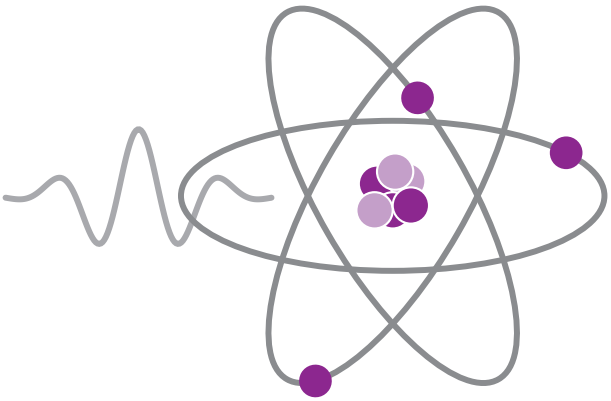
MODERN PHYSICS

PROBLEM BOOK

SAMPLE PAGES

14 chapters · 172 problems · step-by-step solutions



Ertan Sinan Şahin

ertansinansahin.com

How to use this book

- Each chapter opens with a reminder box summarizing the definitions and formulas the problems rely on.
- Difficulty: ★☆☆ basic ★★☆ intermediate ★★★ advanced. Each chapter has a few multiple-choice concept questions; its last problem is an exam-style mixed problem combining several topics.
- The dotted area below each problem is for your own work. Solve first; if stuck, go back to the reminder box and the lecture notes, and look at the solution last.
- All solutions are at the end of the book. The Solution → link next to a problem takes you to its solution, and ← Problem takes you back. Short answers are in the **ANSWER** box. Each solution begins with a short statement of the idea; in particle problems it checks whether relativistic relations are needed.
- **Appendices:** Physical constants, atomic masses and isotopes, electron configurations, and mathematical aids are at the end of the book. Constants are not repeated in the problem statements; the solutions use the four-figure values of the appendix. Shortcuts such as $hc = 1240$ eV·nm are used in the calculations.
- **Advanced chapters: 10, 11, and 14.** Molecules (Chapter 10), statistical physics and lasers (Chapter 11), and nuclear reactions and particles (Chapter 14) are covered in fewer than half of the courses; they are marked ADVANCED in the contents. If your course does not cover these topics, you can skip these chapters; the later chapters do not depend on them.
- **Midterm and final:** Choose the chapters according to your own syllabus. Courses that teach relativity at the end of the term can leave Chapters 1–2 for last; the later chapters use only the relation $E^2 = (pc)^2 + (mc^2)^2$ from relativity. For electrical engineering and materials students, Chapters 1–9 and 12 have priority; for physics teacher education, Chapters 1–2 and 13–14.

Contents

Page numbers refer to the full book. Chapters in color are in this sample; click to go there.

