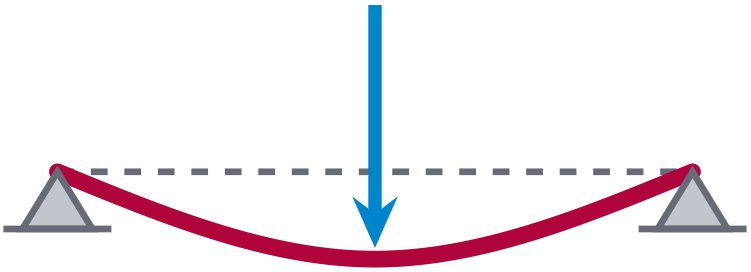
MECHANICS OF MATERIALS 1

PROBLEM BOOK

SAMPLE PAGES

6 chapters · 106 problems · step-by-step solutions



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How to use this book

- Each chapter opens with a reminder box summarizing the definitions and formulas the problems rely on.
- Difficulty: ★☆☆ basic ★★☆ intermediate ★★★ advanced. Each chapter has a few multiple-choice concept questions; each chapter ends with exam-style mixed problems.
- The dotted area below each problem is for your own work. Solve first; if stuck, go back to the reminder box and the lecture notes, and look at the solution last.
- All solutions are at the end of the book. The Solution → link next to a problem takes you to its solution, and ← Problem takes you back. Short answers are in the **ANSWER** box.
- **Units and signs:** SI units throughout; stresses in MPa, moduli of elasticity in GPa; $g = 9.81$ m/s^2. Normal stress and strain are positive in tension. A bending moment is positive when it compresses the top fiber of the beam. In the figures, forces and reactions are red.
- **Shared chapters:** Chapters 5–6 (analysis and design of beams, shearing stresses in beams) also appear in *Mechanics of Materials 2*, for schools whose second course begins with beams.
- **Midterm and final:** Mechanics of Materials I is usually taught in the fall of the second year. The first midterm usually covers stress, axial loading and torsion (Chapters 1–3); the final covers bending and beams (Chapters 4–6). Schools that complete the subject in a single term also need *Mechanics of Materials 2*. Choose the chapters to study according to your own course syllabus.

Contents

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CHAPTER 2

STRESS AND STRAIN: AXIAL LOADING

normal strain	$\varepsilon = \dfrac{\delta}{L},\ \sigma = E\varepsilon$	generalized Hooke's law	$\varepsilon_x = \dfrac{\sigma_x}{E} - \dfrac{\nu}{E}(\sigma_y + \sigma_z)$
axial elongation	$\delta = \dfrac{PL}{AE},\ \delta = \sum_i \dfrac{P_i L_i}{A_i E_i}$	change in volume	$e = \dfrac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$
thermal elongation	$\delta_T = \alpha \Delta T\, L$	shear	$\tau = G\gamma,\ G = \dfrac{E}{2(1+\nu)}$
Poisson's ratio	$\nu = -\dfrac{\varepsilon_{\text{lateral}}}{\varepsilon_{\text{axial}}}$	elastoplastic	$\sigma = E\varepsilon\ (\varepsilon \le \varepsilon_Y),\ \sigma = \sigma_Y\ (\varepsilon \ge \varepsilon_Y)$
stress concentration	$\sigma_{\max} = K\,\sigma_{\text{ave}}$		

Normal stress and elongation are positive in tension. In statically indeterminate problems, equilibrium, compatibility, and force–deformation relations are written separately. Unless stated otherwise, the following constants are used: for steel, $E = 200$ GPa, $\nu = 0.30$ and $\alpha = 12 \times 10^{-6}/°\text{C}$; for aluminum, $E = 70$ GPa, $\nu = 0.33$ and $\alpha = 23 \times 10^{-6}/°\text{C}$. Unless stated otherwise, materials remain in the linear elastic range; the self-weight of members is neglected.

