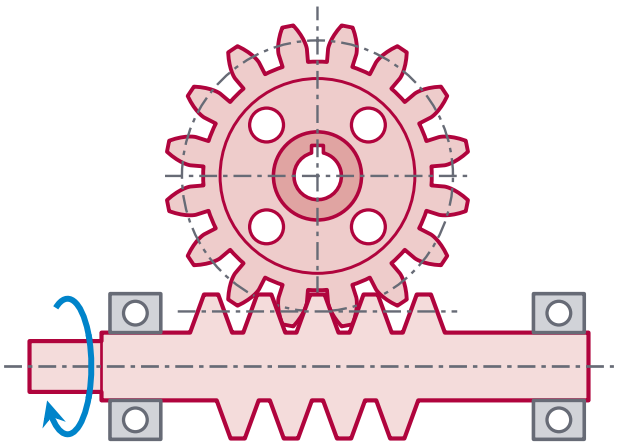
MACHINE ELEMENTS 2

PROBLEM BOOK

SAMPLE PAGES

8 chapters · 148 problems · step-by-step solutions



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How to use this book

- Each chapter opens with a reminder box summarizing the definitions and formulas the problems rely on.
- Difficulty: ★☆☆ basic ★★☆ intermediate ★★★ advanced. Each chapter has a few multiple-choice concept questions; each chapter ends with exam-style mixed problems.
- The dotted area below each problem is for your own work. Solve first; if stuck, go back to the reminder box and the lecture notes, and look at the solution last.
- All solutions are at the end of the book. The Solution → link next to a problem takes you to its solution, and ← Problem takes you back. Short answers are in the **ANSWER** box.
- **Units and signs:** SI units throughout; stresses and strengths in MPa, moduli of elasticity in GPa, forces in N or kN, moments in N·m. Normal stress is positive in tension. In the figures, force, moment and torque arrows are red.
- **Appendices:** The appendices at the end of the book (materials; rolling bearings; lubricants and journal bearings; gears; belts, chains and wire ropes) contain the material data, factors and standard dimensions not given in the problems; the problems are solved with the data in the problem statement and these appendices only.
- **Midterm and final:** In most schools the midterm and final topics are split as follows:

Midterm	Ch. 10–13	rolling bearings, journal bearings, introduction to gears, spur gear strength
Final	Ch. 14–17	helical, bevel and worm gears, belts and chains, clutches and brakes, power transmission

  Chapter 17 is a mixed gear reducer design from start to finish; it builds on the fatigue, shaft and shaft–hub chapters of *Machine Elements 1* (Chapters 4–6). Choose the chapters to study according to your own course syllabus.

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Solutions 95

CHAPTER 12
INTRODUCTION TO GEARS

diameters

$$d = mz,\ \ d_b = d\cos\alpha,\ \ d_a = d + 2m(1+x)$$

root $\quad d_f = d - 2m(1.25 - x)$

pitches $\quad p = \pi m,\ \ p_b = p\cos\alpha$

center distance $\quad a = \tfrac{1}{2}m(z_1 + z_2),\ \ u = z_2/z_1$

involute

$$\operatorname{inv}\alpha = \tan\alpha - \alpha,\ \ \cos\alpha_y = r_b/r_y,\ \ \rho_y = r_b\tan\alpha_y$$

tooth thickness

$$s_y = d_y\left(\frac{\pi/2 + 2x\tan\alpha}{z} + \operatorname{inv}\alpha - \operatorname{inv}\alpha_y\right)$$

contact ratio

$$\varepsilon = \frac{\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a_w\sin\alpha_w}{p_b}$$

undercutting

$$z_g = \frac{2}{\sin^2\alpha} \approx 17,\ \ x_{\min} = 1 - \frac{z\sin^2\alpha}{2}$$

profile shift $\quad a_w\cos\alpha_w = a\cos\alpha$

$$\operatorname{inv}\alpha_w = \operatorname{inv}\alpha + 2\tan\alpha\,\frac{x_1 + x_2}{z_1 + z_2}$$

speed ratio $\quad i = \dfrac{n_{\text{in}}}{n_{\text{out}}} = \dfrac{\prod z_{\text{driven}}}{\prod z_{\text{driver}}}$