1 Special Relativity I: Kinematics ..... 16 problems 4

2 Special Relativity II: Momentum and Energy ..... 14 problems 13

3 **The Particle Nature of Light** ..... 14 problems 21

4 The Nuclear Atom ..... 12 problems 32

5 The Wave Nature of Matter ..... 14 problems 42

6 The Schrödinger Equation ..... 14 problems 51

7 **Barriers, Steps, and Tunneling** ..... 10 problems 60

8 The Hydrogen Atom ..... 12 problems 69

9 Spin and Many-Electron Atoms ..... 12 problems 78

10 Molecules ADVANCED ..... 8 problems 87

11 Statistical Physics and Lasers ADVANCED ..... 10 problems 94

12 Solid-State Physics ..... 12 problems 103

13 **Nuclear Physics** ..... 14 problems 112

14 Nuclear Reactions and Elementary Particles ADVANCED ..... 10 problems 122

Solutions 129

CHAPTER 3

THE PARTICLE NATURE OF LIGHT

Wien's law	$\lambda_{\max} T = b$	cutoff wavelength	$\lambda_{\min} = \dfrac{hc}{eV}$
Stefan–Boltzmann	$I = \sigma T^4$	Bragg condition	$2d \sin\theta = n\lambda$
Planck's law	$I(\lambda, T) = \dfrac{2\pi hc^2}{\lambda^5} \dfrac{1}{e^{hc/\lambda k_B T} - 1}$	Compton shift	$\lambda' - \lambda = \dfrac{h}{m_e c}(1 - \cos\theta)$
photon	$E = hf = \dfrac{hc}{\lambda}, \quad p = \dfrac{h}{\lambda} = \dfrac{E}{c}$	pair threshold	$E_\gamma \geq 2m_e c^2 = 1.022\ \text{MeV}$
photoelectric effect	$K_{\max} = eV_0 = hf - \phi$	annihilation (at rest)	$e^+ + e^- \to 2\gamma, \quad E_\gamma = m_e c^2$

I is the total power emitted per unit area of a blackbody; $I(\lambda, T)$ is the power emitted per unit area per unit wavelength interval (spectral exitance), $I = \int_0^\infty I(\lambda, T)\, d\lambda$. At the peak of the Planck curve, $hc/(\lambda_{\max} k_B T) = 4.965$; the Wien constant b and σ are in the appendix. In the photoelectric effect, ϕ is the work function of the metal; the threshold frequency is $f_0 = \phi/h$, and the slope h/e of the V_0–f line is the same for every metal. The intensity of the light sets the number of photons; the frequency sets the energy of each photon. In the Compton formula, θ is the scattering angle of the photon, and $h/m_e c = \lambda_C = 2.426$ pm is the Compton wavelength of the electron. A photon alone in vacuum cannot create a pair, and a pair cannot annihilate into a single photon, because energy and momentum cannot both be conserved; pair production takes place near a nucleus.

Blackbody radiation

3.1 ★☆☆ Solution →

A small hole in the wall of a furnace radiates like a blackbody. The absolute temperature of the furnace is raised from T to $2T$. Which of the following is true?

A) The total power emitted from the hole doubles.

B) The spectral exitance increases at every wavelength, and the wavelength of the peak of the spectrum is halved.

C) The wavelength of the peak of the spectrum doubles; the hole looks redder.

D) The average energy of the emitted photons does not change; only the number of photons increases.

E) According to classical physics, too, the total power is finite and proportional to T^4; Planck's law only corrects the proportionality constant.

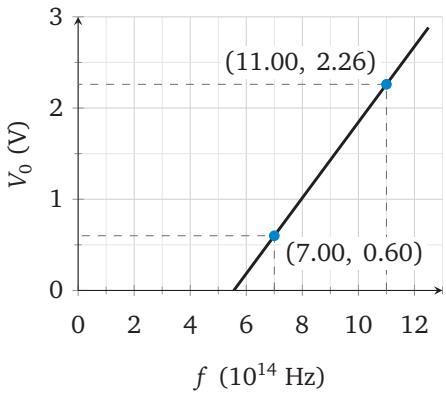
The photoelectric effect

3.5

★★☆ Solution →

The figure gives the stopping potential measured for the cathode metal of a phototube as a function of the frequency of the light; two measured points are marked. The work functions of some metals are as follows:

metal	Cs	K	Na	Ca	Mg
ϕ (eV)	1.95	2.29	2.75	2.87	3.66



a. Find h from the graph. By what percentage does it differ from the value in the appendix?

b. Find the work function, the threshold frequency, and the threshold wavelength of the cathode. Which metal could the cathode be?

c. If the cathode is replaced by magnesium, what does the V_0–f line look like? Sketch it on the graph (use h from the appendix), and find the threshold wavelength.

d. Which of the three lasers, with wavelengths 405 nm, 450 nm, and 650 nm, eject electrons from the metal of part b (with the value in the table) and from magnesium? For those that do, find K_{max}.