Strain and Hooke's law

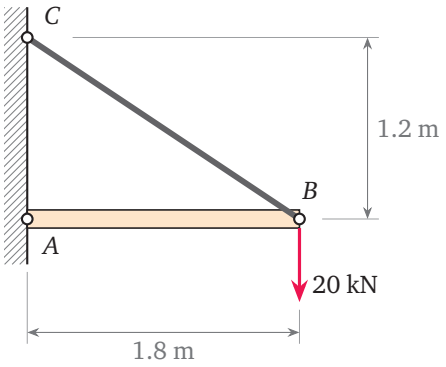
2.4 ★★☆ Solution →

The horizontal rigid arm AB is pinned to the wall at A; end B is held by a steel rod BC of 15 mm diameter running from point C, which is 1.2 m above A. $AB = 1.8$ m, and a load of 20 kN hangs from B.

a. Determine the stress and the elongation in rod BC.

b. Determine the vertical displacement of point B.

c. If the vertical displacement of B is not to exceed 3 mm, what must be the minimum diameter of the rod?



Statically indeterminate problems

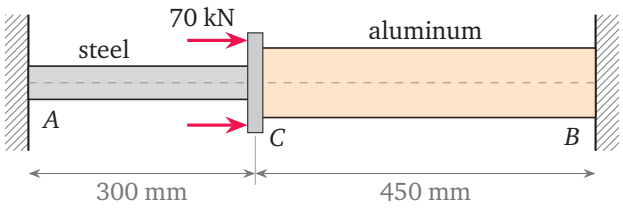
2.6 ★★☆ Solution →

The bar shown in the figure is fixed at both ends to rigid walls. Portion AC is steel of 22 mm diameter and portion CB is aluminum of 46 mm diameter. A total force of 70 kN toward B is applied to the collar at C.

a. Determine the wall reactions and the stress in each portion.

b. Determine the displacement of section C.

c. If the allowable stresses are 120 MPa in the steel and 80 MPa in the aluminum, what is the maximum force that can be applied at C?



Temperature change

2.8 ★☆☆ Solution →

A prismatic bar fixed without stress between two rigid walls is heated by ΔT. If the length and the cross-sectional area of the bar were doubled and the same heating were applied, what could be said about the stress in the bar?

A) Since the length doubles, the stress also doubles.

B) Since the cross-sectional area doubles, the stress is halved.

C) The effects of length and area multiply; the stress becomes four times as large.

D) The force on the walls does not change, so the stress is halved.

E) The stress does not change; its magnitude depends only on E, α, and ΔT.

CHAPTER 4

PURE BENDING

normal stress $\sigma_x = -\dfrac{My}{I},\ \sigma_m = \dfrac{Mc}{I} = \dfrac{M}{S}$

rectangular section $I = \frac{1}{12}bh^3,\ S = \frac{1}{6}bh^2$

curvature $\dfrac{1}{\rho} = \dfrac{M}{EI},\ \varepsilon_x = -\dfrac{y}{\rho},\ \rho' = \dfrac{\rho}{\nu}$

composite section $n = \dfrac{E_2}{E_1},\ \sigma_1 = -\dfrac{My}{I_{\text{tr}}},\ \sigma_2 = -n\dfrac{My}{I_{\text{tr}}}$

stress concentration $\sigma_m = K\dfrac{Mc}{I}$

eccentric axial load $\sigma_x = \dfrac{P}{A} - \dfrac{My}{I},\ M = Pe$

unsymmetric bending $\sigma_x = -\dfrac{M_z y}{I_z} + \dfrac{M_y z}{I_y}$

neutral axis $\tan\phi = \dfrac{I_z}{I_y}\tan\theta$

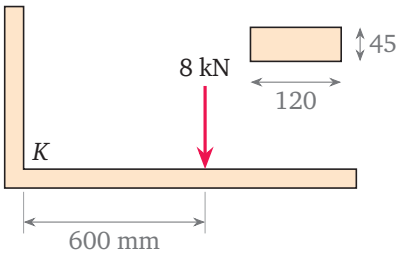
elastoplastic rectangle $M_p = \frac{3}{2}M_Y,\ M = \frac{3}{2}M_Y\left(1 - \dfrac{y_Y^2}{3c^2}\right)$

The sections have at least one axis of symmetry and the material operates in the linear elastic range (unless stated otherwise). The x axis is along the beam axis; in the cross section, y is upward and z is in the horizontal direction, and the neutral axis passes through the centroid. A bending moment is positive when it compresses the top fiber; for normal stress, tension is positive and compression is negative. In the figures, couples are shown by red curved arrows and moment vectors (right-hand rule) by red double-headed arrows. θ is the angle of the moment vector, and ϕ the angle of the neutral axis, with the z axis. Unless otherwise given, take $E = 200$ GPa for steel, $E = 70$ GPa and $\nu = 0.33$ for aluminum, and $E = 12$ GPa for wood.