Willis $\quad \dfrac{n_k - n_T}{n_1 - n_T} = \pm\dfrac{\prod z_{\text{driver}}}{\prod z_{\text{driven}}}$

planetary $\quad z_r = z_s + 2z_p,\ \ (z_s + z_r)/N$ integer

forces $\quad F_t = \dfrac{2T}{d},\ \ F_r = F_t\tan\alpha_w,\ \ F_n = \dfrac{F_t}{\cos\alpha_w}$

power, speed $\quad P = T\omega = F_t v,\ \ v = \pi d n$

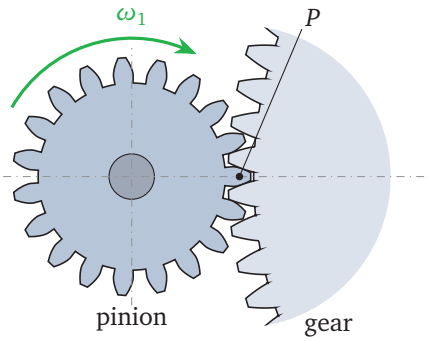
ISO 54 module series (mm) Series 1 (preferred): 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20 Series 2: 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18

Unless stated otherwise, the following assumptions apply. The gears are spur gears with involute profiles and the ISO standard basic rack: pressure angle $\alpha = 20°$, addendum m, dedendum $1.25\,m$, bottom clearance $0.25\,m$; there is no profile shift ($x = 0$), and the center distance is standard ($a_w = a$, $\alpha_w = \alpha$). Backlash and friction are neglected; if no efficiency is given, there are no losses. The speed ratio is defined as $i = n_{\text{in}}/n_{\text{out}}$. In the Willis equation T is the carrier; the sign is plus if the first and last members of the train turn in the same direction with the carrier held, and minus if they turn in opposite directions. In kinematic diagrams, gears are shown from the side as rectangles whose height is the pitch diameter. Dimensions in drawings are in mm.

Spur gear forces

12.15 ★☆☆ Solution →

The final stage of the drive reducer of a bucket elevator is a spur gear pair: pinion $z_1 = 19$, gear $z_2 = 74$, $m = 6$ mm. The pinion turns clockwise at 238 rpm and transmits 9.5 kW (see figure).



a. Find the pinion torque, the pitch-line velocity, and the components of the gear force (F_t, F_r, F_n).

b. Sketch the force components acting on the gear at the contact point with their correct directions.

c. Find the speed and the torque of the gear; verify the power with $P = F_t v$.

Mixed problems

12.20 ★★★ Solution →

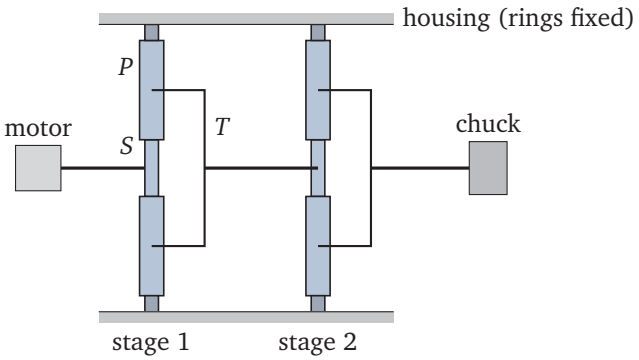
The two-stage planetary gearbox of a cordless drill is to be designed (see figure). The two stages are identical: ring fixed to the housing, sun input, carrier output; each stage has 3 equally spaced planets; module $m = 0.5$ mm. The stage ratio must be about 5.5, and the sun must have at least 10 teeth.

a. Find the tooth numbers that satisfy the geometry and assembly conditions and give the ratio closest to 5.5.

b. With the motor at 16 500 rpm, find the chuck speed.

c. A torque of 15 N·m is required at the chuck; the stage efficiency is 0.95. Find the motor torque, the torque of the second-stage sun, and the tangential force at one sun–planet mesh in this stage.

d. Neglecting losses, find the force on one planet shaft in the second stage and the torque that the ring transmits to the housing.