CHAPTER 7

BARRIERS, STEPS, AND TUNNELING

wave numbers $k = \dfrac{\sqrt{2mE}}{\hbar}, \quad \kappa = \dfrac{\sqrt{2m(U_0 - E)}}{\hbar}$

step, $E > U_0$ $R = \left(\dfrac{k_1 - k_2}{k_1 + k_2}\right)^2$

$T = \dfrac{4k_1k_2}{(k_1 + k_2)^2}, \quad R + T = 1$

step, $E < U_0$ $R = 1, \quad \psi \propto e^{-\kappa x}\ (x > 0)$

transmission coefficient $T = \dfrac{|\psi_{\text{tr}}|^2 v_{\text{tr}}}{|\psi_{\text{inc}}|^2 v_{\text{inc}}}$

barrier, $E < U_0$ (exact) $T = \left[1 + \dfrac{U_0^2 \sinh^2(\kappa L)}{4E(U_0 - E)}\right]^{-1}$

thick barrier, $\kappa L \gg 1$ $T \approx 16 \dfrac{E}{U_0}\left(1 - \dfrac{E}{U_0}\right) e^{-2\kappa L}$

barrier, $E > U_0$ (exact) $T = \left[1 + \dfrac{U_0^2 \sin^2(k'L)}{4E(E - U_0)}\right]^{-1}$

$k' = \dfrac{\sqrt{2m(E - U_0)}}{\hbar}$

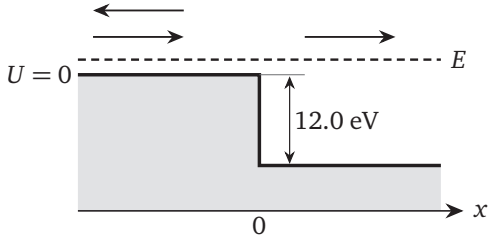
tunneling current $I \propto e^{-2\kappa d}$

At a step or a barrier the particle comes from the left; T and R are the fractions of the incident particles that are transmitted and reflected. In the transmission coefficient the square of the amplitude of the wave function is multiplied by the speed of the particle: if the speeds on the two sides differ, $|C/A|^2$ alone is not T. If the potential is the same on both sides of a barrier, the transmitted particle has the same energy and wavelength as the incident one. The region where $E < U_0$ is the classically forbidden region; there ψ does not oscillate but has the form $e^{\pm\kappa x}$. The thick-barrier approximation is used when κL is larger than a few units; every solution states which expression is used. For an electron, with $\hbar c = 197.3$ eV·nm and $mc^2 = 0.5110$ MeV, $\kappa = \sqrt{2mc^2(U_0 - E)}/(\hbar c)$.

The potential step

7.2 ★☆☆ Solution →

Electrons of kinetic energy 2.00 eV approach from the left a region where the potential energy drops suddenly by 12.0 eV at $x = 0$ (figure).



a. Find the wave number and the de Broglie wavelength of the electron on the two sides of the step.

b. What percentage of the electrons are reflected, and what percentage are transmitted?

c. If electrons of kinetic energy 14.0 eV approached the same step from the right, what would the reflection coefficient be?

The rectangular barrier

7.6 ★★★ Solution →

An electron approaches a rectangular barrier of height $U_0 = 3.00$ eV and width $L = 0.600$ nm.

a. By taking the limit $E \to U_0$ in the exact transmission coefficient for $E < U_0$, show that

$$T(E = U_0) = \left[1 + \frac{mU_0L^2}{2\hbar^2}\right]^{-1}$$

and calculate it for this barrier. What does classical physics predict at this energy?

b. According to the exact expression for $E > U_0$, write the condition on the energies at which $T = 1$ (the barrier is completely transparent). Find the two lowest energies that satisfy this condition.

c. Sketch the graph of $T(E)$ qualitatively in the range $0 < E < 8$ eV; mark the points from parts a and b on the graph.