Stresses and strains in pure bending

4.2

★☆☆ Solution →



The rectangular steel forks of a forklift are 120 mm wide and 45 mm thick. A pallet weighing 16 kN is shared equally by the two forks; the resultant load on each fork is 600 mm from the front face of the vertical shank of the fork. Neglect the self-weight of the fork, and take the root section K at the level of the front face of the shank.

a. Find the maximum normal stress at section K. Which face is in tension?

b. If the fork thickness is reduced by 10% through wear, by what percentage does the maximum stress increase under the same load?

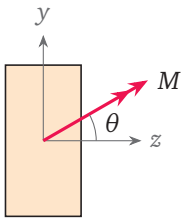
c. If the allowable stress is 180 MPa, what is the heaviest pallet, in kN, that can be lifted with the worn forks?

Unsymmetric bending

4.13

★☆☆ Solution →

A moment M whose vector makes an angle θ $(0 < \theta < 90°)$ with the z axis acts on a rectangular section whose height is twice its width. Which statement is correct for the location of the neutral axis?



A) The neutral axis coincides with the moment vector.

B) The neutral axis is perpendicular to the moment vector.

C) The neutral axis lies between the moment vector and the y axis $(\phi > \theta)$.

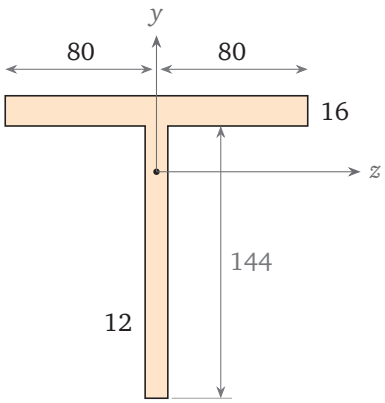
D) The neutral axis lies between the moment vector and the z axis $(\phi < \theta)$.

E) The neutral axis coincides with the z axis, independent of θ.

4.15

★★★ Solution →

The edge beam of a walkway passing over the basins in a treatment plant is a welded steel beam of T section: top flange 160 × 16, web 12 × 144 (mm). At the most critical section the moment of the vertical walkway loads is $M_z = 14\ \text{kN}\cdot\text{m}$ (it compresses the top fiber). Railing posts welded to the top flange transfer the horizontal push of the crowd leaning on the railing to the beam; this push produces an additional moment $M_y = 1.4\ \text{kN}\cdot\text{m}$ that bends the flange in the horizontal direction, compressing the left end of the flange and stretching the right end.



a. Find the maximum stresses at the two ends of the flange and at the bottom end of the web.

b. Find the angle the neutral axis makes with the horizontal.

c. By what percentage does the horizontal moment increase the maximum compressive stress at the flange ends?

CHAPTER 5

ANALYSIS AND DESIGN OF BEAMS FOR BENDING

load, shear, moment $\dfrac{dV}{dx} = -w, \quad \dfrac{dM}{dx} = V$

area relations

$$V_D - V_C = -\int_C^D w\,dx, \quad M_D - M_C = \int_C^D V\,dx$$

jumps

down P : $\Delta V = -P$; clockwise M_0 : $\Delta M = +M_0$

normal stress $\sigma_m = \dfrac{|M|_{\max} c}{I} = \dfrac{|M|_{\max}}{S}$

design $S_{\min} = \dfrac{|M|_{\max}}{\sigma_{\text{all}}}$

rectangle, circle $S = \frac{1}{6}bh^2, \quad S = \dfrac{\pi d^3}{32}$

singularity function $\langle x-a\rangle^n = \begin{cases} (x-a)^n & x \geq a \\ 0 & x < a \end{cases}$

integral $\displaystyle\int \langle x-a\rangle^n dx = \frac{\langle x-a\rangle^{n+1}}{n+1}$

concentrated load

$$w = P\langle x-a\rangle^{-1}, \quad \int \langle x-a\rangle^{-1} dx = \langle x-a\rangle^0$$