CHAPTER 14

HELICAL, BEVEL AND WORM GEARS

helical geometry

$$m_t = \frac{m_n}{\cos\beta},\quad d = \frac{z\,m_n}{\cos\beta},\quad \tan\alpha_t = \frac{\tan\alpha_n}{\cos\beta}$$

equivalent spur gear $$z_n = \frac{z}{\cos^3\beta},\quad \varepsilon_\beta = \frac{b\sin\beta}{\pi m_n}$$

helical forces

$$F_t = \frac{2T}{d},\quad F_r = \frac{F_t\tan\alpha_n}{\cos\beta},\quad F_a = F_t\tan\beta$$

bevel ($\Sigma = 90°$)

$$\tan\delta_1 = \frac{z_1}{z_2},\quad R_e = \frac{d_e}{2\sin\delta},\quad d_m = d_e\left(1-\frac{b}{2R_e}\right)$$

bevel forces

$$F_t = \frac{2T}{d_m},\quad F_{r1} = F_t\tan\alpha\cos\delta_1,\quad F_{a1} = F_t\tan\alpha\sin\delta_1$$

bevel equivalent

$$z_v = \frac{z}{\cos\delta},\quad d_v = \frac{d_m}{\cos\delta},\quad m_m = \frac{d_m}{z}$$

heat balance $$P_1(1-\eta) = hA(t_{\text{oil}} - t_{\text{a}})$$

tooth root $$\sigma_F = \frac{F_t}{b\,m_n}Y_F Y_S Y_\varepsilon Y_\beta K_A K_V K_{F\beta} K_{F\alpha}$$

surface $$\sigma_H = Z_H Z_E Z_\varepsilon Z_\beta\sqrt{\frac{F_t}{b\,d_1}\frac{u+1}{u}K_A K_V K_{H\beta} K_{H\alpha}}$$

helix factors

$$Y_\beta = 1-\varepsilon_\beta\frac{\beta}{120°}\ (\varepsilon_\beta \le 1),\quad Z_\beta = \sqrt{\cos\beta}$$

safety $$S_F = \sigma_{FG}/\sigma_F,\quad S_H = \sigma_{HG}/\sigma_H$$

worm $$\tan\gamma = \frac{z_1 m}{d_1} = \frac{z_1}{q},\quad i = \frac{z_2}{z_1},\quad F_{t2} = F_{a1}$$

worm forces

$$F_{t1} = W(\cos\alpha_n\sin\gamma + \mu\cos\gamma),\quad F_r = W\sin\alpha_n$$

$$F_{a1} = W(\cos\alpha_n\cos\gamma - \mu\sin\gamma)$$

efficiency

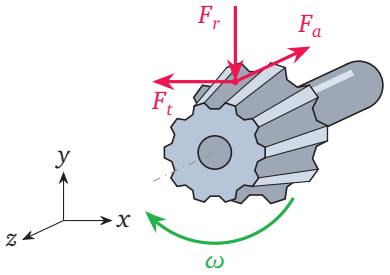
$$\eta = \frac{\cos\alpha_n - \mu\tan\gamma}{\cos\alpha_n + \mu\cot\gamma},\quad \eta' = \frac{\cos\alpha_n - \mu\cot\gamma}{\cos\alpha_n + \mu\tan\gamma}$$

Unless stated otherwise, the following assumptions apply. The normal pressure angle is $\alpha_n = 20°$. Bevel gears are straight bevel gears, their axes intersect at right angles ($\Sigma = 90°$), and the forces act at the mean diameter (d_m). For helical and bevel gears, friction losses are neglected in the force calculation; for worm gearing μ is given in the problem, and η' is the efficiency of power flow from the gear to the worm (if $\eta' \le 0$, the drive is self-locking). For a helical driving gear, the direction of the axial force is found with the hand rule: the fingers of the right hand for a right-hand helix, or of the left hand for a left-hand helix, are curled in the direction of rotation, and the thumb points in the direction of the axial force on the driver; on the driven gear the force is opposite. The factors (Y, Z, K) and the endurance limits are given in the problems; the strengths are $\sigma_{FG} = \sigma_{F\lim} Y_{ST} Y_{NT}$ and $\sigma_{HG} = \sigma_{H\lim} Z_{NT}$, with $Y_{ST} = 2$ and $Y_{NT} = Z_{NT} = 1$. Pictorial figures are not to scale; dimensions in drawings are in mm.