CHAPTER 13

NUCLEAR PHYSICS

nuclear radius $R = R_0 A^{1/3}, \quad R_0 \approx 1.2 \text{ fm}$

binding energy $B = \left[Z m_{\mathrm{H}} + N m_n - M(A,Z) \right] c^2$

separation energy

$S_n = \left[M(A-1,Z) + m_n - M(A,Z) \right] c^2$

mass formula $B = a_V A - a_S A^{2/3} - a_C \dfrac{Z(Z-1)}{A^{1/3}} - a_A \dfrac{(A-2Z)^2}{A} \pm \delta$

disintegration energy

$Q = \left(\sum M_{\text{initial}} - \sum M_{\text{final}} \right) c^2$

decay law $N = N_0 e^{-\lambda t}, \quad R = \lambda N = R_0 e^{-\lambda t}$

half-life, mean life $T_{1/2} = \dfrac{\ln 2}{\lambda}, \quad \tau = \dfrac{1}{\lambda}$

$N = N_0 \, 2^{-t/T_{1/2}}$

α $Q = \left[M_P - M_D - M(^4\mathrm{He}) \right] c^2$

β^-, EC $Q = (M_P - M_D) c^2$

β^+ $Q = (M_P - M_D - 2m_e) c^2$

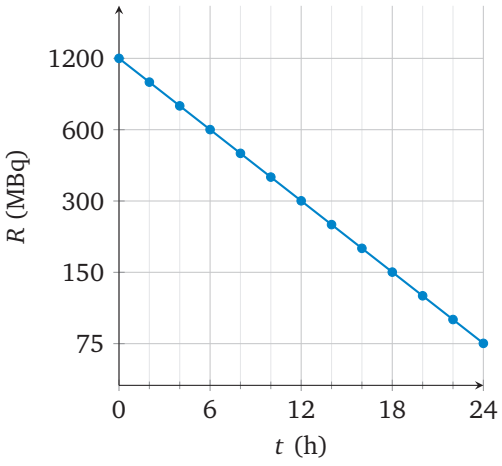
unit of activity $1 \text{ Ci} = 3.7 \times 10^{10} \text{ Bq}$

Masses are those of neutral atoms; m_{H} is the mass of the hydrogen atom, so the electron masses cancel between $Z m_{\mathrm{H}}$ and $M(A,Z)$. M_P is the mass of the parent atom and M_D that of the daughter; the $2m_e$ term in β^+ decay comes from this choice of atomic masses. $1 \text{ u} = 931.5 \text{ MeV}/c^2$. The probability that a nucleus decays does not depend on its age; hence the activity decreases exponentially. If the half-life of the parent is much longer than that of the daughter, the two activities become equal after a long time (secular equilibrium).

The law of radioactive decay

13.9 ★★☆ Solution →

$^{99\mathrm{m}}$Tc (a metastable excited state of technetium), used in medical imaging, decays to its ground state by emitting a gamma ray. The activity of a prepared dose was measured every two hours and plotted on a logarithmic axis as shown.



a. What does it mean that the points lie on a straight line on the logarithmic axis? Read the half-life from the graph.

b. Find the decay constant and the mean life.

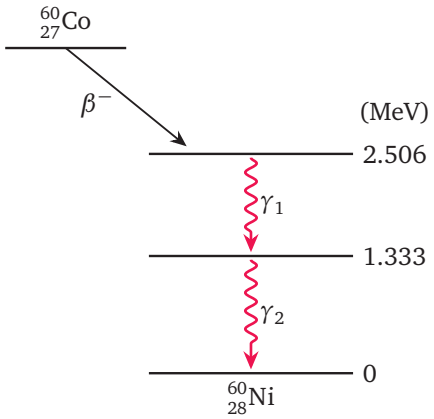
c. If the patient is to receive 400 MBq, how many hours after preparation should the injection be made? How many $^{99\mathrm{m}}$Tc nuclei are in the dose at the time of preparation?

Mixed problems

13.14

★★★ Solution →

A ^{60}Co source used in cancer treatment has an activity of 4.00×10^{14} Bq. In the decay scheme shown, ^{60}Co goes by β^- decay to the 2.506 MeV excited level of ^{60}Ni; this level decays to the ground state by emitting two gamma rays. The average kinetic energy of the electrons in a β^- decay is 0.0958 MeV.



a. Find the Q value of the decay with the masses in the appendix. What are the largest kinetic energy of the electrons and the energies of the two gamma rays?

b. Find the number and mass of ^{60}Co nuclei in the source.

c. Calculate the power the source emits as gamma rays and the thermal power produced by the electrons absorbed in the capsule. By what factor does the treatment time needed for the same radiation dose increase after 2.00 years?