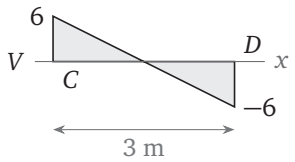
Beams are prismatic and act in the linear elastic range; their own weight is neglected unless stated otherwise. The x axis runs from the left end of the beam to the right; the distributed load w is positive downward. Sign convention: the shear V is positive if the resultant of the external forces acting on the part of the beam to the left of the section is directed upward; the bending moment M is positive when the top fiber of the beam is in compression and the bottom fiber is in tension. In the diagrams, positive values are plotted above the axis. In the figures, the triangle symbol is a pin support and the wheel symbol is a roller. $\langle x-a\rangle^0$ is 1 for $x \geq a$ and 0 for $x < a$.

Relations among load, shear, and moment

5.6

★☆☆ Solution →

The shear diagram of a beam over the interval CD is as shown in the figure: it decreases linearly from +6 kN at C to −6 kN at D. There is no concentrated load or couple in this interval. Which statement is correct for this interval?



A) M decreases linearly over the interval.

B) M is constant over the interval.

C) M takes its minimum value at the middle of the interval.

D) M takes its maximum value at the middle of the interval; the moments at C and D are equal.

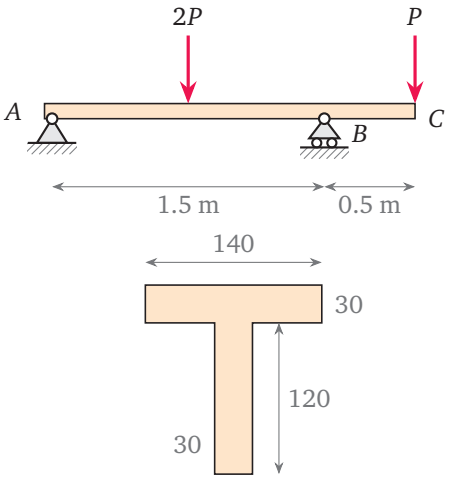
E) A uniformly distributed load acting upward is applied over the interval.

Mixed problems

5.17

★★★ Solution →

A cast-iron machine-support beam is pinned at A and supported by a roller at B: $AB = 1.5$ m, overhang $BC = 0.5$ m. A load $2P$ acts at the middle of AB and a vertical load P acts at end C. The cross section is a T with the flange on top: flange 140×30, web 30×120 (mm). The allowable stresses of the cast iron are 35 MPa in tension and 105 MPa in compression.



a. Draw the moment diagram in terms of P.

b. Find the maximum value of P. Which section and which fiber govern?

c. Can P be increased if the section is inverted (flange at the bottom)? Explain.

SOLUTIONS

2 STRESS AND STRAIN: AXIAL LOADING

2.4

← Problem

The length BC is $\sqrt{1.8^2+1.2^2}=2.163$ m; for the angle the rod makes with the horizontal, $\sin\theta = 1.2/2.163 = 0.5547$.

a. Moment about A: the vertical component of the rod force balances the load.

$$(T\sin\theta)(1.8) = (20)(1.8) \;\Rightarrow\; T = \frac{20}{0.5547} = 36.06 \text{ kN}.$$

$$\sigma = \frac{36.06\times10^3}{\frac{\pi}{4}(15)^2} = \frac{36.06\times10^3}{176.7} = 204 \text{ MPa},$$

$$\delta = \frac{\sigma L}{E} = \frac{(204.0)(2163)}{200\times10^3} = 2.207 \approx 2.21 \text{ mm}.$$

b. The arm rotates about A through a small angle; B moves down a vertical distance v_B. The elongation of the rod is the component of the displacement of B along the rod direction:

$$\delta = v_B\sin\theta \;\Rightarrow\; v_B = \frac{2.207}{0.5547} = 3.98 \text{ mm}.$$