Bevel gears

14.12 ★☆☆ Solution →

In an angle grinder, the motor delivers 1.4 kW at 33 000 rpm. The bevel pinion ($z_1 = 10$) at the end of the motor shaft turns the gear ($z_2 = 31$) on the spindle; outer module $m_e = 1.25$ mm, $b = 6$ mm. The gears are to be treated as straight bevel gears. The figure shows the pinion and the forces acting on it at the contact point; the gear is above the pinion.



a. Find the cone angle and the mean diameter of the pinion.

b. Find the forces F_t, F_r, F_a on the pinion.

c. What are the axial and radial forces on the gear? At what speed (rev/min) does the spindle rotate?

Mixed problems

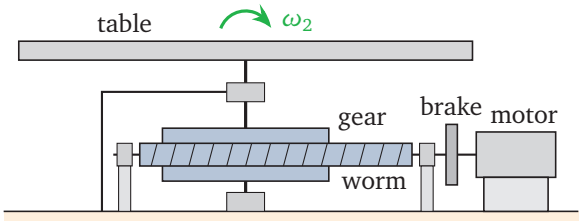
14.20 ★★☆ Solution →

The revolving stage of a theater requires a torque of $T_2 = 2400$ N·m and a speed of $n_2 = 1.5$ rpm at the worm gear shaft. The worm gear has $z_2 = 50$ teeth; $m = 8$ mm, $q = 10$. In operation $\mu = 0.06$, and at rest $\mu_0 = 0.10$. When the stage stops, the scenery load and the actors tend to turn the table; the designer wants the worm to be self-locking in addition to the brake.

a. Find the lead angles for a single-start and a double-start worm. Which one is self-locking at rest?

b. For the self-locking option, find the operating efficiency and the speed, power, and torque of the worm shaft.

c. If the other option were selected, what would the worm power be? Evaluate the decision.



CHAPTER 15

BELTS, CHAINS AND WIRE ROPES

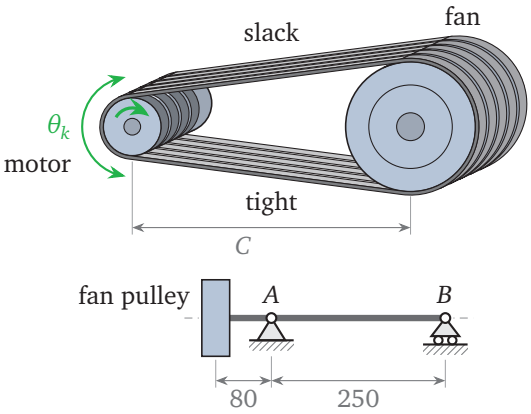
open belt	$\theta_{k,b} = \pi \mp 2\arcsin\dfrac{D-d}{2C}$	sprocket	$d = \dfrac{p}{\sin(180°/z)},\ v = \dfrac{z\,p\,n}{60\,000}$
belt length	$L = \sqrt{4C^2-(D-d)^2} + \frac{1}{2}(D\theta_b + d\theta_k)$	chain length (pitches)	$L_p = \dfrac{2C}{p} + \dfrac{z_1+z_2}{2} + \left(\dfrac{z_2-z_1}{2\pi}\right)^2 \dfrac{p}{C}$
Euler–Eytelwein	$\dfrac{F_1-F_c}{F_2-F_c} = e^{\mu\theta},\ F_c = m'v^2$	center distance	$C = \dfrac{p}{4}\left[A + \sqrt{A^2 - 8\left(\dfrac{z_2-z_1}{2\pi}\right)^2}\right]$
power, preload	$P = (F_1-F_2)\,v,\ F_i = \frac{1}{2}(F_1+F_2)$		$A = L_p - \frac{1}{2}(z_1+z_2)$
belt stresses	$\sigma_1 = \dfrac{F_1}{bt},\ \sigma_c = \rho v^2,\ \sigma_b = \dfrac{E_b\,t}{d}$	chordal action	$\dfrac{v_{\min}}{v_{\max}} = \cos\dfrac{180°}{z}$
V-belt	$\mu' = \dfrac{\mu}{\sin(\beta/2)},\ N = \dfrac{P\,K_s}{P_r K_\theta K_L}$	wire rope	$\sigma_b = \dfrac{E_r\,d_w}{D},\ F_b = \sigma_b A_m,\ n = \dfrac{F_B - F_b}{F_t}$
shaft load	$F_s = \sqrt{F_1'^2 + F_2'^2 + 2F_1'F_2'\cos(\pi-\theta)}$	rope force	$F_t = (m+wL)(g+a)$
	$F' = F - F_c$		