SOLUTIONS

3 THE PARTICLE NATURE OF LIGHT

3.1

← Problem

By Wien's law, $\lambda_{\max} \propto 1/T$; in Planck's law, as T increases, $e^{hc/\lambda k_B T}$ decreases at every λ, so $I(\lambda, T)$ increases. Planck curves for different temperatures never cross. The peak shifts to $\lambda_{\max}/2$. **B** is true.

- (A): $I = \sigma T^4$; when the temperature doubles, the total power becomes $2^4 = 16$ times as large.
- (C): The direction of Wien's law is reversed. A hotter body radiates at shorter wavelengths; its color shifts from red toward white and blue.
- (D): The typical photon energy is proportional to T (at the peak, $hc/\lambda_{\max} = 4.965\,k_B T$); the average photon energy also doubles. Since the power becomes 16 times as large, the number of photons becomes 8 times as large.
- (E): Classical theory (Rayleigh–Jeans) gives $I(\lambda, T) \propto T/\lambda^4$ at short wavelengths; the integral of this expression is infinite (the ultraviolet catastrophe). A finite power and the T^4 dependence follow only from Planck's law, in which the energy is quantized.

ANSWER B

3.5

← Problem

The relation $eV_0 = hf - \phi$ is the straight line $V_0 = (h/e)f - \phi/e$: the slope is h/e, and the line crosses the f axis at the threshold frequency.

a. The slope from the two points:

$$\frac{h}{e} = \frac{2.26 - 0.60}{(11.00 - 7.00) \times 10^{14}} = 4.150 \times 10^{-15}\ \text{V·s}$$

$$h = (4.150 \times 10^{-15})(1.602 \times 10^{-19})$$

$$h = 6.6483 \times 10^{-34} \approx 6.65 \times 10^{-34}\ \text{J·s}$$

$$\frac{6.6483 - 6.626}{6.626} = 0.0034$$

The measured value is about 0.3% larger than the value in the appendix.

b. Substituting the first point into the equation of the line (ϕ in eV):

$$\phi = \frac{h}{e} f_1 - V_1 = (4.150 \times 10^{-15})(7.00 \times 10^{14}) - 0.60$$

$$\phi = 2.905 - 0.60 = 2.305 \approx 2.31\ \text{eV}$$

$$f_0 = \frac{\phi}{h/e} = \frac{2.305}{4.150 \times 10^{-15}}$$

$$f_0 = 5.554 \times 10^{14} \approx 5.55 \times 10^{14}\ \text{Hz}$$

$$\lambda_0 = \frac{c}{f_0} = \frac{2.998 \times 10^8}{5.554 \times 10^{14}} = 5.398 \times 10^{-7}\ \text{m} \approx 540\ \text{nm}$$

The value in the table closest to 2.31 eV is that of potassium (2.29 eV); the cathode could be potassium.

c. The slope h/e does not depend on the metal; the magnesium line is parallel to the potassium line and shifted to the right. It crosses the f axis at

$$f_0 = \frac{\phi}{h} = \frac{3.66}{4.136 \times 10^{-15}} = 8.849 \times 10^{14}\ \text{Hz}$$

The line starts at $f = 8.85 \times 10^{14}$ Hz and reaches $V_0 = 1.30$ V at the right edge of the graph ($f = 12 \times 10^{14}$ Hz).

$$\lambda_0 = \frac{hc}{\phi} = \frac{1240}{3.66} = 338.8 \approx 339\ \text{nm}$$

d. The photon energies are $E = 1240/\lambda$:

$$E_{405} = 3.062\ \text{eV}, \quad E_{450} = 2.756\ \text{eV}, \quad E_{650} = 1.908\ \text{eV}$$

Potassium ($\phi = 2.29$ eV): 650 nm is below the threshold and ejects no electrons;

$$K_{\max}(405) = 3.062 - 2.29 = 0.772\ \text{eV}$$

$$K_{\max}(450) = 2.756 - 2.29 = 0.466\ \text{eV}$$

Magnesium ($\phi = 3.66$ eV, threshold 339 nm): all three photon energies are less than 3.66 eV; none ejects electrons. Magnesium needs ultraviolet light.