B moves down almost twice as much as the rod elongates; the inclined rod resists the vertical motion only partially.

c. Condition $v_B = \delta/\sin\theta \le 3$ mm:

$$\delta = \frac{TL}{AE} \le (3)(0.5547) = 1.664 \text{ mm},$$

$$A \ge \frac{(36.06\times10^3)(2163)}{(200\times10^3)(1.664)} = 234.4 \text{ mm}^2,$$

$$d \ge 17.3 \text{ mm}.$$

ANSWER a. 204 MPa, 2.21 mm b. 3.98 mm downward c. 17.3 mm

2.6

← Problem

The x axis runs from A to B. The reactions R_A and R_B are unknown and there is only one equilibrium equation: one degree of indeterminacy. Portion AC is stretched by $N_1 = R_A$, and portion CB is compressed by $N_2 = -R_B$.

a. Equilibrium: $R_A + R_B = 70$ kN (both in the $-x$ direction). Compatibility: the distance between the walls does not change, $\delta_{AC} + \delta_{CB} = 0$; that is, the elongation of AC equals the shortening of CB. Force–deformation $\delta = N/k$, $k = AE/L$:

$$k_1 = \frac{(380.1)(200\times10^3)}{300} = 253\,420 \text{ N/mm},$$

$$k_2 = \frac{(1661.9)(70\times10^3)}{450} = 258\,520 \text{ N/mm}.$$

$$\frac{R_A}{k_1} = \frac{R_B}{k_2} \;\Rightarrow\; R_A = 70\,\frac{k_1}{k_1+k_2} = 34.652 \text{ kN},$$

$$R_B = 70 - 34.652 = 35.348 \text{ kN}.$$

To three significant figures, $R_A = 34.7$, $R_B = 35.3$ kN; both reactions are opposite to the applied force.

$$\sigma_{AC} = \frac{34.652\times10^3}{380.1} = 91.2 \text{ MPa},$$

$$\sigma_{CB} = -\frac{35.348\times10^3}{1661.9} = -21.3 \text{ MPa}.$$

b. C moves toward B by the elongation of AC:

$$u_C = \frac{34.652\times10^3}{253\,420} = 0.137 \text{ mm}.$$

c. The stresses are proportional to the force:

$$P \le 70\,\frac{120}{91.16} = 92.1 \text{ kN (steel)},$$

$$P \le 70\,\frac{80}{21.27} = 263 \text{ kN (aluminum)}.$$

The steel portion governs: $P_{\max} = 92.1$ kN. Because the stiffnesses of the two portions are nearly equal, the force is shared about equally, but the stress in the thin steel portion is more than four times as large.

ANSWER a. $R_A = 34.7$, $R_B = 35.3$ kN (both opposite to the force); $\sigma_{AC} = 91.2$, $\sigma_{CB} = -21.3$ MPa b. 0.137 mm, toward B c. 92.1 kN

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2.8 ← Problem

Compatibility: the free thermal elongation is completely canceled by the shortening caused by the force:

$$\alpha\,\Delta T\,L - \frac{PL}{AE} = 0 \;\Rightarrow\; \sigma = -\frac{P}{A} = -E\alpha\,\Delta T.$$

L cancels, and A does not appear in the relation. **E** is correct.

- (A) and (C): When the length doubles, the prevented elongation also doubles, but the strain (elongation/length) stays the same.
- (B) and (D): When the area doubles, the wall force also doubles ($P = E\alpha\,\Delta T\,A$); the stress is the same. The assumption in (D) that "the force does not change" is wrong.