Unless stated otherwise, the following assumptions apply. In belt calculations the wrap angle is the angle on the small pulley (θ_k) and is used in radians. The centrifugal force F_c is the same in both spans of the belt; since it is balanced within the belt itself, it is not transmitted to the shaft. Belt and chain masses are given per meter (m'), and rope mass per meter (w). V-belt power ratings, the chain power table, and rope breaking forces are fictitious values given in the problem. Pictorial drawings are not to scale; in layout drawings the wrap angles are true. $g = 9.81\ \text{m/s}^2$.

V-belts and timing belts

15.7 ★★☆ Solution →

In an air handling unit, an 11 kW, 1460 rpm motor drives the $d_2 = 280$ mm fan pulley with V-belts through a $d_1 = 125$ mm pulley; $C = 600$ mm. The service factor is $K_s = 1.2$; for the selected section, the power rating of one belt at this diameter and speed is $P_r = 3.1$ kW, the wrap correction is $K_\theta = 0.96$, and the length correction is $K_L = 0.98$. The fan pulley is at the end of the fan shaft, 80 mm outboard of bearing A; the A–B spacing is 250 mm.



a. Find the wrap angle on the small pulley and the belt speed.

b. Find the required number of belts.

c. If $F_1/F_2 = 5$ is used in the design (total for all belts), find the load on the fan shaft and the bearing reactions.

15.8 ★☆☆ Solution →

Which of the following statements about V-belts and timing belts is correct?

A) A V-belt works by seating on the groove bottom; contact with the groove bottom increases friction.

B) Since a timing belt engages through teeth, it does not slip; the speed ratio is constant, and no high preload is needed for friction.

C) The equivalent coefficient of friction of a V-belt is $\mu \sin(\beta/2)$; it increases as the groove angle grows.

D) In a timing belt, power is transmitted by friction between the back of the belt and the pulley surface.

E) In a multiple V-belt drive the load divides equally among the belts by itself; the belts need not be replaced as a set.

Roller chains

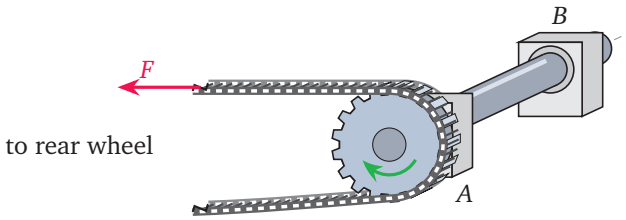
15.13 ★★★ Solution →

On the gearbox output shaft of a motorcycle there is a $z_1 = 17$ tooth front sprocket; $p = 15.875$ mm, the chain mass per meter is 0.80 kg/m, and the breaking force is 37 kN. The sprocket is at the end of the shaft, 35 mm outboard of bearing A; the A–B spacing is 110 mm (see figure). The force in the slack span is to be neglected.

a. At maximum power (38 kW, sprocket at 3400 rpm), find the chain speed, the tight-span force, the centrifugal force, and the factor of safety.

b. In first gear the largest torque on the sprocket is 560 N·m. Find the chain force and the factor of safety.

c. Find the bearing reactions for the first-gear case.