ANSWER **a.** $h = 6.65 \times 10^{-34}$ J·s (0.3% larger) **b.** $\phi = 2.31$ eV, $f_0 = 5.55 \times 10^{14}$ Hz, $\lambda_0 = 540$ nm; potassium **c.** parallel line, $f_0 = 8.85 \times 10^{14}$ Hz, $\lambda_0 = 339$ nm **d.** K: 405 nm (0.772 eV), 450 nm (0.466 eV); Mg: none

ertansinansahin.com

7 BARRIERS, STEPS, AND TUNNELING

7.2

← Problem

At the step ψ and $d\psi/dx$ must be continuous; between the oscillating waves of the two regions, both with $E > U$, these conditions give $R = [(k_1 - k_2)/(k_1 + k_2)]^2$. Even though the step goes down, there is reflection because $k_1 \neq k_2$. **Assumption:** the energies are much smaller than $mc^2 = 511$ keV; nonrelativistic relations.

a. The kinetic energy is 2.00 eV on the left and $2.00 + 12.0 = 14.0$ eV on the right:

$$k_1 = \frac{\sqrt{2(511\,000)(2.00)}}{197.3} = \frac{1430}{197.3} = 7.246\ \text{nm}^{-1}$$

$$k_2 = \frac{\sqrt{2(511\,000)(14.0)}}{197.3} = 19.17\ \text{nm}^{-1}$$

$$\lambda_1 = \frac{2\pi}{k_1} = 0.8671 \approx 0.867\ \text{nm}$$

$$\lambda_2 = \frac{2\pi}{k_2} = 0.3277 \approx 0.328\ \text{nm}$$

b. $k_2/k_1 = \sqrt{14.0/2.00} = \sqrt{7} = 2.646$:

$$R = \left(\frac{1 - 2.646}{1 + 2.646}\right)^2 = (-0.4514)^2 = 0.2038 \approx 0.204$$

$$T = 1 - R = 0.796$$

About 20.4% of the electrons are reflected and 79.6% are transmitted; classically all of them would pass.

c. An electron of 14.0 eV coming from the right enters the 2.00 eV region after the step; k_1 and k_2 are interchanged. Since the expression for R contains $(k_1 - k_2)^2$, it does not change:

$$R = \left(\frac{2.646 - 1}{2.646 + 1}\right)^2 = 0.204$$

Check: a similar situation occurs when slow electrons strike a metal surface: the potential energy is lower inside the metal, and some of the slow electrons are reflected from the surface.

ANSWER **a.** $k_1 = 7.246\ \text{nm}^{-1}$, $\lambda_1 = 0.867$ nm; $k_2 = 19.17\ \text{nm}^{-1}$, $\lambda_2 = 0.328$ nm **b.** $R = 0.204$ (20.4%), $T = 0.796$ (79.6%) **c.** $R = 0.204$ (the same)

7.6

← Problem

At $E \to U_0$, $\kappa \to 0$; writing $\sinh(\kappa L) \approx \kappa L$, the factors $U_0 - E$ that go to zero cancel. For $E > U_0$, $T = 1$ requires $\sin(k'L) = 0$: the waves reflected from the two edges of the barrier cancel each other.

a. For small κL, $\sinh^2(\kappa L) \approx \kappa^2 L^2 = 2m(U_0 - E)L^2/\hbar^2$:

$$\frac{U_0^2 \sinh^2(\kappa L)}{4E(U_0 - E)} \approx \frac{U_0^2}{4E(U_0 - E)} \cdot \frac{2m(U_0 - E)L^2}{\hbar^2} = \frac{mU_0^2 L^2}{2\hbar^2 E}$$