ANSWER E

4 PURE BENDING

4.2 ← Problem

a. The moment at K is the load on one fork times its moment arm:

$$M = (8\times10^3)(600) = 4.8\times10^6\ \text{N}\cdot\text{mm},$$

$$S = \tfrac{1}{6}(120)(45)^2 = 40\,500\ \text{mm}^3,$$

$$\sigma_m = \frac{M}{S} = \frac{4.8\times10^6}{40\,500} = 118.5 \approx 119\ \text{MPa}.$$

The fork bends downward like a cantilever; the moment is negative (the top fiber is stretched). The top face is in tension at 119 MPa, and the bottom face is in compression of the same magnitude.

b. $S \propto h^2$; the thickness decreases to 40.5 mm:

$$\sigma_m' = 118.5\left(\frac{45}{40.5}\right)^2 = 146.3\ \text{MPa},$$

$$\frac{146.3}{118.5} = 1.235 \;\Rightarrow\; 23.5\%\ \text{increase}.$$

A small loss in thickness increases the stress through the square of the thickness; this is why worn forks are taken out of service.

c. The stress is proportional to the load:

$$W_{\max} = 16\,\frac{180}{146.3} = 19.7\ \text{kN}.$$

ANSWER a. 119 MPa; top face in tension
b. 23.5% c. 19.7 kN

4.13 ← Problem

y and z are axes of symmetry and therefore principal axes. The neutral axis is given by $\tan\phi = (I_z/I_y)\tan\theta$. For $h = 2b$, $I_z/I_y = (bh^3)/(hb^3) = (h/b)^2 = 4 > 1$; hence $\tan\phi > \tan\theta$, that is, $\phi > \theta$: the neutral axis is steeper than the moment vector and lies between the vector and the y axis. **C** is correct.

- (A): It is true only when $I_y = I_z$ (square, circle) or when $\theta = 0,\ 90°$.
- (B): Even in symmetric bending the neutral axis is parallel to the moment vector, not perpendicular.
- (D): It would be true if $I_z/I_y < 1$ (if the section lay on its side).
- (E): The neutral axis rotates with θ; it coincides with the z axis only for $\theta = 0$.

ANSWER C

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4.15

← Problem

Since the section is symmetric about the y axis, y and z are principal axes; $\sigma_x = -M_z y/I_z + M_y z/I_y$ is used. Let us take the sign of M_y as positive such that the right end $(+z)$ is in tension.

a. Centroid (from the bottom):

$$\bar{y} = \frac{1728(72) + 2560(152)}{4288} = 119.8 \text{ mm}.$$

The top fiber is 40.24 mm above, and the bottom end of the web is 119.8 mm below:

$$I_z = \tfrac{1}{12}(12)(144)^3 + 1728(47.76)^2$$
$$+ \tfrac{1}{12}(160)(16)^3 + 2560(32.24)^2,$$
$$I_z = 9.643 \times 10^6 \text{ mm}^4.$$

$$I_y = \tfrac{1}{12}(16)(160)^3 + \tfrac{1}{12}(144)(12)^3 = 5.482 \times 10^6 \text{ mm}^4.$$

Component stresses:

$$\frac{M_z(40.24)}{I_z} = 58.42, \quad \frac{M_y(80)}{I_y} = 20.43 \text{ MPa},$$

$$\frac{M_z(119.8)}{I_z} = 173.9, \quad \frac{M_y(6)}{I_y} = 1.532 \text{ MPa}.$$

Flange ends:

$$\sigma_{\text{left}} = -58.42 - 20.43 = -78.9 \text{ MPa},$$

$$\sigma_{\text{right}} = -58.42 + 20.43 = -38.0 \text{ MPa}.$$

At the bottom end of the web (right corner):

$$\sigma = 173.9 + 1.532 = 175 \text{ MPa (tension)}.$$

b. $\tan\theta = M_y/M_z = 0.1$, $\theta = 5.71°$:

$$\tan\phi = \frac{9.643}{5.482}(0.1) = 0.1759 \Rightarrow \phi = 9.98°.$$

c. Under the vertical moment alone, the flange ends are at −58.4 MPa:

$$\frac{78.85}{58.42} = 1.350 \Rightarrow 35.0\% \text{ increase}.$$

A horizontal moment of only 10% of the vertical moment increases the compression at the flange end by 35%: in horizontal bending the resistance of the section comes almost entirely from the flange ($I_y < I_z$), and the flange end is far from the z axis. The narrow bottom end of the web, on the other hand, is hardly affected by the horizontal moment.