SOLUTIONS

12 INTRODUCTION TO GEARS

12.15

← Problem

a. $d_1 = 6(19) = 114$ mm:

$$T_1 = \frac{P}{\omega_1} = \frac{9500}{2\pi(238)/60} = 381.2 \approx 381 \text{ N·m},$$

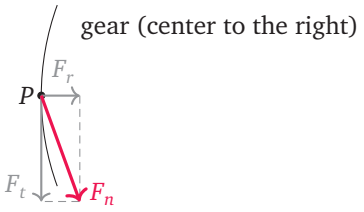
$$v = \frac{\pi(114)(238)}{60\,000} = 1.421 \approx 1.42 \text{ m/s},$$

$$F_t = \frac{2T_1}{d_1} = \frac{2(381.2 \times 10^3)}{114} = 6687 \text{ N} \approx 6.69 \text{ kN},$$

$$F_r = 6687 \tan 20^\circ = 2434 \text{ N} \approx 2.43 \text{ kN},$$

$$F_n = \frac{6687}{\cos 20^\circ} = 7116 \text{ N} \approx 7.12 \text{ kN}.$$

b. Since the pinion turns clockwise, its periphery moves downward at the contact point (P, to the right of the pinion). The tangential force on the driven gear is in the direction of rotation of the gear, that is, downward; the gear turns counterclockwise. The radial force points toward the center of the gear (to the right). The forces on the pinion are opposite to these: F_t up (against the rotation), F_r to the left (toward the pinion center).



c. Gear:

$$n_2 = 238 \cdot \frac{19}{74} = 61.11 \approx 61.1 \text{ rpm},$$

$$T_2 = F_t \frac{d_2}{2} = 6687(0.222) = 1485 \text{ N·m},$$

$$P = F_t v = 6687(1.421) = 9502 \text{ W} \approx 9.50 \text{ kW}.$$

$T_2/T_1 = 74/19 = 3.895$: in a lossless train the torque increases by the speed ratio.

ANSWER a. $T_1 = 381$ N·m, $v = 1.42$ m/s, $F_t = 6.69$ kN, $F_r = 2.43$ kN, $F_n = 7.12$ kN b. on the gear F_t down, F_r toward the gear center c. $n_2 = 61.1$ rpm, $T_2 = 1485$ N·m; 9.50 kW

12.20

← Problem

a. For $i = 1 + z_r/z_s = 5.5$, $z_r = 4.5\,z_s$; with $z_r = z_s + 2z_p$, $z_p = 1.75\,z_s$. For integers, z_s must be a multiple of four: $z_s = 12$, $z_p = 21$, $z_r = 54$. Assembly:

$$\frac{z_s + z_r}{3} = \frac{66}{3} = 22 \text{ (integer)},$$

$$i = 1 + \frac{54}{12} = 5.500 \approx 5.50.$$

($z_s = 11$, $z_p = 19$, $z_r = 49$ also satisfy the assembly condition, but $i = 5.455$; $z_s = 12$ gives exactly 5.5.)

b. Overall ratio $5.5^2 = 30.25$:

$$n = \frac{16\,500}{30.25} = 545.45 \approx 545 \text{ rpm}.$$

c. Motor torque and the second-stage sun (the carrier of the first stage):

$$T_m = \frac{15}{30.25(0.95)^2} = 0.5494 \approx 0.549 \text{ N·m},$$

$$T_{s2} = T_m\, i\, \eta = 0.5494(5.5)(0.95) = 2.871 \approx 2.87 \text{ N·m}.$$

The pitch radius of the sun is $r_s = 0.5(12)/2 = 3$ mm; the torque is shared equally by the three planets:

$$F_t = \frac{T_{s2}}{3r_s} = \frac{2871}{3(3)} = 319.0 \approx 319 \text{ N}.$$

d. With losses neglected, the torque balance of the planet shows that the tangential forces at the sun and ring meshes are equal and act in the same direction; the planet shaft receives their sum:

$$F_{\text{pin}} = 2F_t = 2(319.0) = 638.0 \approx 638 \text{ N}.$$

Ring torque ($r_r = 0.5(54)/2 = 13.5$ mm):

$$T_r = 3F_t r_r = 3(319.0)(13.5) = 12.92 \approx 12.9 \text{ N·m}.$$

Check: $T_{s2} + T_r = 2.871 + 12.92 = 15.79$ N·m $= T_{s2}\, i$ (lossless carrier torque). Since losses are not modeled, the planet forces are approximate.