At $E = U_0$ this becomes $mU_0L^2/(2\hbar^2)$, which gives the required expression. Numerically:

$$\frac{mU_0L^2}{2\hbar^2} = \frac{mc^2U_0L^2}{2(\hbar c)^2} = \frac{(511\,000)(3.00)(0.600)^2}{2(197.3)^2}$$

$$\frac{mU_0L^2}{2\hbar^2} = \frac{551\,880}{77\,855} = 7.089$$

$$T = \frac{1}{1 + 7.089} = 0.1236$$

In classical physics $E = U_0$ is the borderline case: with slightly less energy the particle turns back ($T = 0$), and with slightly more it passes ($T = 1$). In quantum mechanics only 12.4% of the electrons are transmitted at $E = U_0$, and T varies continuously with energy.

b. For $T = 1$, $\sin(k'L) = 0$, that is, $k'L = n\pi$ ($n = 1, 2, \ldots$):

$$E - U_0 = \frac{n^2\pi^2\hbar^2}{2mL^2} = n^2 \frac{\pi^2(197.3)^2}{2(511\,000)(0.600)^2}$$

$$E - U_0 = n^2(1.044\ \text{eV})$$

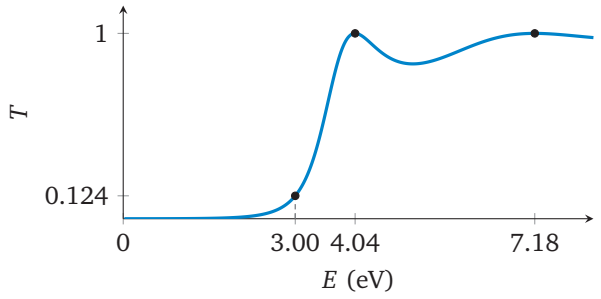
$$E_1 = 3.00 + 1.044 = 4.044 \approx 4.04\ \text{eV}$$

$$E_2 = 3.00 + 4(1.044) = 7.177 \approx 7.18\ \text{eV}$$

At these energies a whole number of half-wavelengths fits inside the barrier ($L = n\lambda'/2$).

c. As $E \to 0$, $T \to 0$; for $E < U_0$, T grows exponentially (for example, at 1 eV, T is about 6×10^{-4}) and reaches 0.124 at $E = U_0$. For $E > U_0$, T rises quickly to 1 at 4.04 eV, then dips slightly (its smallest value, near 5.05 eV, is about 0.84), and returns to 1 at 7.18 eV. As the energy grows, the oscillations shrink and $T \to 1$.

ertansinansahin.com



ANSWER **a.** $T = 0.1236$ (classical: borderline case) **b.** $k'L = n\pi$; $E_1 = 4.04$ eV, $E_2 = 7.18$ eV **c.** graph: T grows exponentially, 0.124 at $E = U_0$, 1 at 4.04 and 7.18 eV

13 NUCLEAR PHYSICS

13.9

← Problem

$\ln R = \ln R_0 - \lambda t$: on a logarithmic axis an exponential decrease is a straight line; the time for the activity to halve is the same everywhere.

a. The straight line shows that the activity decreases exponentially with a constant half-life. At $t = 0$, $R = 1200$ MBq; it falls to 600 MBq at $t \approx 6.0$ hours and to 300 MBq at $t \approx 12$ hours: $T_{1/2} \approx 6.0$ hours.