ANSWER a. left end −78.9 MPa, right end −38.0 MPa, bottom end of web 175 MPa (tension) b. 9.98° c. 35.0%

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5 ANALYSIS AND DESIGN OF BEAMS FOR BENDING

5.6

← Problem

$dM/dx = V$: the slope of M is +6 at C, 0 at the middle, and −6 at D. M increases up to the middle and then decreases; it takes its maximum value at the middle. $M_D - M_C$ is the area of the V diagram: $\frac{1}{2}(6)(1.5) - \frac{1}{2}(6)(1.5) = 0$, so $M_C = M_D$. **D** is correct.

- (A) and (B): If V were constant, M would be linear, and if $V = 0$, M would be constant; here V is linear, so M is parabolic.
- (C): At the middle, dM/dx changes from positive to negative; this is a maximum, not a minimum.
- (E): $dV/dx = -w = -4$ kN/m, so $w = 4$ kN/m and the load acts *downward*.

ANSWER D

5.17

← Problem

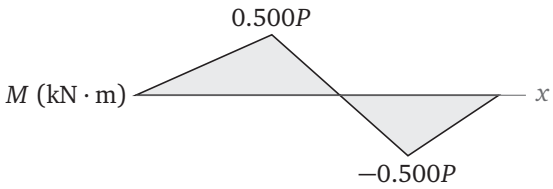
a. Reactions ($\sum M_A = 0$):

$$1.5B = 2P(0.75) + P(2) \Rightarrow B = 2.333P,$$

$$A = 3P - 2.333P = 0.6667P.$$

$$M_{\text{mid}} = 0.6667P(0.75) = +0.500P,$$

$$M_B = -P(0.5) = -0.500P \ \text{(kN·m)}.$$



b. Centroid (from the top):

$$\bar{y} = \frac{4200(15) + 3600(90)}{7800} = 49.62 \text{ mm},$$

the bottom fiber is $150 - 49.62 = 100.38$ mm away.

$$I = 315\,000 + 4200(34.62)^2$$
$$+ 4.320 \times 10^6 + 3600(40.38)^2,$$
$$I = 15.54 \times 10^6 \text{ mm}^4.$$

A positive moment (at midspan) puts the bottom in tension; a negative moment (at B) puts the top in

tension. Four limits:

$$M^{+}_{\text{bottom, tension}} = \frac{35(15.54 \times 10^6)}{100.38} = 5.418 \text{ kN} \cdot \text{m},$$

$$M^{+}_{\text{top, compression}} = \frac{105(15.54 \times 10^6)}{49.62} = 32.88 \text{ kN} \cdot \text{m},$$

$$M^{-}_{\text{top, tension}} = \frac{35(15.54 \times 10^6)}{49.62} = 10.96 \text{ kN} \cdot \text{m},$$

$$M^{-}_{\text{bottom, compression}} = \frac{105(15.54 \times 10^6)}{100.38} = 16.26 \text{ kN} \cdot \text{m}.$$

At both sections $|M| = 0.5P$; the smallest limit governs:

$$0.5P \le 5.418 \Rightarrow P_{\max} = 10.8 \text{ kN}.$$

Tension in the far bottom fiber at the midspan section governs.

c. When the section is inverted, the far fiber moves to the top. This time the negative moment at B puts the far top fiber in tension: the limit is again $35I/100.38 = 5.418$ kN · m and $P_{\max} = 10.8$ kN. Since the positive and negative moments are equal in magnitude, flipping the section gives no benefit; a cast-iron T section is used efficiently only under a moment of a single sign, with the flange on the tension side.

ANSWER a. $M_{\text{mid}} = 0.500P$, $M_B = -0.500P$ b. $P_{\max} = 10.8$ kN; bottom fiber (tension) at the midspan section c. no, again 10.8 kN

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- 6 chapters and 106 problems, all with step-by-step solutions.
- A reminder box at the start of each chapter, summarizing the relations the problems rely on.
- Basic, intermediate and advanced problems, each with a dotted area for your own work.
- One-click links from each problem to its solution and back.

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