ANSWER a. $z_s = 12$, $z_p = 21$, $z_r = 54$; $i = 5.50$ b. 545 rpm c. $T_m = 0.549$ N·m, $T_{s2} = 2.87$ N·m, $F_t = 319$ N d. 638 N; $T_r = 12.9$ N·m

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14 HELICAL, BEVEL AND WORM GEARS

14.12

← Problem

a. Geometry:

$$\delta_1 = \arctan\frac{10}{31} = 17.88 \approx 17.9^\circ,$$

$$d_{e1} = 10(1.25) = 12.5 \text{ mm},$$

$$R_e = \frac{12.5}{2\sin 17.88^\circ} = 20.36 \text{ mm},$$

$$d_{m1} = 12.5\left(1 - \frac{6}{2(20.36)}\right) = 10.66 \approx 10.7 \text{ mm}.$$

b. Torque and forces:

$$T_1 = \frac{1400}{2\pi(33\,000)/60} = 0.4051 \text{ N·m},$$

$$F_t = \frac{2(405.1)}{10.66} = 76.02 \approx 76.0 \text{ N},$$

$$F_{r1} = 76.02\tan 20^\circ \cos 17.88^\circ = 26.33 \text{ N},$$

$$F_{a1} = 76.02\tan 20^\circ \sin 17.88^\circ = 8.495 \text{ N}.$$

c. With perpendicular axes $F_{a2} = F_{r1} = 26.3$ N (it pushes the spindle up, away from the cone apex of the gear), $F_{r2} = F_{a1} = 8.50$ N. Spindle:

$$n_2 = 33\,000\,\frac{10}{31} = 10\,645 \approx 10\,650 \text{ rpm}.$$

The forces are small, but the speed is very high: in these gears the problem is heating, noise, and wear rather than strength; this is why spiral bevel gears are used in such machines.

ANSWER a. $\delta_1 = 17.9^\circ$, $d_{m1} = 10.7$ mm
b. $F_t = 76.0$ N, $F_r = 26.3$ N, $F_a = 8.50$ N
c. $F_{a2} = 26.3$ N, $F_{r2} = 8.50$ N; 10 650 rpm

14.20

← Problem

a. Lead angles and the static friction angle:

$$z_1 = 1:\ \gamma = \arctan 0.1 = 5.711 \approx 5.71^\circ,$$

$$z_1 = 2:\ \gamma = \arctan 0.2 = 11.31 \approx 11.3^\circ,$$

$$\rho_0' = \arctan\frac{0.10}{\cos 20^\circ} = 6.074^\circ.$$

Reverse efficiency (with μ_0):

$$z_1 = 1:\ \eta' = \frac{0.9397 - 0.10(10)}{0.9397 + 0.10(0.1)} = -0.0635,$$

$$z_1 = 2:\ \eta' = \frac{0.9397 - 0.10(5)}{0.9397 + 0.10(0.2)} = 0.4582.$$

Only the single-start worm is self-locking ($\gamma < \rho_0'$, $\eta' < 0$).

b. Single start, $i = 50$:

$$\eta = \frac{0.9397 - 0.06(0.1)}{0.9397 + 0.06(10)} = 0.6064 \approx 0.606,$$

$$n_1 = 50(1.5) = 75 \text{ rpm},$$

$$P_2 = 2400\,\frac{2\pi(1.5)}{60} = 377.0 \text{ W},$$

$$P_1 = \frac{377.0}{0.6064} = 621.7 \approx 622 \text{ W},$$

$$T_1 = \frac{2400}{50(0.6064)} = 79.15 \approx 79.2 \text{ N·m}.$$

c. With a double start:

$$\eta = \frac{0.9397 - 0.06(0.2)}{0.9397 + 0.06(5)} = 0.7483,$$

$$P_1 = \frac{377.0}{0.7483} = 503.8 \approx 504 \text{ W}.$$

Self-locking requires about 118 W (23%) more power. On a revolving stage the power is small and the motion is short; since safety is more important, the single-start worm is the right choice. Even so, self-locking cannot be relied on under vibration; the brake stays.