b. Decay constant and mean life:

$$\lambda = \frac{0.6931}{6.0\text{ h}} = 0.1155\text{ h}^{-1}$$

$$\lambda = \frac{0.1155}{3600\text{ s}} = 3.209 \times 10^{-5} \approx 3.21 \times 10^{-5}\text{ s}^{-1}$$

$$\tau = \frac{1}{\lambda} = \frac{1}{0.1155} = 8.66\text{ h}$$

c. To fall from 1200 MBq to 400 MBq, $2^{-t/T_{1/2}} = 1/3$:

$$t = T_{1/2}\log_2 3 = 6.0\,(1.585) = 9.51\text{ h}$$

This time lies between 6 and 12 hours, which correspond to the 600 and 300 MBq lines; it is consistent. The number of nuclei at the time of preparation:

$$N_0 = \frac{R_0}{\lambda} = \frac{1.200 \times 10^9}{3.209 \times 10^{-5}} = 3.739 \times 10^{13} \approx 3.74 \times 10^{13}$$

Check: after one day (4 half-lives) 1/16 of the dose remains; the short half-life keeps the total dose received by the patient small. The measured half-life is 6.007 hours.

ANSWER **a.** exponential decay; $T_{1/2} \approx 6.0$ h **b.** $\lambda = 3.21 \times 10^{-5}$ s^{-1}, $\tau = 8.66$ h **c.** $t = 9.51$ h; $N_0 = 3.74 \times 10^{13}$

13.14

← Problem

The Q of the β^- decay is shared among the kinetic energies of the electron and the antineutrino and the excitation energy of the daughter nucleus; the excited level then gives up its energy as gamma rays.

a. For β^-, the difference of the atomic masses:

$$Q = (59.933816 - 59.930785)\,(931.5) = 2.823\text{ MeV}$$

$$K_{\max} = Q - 2.506 = 0.317\text{ MeV}$$

$$E_{\gamma_1} = 2.506 - 1.333 = 1.173\text{ MeV}$$

$$E_{\gamma_2} = 1.333\text{ MeV}$$

b. From the appendix, $T_{1/2} = 5.271$ yr:

$$\lambda = \frac{0.6931}{5.271\,(3.156 \times 10^7)} = 4.167 \times 10^{-9}\text{ s}^{-1}$$

$$N = \frac{4.00 \times 10^{14}}{4.167 \times 10^{-9}} = 9.600 \times 10^{22} \approx 9.60 \times 10^{22}$$

$$m = \frac{9.600 \times 10^{22}}{6.022 \times 10^{23}}\,(59.934) = 9.554 \approx 9.55\text{ g}$$

c. In each decay the two gamma rays carry a total of 2.506 MeV:

$$P_\gamma = (4.00 \times 10^{14})(2.506 \times 10^6)(1.602 \times 10^{-19})$$

$$P_\gamma = 160.6 \approx 161\text{ W}$$

$$P_\beta = (4.00 \times 10^{14})(0.0958 \times 10^6)(1.602 \times 10^{-19})$$

$$P_\beta = 6.14\text{ W}$$

The dose rate is proportional to the activity; for the same dose the time is proportional to $1/R$:

$$\frac{t_2}{t_1} = \frac{R_0}{R} = 2^{2.00/5.271} = 1.301 \approx 1.30$$

Check: the rest of the average energy per decay (2.823−2.506−0.0958 = 0.221 MeV) leaves the source with the antineutrino; it contributes neither to heat nor to dose.

ANSWER **a.** $Q = 2.823$ MeV; $K_{\max} = 0.317$ MeV; $E_\gamma = 1.173$, 1.333 MeV **b.** $N = 9.60 \times 10^{22}$, $m = 9.55$ g **c.** $P_\gamma = 161$ W, $P_\beta = 6.14$ W; 1.30 times

ertansinansahin.com

IN THE FULL BOOK

- 14 chapters and 172 problems, all with step-by-step solutions.
- A reminder box at the start of each chapter, summarizing the relations the problems rely on.
- Basic, intermediate and advanced problems, each with a dotted area for your own work.
- One-click links from each problem to its solution and back.
- An Appendix of our own at the end of the book: CODATA 2022 physical constants, a table of atomic masses and half-lives for 90 nuclides (AME2020/NUBASE2020), electron configurations, and mathematical aids (integrals, harmonic oscillator and hydrogen wave functions).



ertansinansahin.com