ANSWER a. 5.71° and 11.3°; the single-start worm is self-locking b. $\eta = 0.606$, $n_1 = 75$ rpm, $P_1 = 622$ W, $T_1 = 79.2$ N·m c. 504 W; single start (safety)

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15 BELTS, CHAINS AND WIRE ROPES

15.7

← Problem

a. $\arcsin[155/1200] = 7.42°$:

$$\theta_k = 180° - 2(7.42°) = 165.2°,$$

$$v = \frac{\pi(125)(1460)}{60\,000} = 9.556 \approx 9.56 \text{ m/s}.$$

b. Corrected power of one belt $3.1(0.96)(0.98) = 2.916$ kW:

$$N = \frac{11(1.2)}{2.916} = 4.526 \approx 4.53,$$

that is, 5 belts. **c.** Total force difference and span forces:

$$F_1 - F_2 = \frac{11\,000}{9.556} = 1151 \text{ N},$$

$$F_2 = \frac{1151}{4} = 287.8 \text{ N}, \quad F_1 = 1439 \text{ N}.$$

Angle between the spans $180° - 165.2° = 14.84°$:

$$F_s = \sqrt{1439^2 + 287.8^2 + 2(1439)(287.8)\cos 14.84°},$$

$$F_s = 1719 \text{ N} \approx 1.72 \text{ kN}.$$

$$R_A = 1719\left(1 + \frac{80}{250}\right) = 2269 \text{ N} \approx 2.27 \text{ kN},$$

$$R_B = 1719\,\frac{80}{250} = 550.0 \approx 550 \text{ N}$$

(in the opposite direction). The shaft load is about 1.5 times the transmitted force (1151 N); bearing A, close to the pulley, is the most heavily loaded element. Moving the pulley closer to the bearing reduces this load directly.

ANSWER a. 165.2°, 9.56 m/s b. 4.53 ⇒ 5 belts c. $F_s = 1.72$ kN, $R_A = 2.27$ kN, $R_B = 550$ N

15.8

← Problem

In a timing belt, power is transmitted by positive engagement between the belt teeth and the pulley teeth; there is no slip, preload is needed only to keep the teeth from jumping out of the pulley, and the shaft load is small. **B** is correct.

- (A): A V-belt works on its flanks; a belt that touches the groove bottom is worn, loses the wedge effect, and slips.
- (C): $\mu' = \mu/\sin(\beta/2)$; the equivalent friction increases as the groove angle decreases.
- (D): In a timing belt there is tooth engagement, not friction.
- (E): Among belts of slightly different lengths, the shortest carries most of the load; the belts are installed as a matched set and replaced together.

ANSWER B

15.13

← Problem

a. $d_1 = 15.875/\sin(180°/17) = 86.39$ mm:

$$v = \frac{17(15.875)(3400)}{60\,000} = 15.29 \approx 15.3 \text{ m/s},$$

$$F = \frac{38\,000}{15.29} = 2485 \text{ N} \approx 2.49 \text{ kN},$$

$$F_c = 0.80(15.29)^2 = 187.1 \approx 187 \text{ N},$$

$$n = \frac{37\,000}{2485 + 187.1} = 13.848 \approx 13.8.$$

b. In first gear the speed is low, and F_c can be neglected:

$$F = \frac{2(560 \times 10^3)}{86.39} = 12\,964 \text{ N} \approx 13.0 \text{ kN},$$

$$n = \frac{37\,000}{12\,964} = 2.854 \approx 2.85.$$

The chain is stressed most not by maximum power but by maximum torque in low gear; this is why the safety against breaking is low in motorcycle chains, and the chain is also checked for fatigue.

c. Only the tight-span force is passed to the shaft (the centrifugal force is balanced in the chain). Overhung sprocket, moments about A:

$$R_A = 12\,964\left(1 + \frac{35}{110}\right) = 17\,089 \text{ N} \approx 17.1 \text{ kN},$$

$$R_B = 12\,964\,\frac{35}{110} = 4125 \text{ N} \approx 4.12 \text{ kN}.$$

R_B acts opposite to F. Moving the sprocket closer to bearing A reduces the load on this bearing directly.

ANSWER a. 15.3 m/s, $F = 2.49$ kN, $F_c = 187$ N, $n = 13.8$ b. 13.0 kN, $n = 2.85$ c. $R_A = 17.1$ kN, $R_B = 4.12$ kN (in the opposite direction)

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