MECHANICS OF MATERIALS 2

PROBLEM BOOK

SAMPLE PAGES

7 chapters · 112 problems · step-by-step solutions



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How to use this book

- Each chapter opens with a reminder box summarizing the definitions and formulas the problems rely on.
- Difficulty: ★☆☆ basic ★★☆ intermediate ★★★ advanced. Each chapter has a few multiple-choice concept questions; each chapter ends with exam-style mixed problems.
- The dotted area below each problem is for your own work. Solve first; if stuck, go back to the reminder box and the lecture notes, and look at the solution last.
- All solutions are at the end of the book. The Solution → link next to a problem takes you to its solution, and ← Problem takes you back. Short answers are in the **ANSWER** box.
- **Units and signs:** SI units throughout; stresses in MPa, moduli of elasticity in GPa; $g = 9.81$ m/s^2. Normal stress and strain are positive in tension. A bending moment is positive when it compresses the top fiber of the beam. In the figures, forces and reactions are red.
- **Shared chapters:** Chapters 5–6 (analysis and design of beams, shearing stresses in beams) are the same as in *Mechanics of Materials 1*. They are included here for schools whose second course begins with beams; for others they serve as review.
- **Midterm and final:** Mechanics of Materials II is usually taught in the spring of the second year. The midterm usually covers stress transformations, Mohr's circle and combined loadings (Chapters 7–8); the final covers beam deflection, columns and energy methods (Chapters 9–11). Choose the chapters to study according to your own course syllabus.

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11 Energy Methods ..... 14 problems 80

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CHAPTER 7

TRANSFORMATIONS OF STRESS AND STRAIN

transformation	$\sigma_{x'} = \sigma_{\text{ave}} + d\cos 2\theta + \tau_{xy}\sin 2\theta$	Tresca	$\max\lvert\sigma_i - \sigma_j\rvert \le \sigma_Y$
	$\tau_{x'y'} = -d\sin 2\theta + \tau_{xy}\cos 2\theta$	von Mises	$\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 \le \sigma_Y^2$
abbreviations	$\sigma_{\text{ave}} = \dfrac{\sigma_x + \sigma_y}{2},\ d = \dfrac{\sigma_x - \sigma_y}{2}$	cylindrical vessel	$\sigma_1 = \dfrac{pr}{t},\ \sigma_2 = \dfrac{pr}{2t}$
principal stress	$\sigma_{1,2} = \sigma_{\text{ave}} \pm R,\ R = \sqrt{d^2 + \tau_{xy}^2}$	spherical vessel	$\sigma_1 = \sigma_2 = \dfrac{pr}{2t}$
principal direction	$\tan 2\theta_p = \dfrac{\tau_{xy}}{d}$	strain	$\sigma \to \varepsilon,\ \tau_{xy} \to \gamma_{xy}/2$
maximum shear	$\tau_{\max} = R,\ \theta_s = \theta_p \pm 45°$	45° rosette	$\gamma_{xy} = 2\varepsilon_b - \varepsilon_a - \varepsilon_c$

Normal stress is positive in tension. The shearing stress τ_{xy} is positive when it acts in the $+y$ direction on the $+x$ face of the element. The rotation angle θ is positive when measured counterclockwise from the x axis. **Mohr's circle** convention: the horizontal axis is σ (positive to the right), the vertical axis is τ, and τ is plotted **positive downward**; the x face is the point $X(\sigma_x, \tau_{xy})$ and the y face is the point $Y(\sigma_y, -\tau_{xy})$. With this convention, a rotation of 2θ on the circle has the same sense as a rotation of θ on the element. On the strain circle the horizontal axis is ε and the vertical axis is $\gamma/2$ (positive downward). In plane stress $\sigma_z = 0$ is taken; in the Tresca criterion σ_i and σ_j are any two of the three principal stresses (including σ_z). Unless otherwise stated, $E = 200$ GPa and $\nu = 0.30$ are used for steel; in thin-walled vessels r is the inner radius.

Plane-stress transformation and principal stresses

7.1 ★☆☆ Solution →

At a point, $\sigma_x = 70$ MPa and $\sigma_y = -30$ MPa. On a rotated element at the same point, the normal stress on the x' face is measured as $\sigma_{x'} = 50$ MPa. What is the normal stress $\sigma_{y'}$ on the y' face of this element?

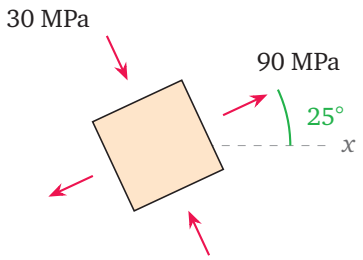
A) 10 MPa **B)** −10 MPa **C)** −30 MPa **D)** 20 MPa **E)** θ is required

Mohr's circle for plane stress

7.6

★★☆ Solution →

At a point on the elbow of a pressurized pipe the principal stresses are $\sigma_1 = 90$ MPa and $\sigma_2 = -30$ MPa. The direction of σ_1 makes an angle of 25° counterclockwise with the x axis.



a. Using Mohr's circle, find σ_x, σ_y and τ_{xy} with respect to the x-y axes.

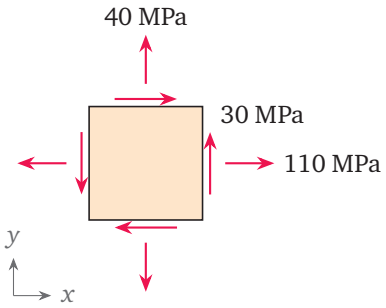
b. Explain how you read the sign of τ_{xy} from the circle.

Three-dimensional analysis and yield criteria

7.9

★★☆ Solution →

At a point on the surface of an aircraft wing skin there is a state of plane stress: $\sigma_x = 110$ MPa, $\sigma_y = 40$ MPa, $\tau_{xy} = 30$ MPa ($\sigma_z = 0$).



a. Find the principal stresses, the maximum in-plane shearing stress, and the absolute maximum shearing stress.

b. What would the absolute maximum shearing stress be if $\sigma_y = -40$ MPa? Compare the two cases.

CHAPTER 9
DEFLECTION OF BEAMS

elastic curve	$EI\dfrac{d^2y}{dx^2}=M(x),\ \ \theta=\dfrac{dy}{dx}$	cantilever, end P	$\lvert y\rvert_{\max}=\dfrac{PL^3}{3EI},\ \lvert\theta\rvert_{\max}=\dfrac{PL^2}{2EI}$
distributed load	$EI\dfrac{d^4y}{dx^4}=-w(x)$	cantilever, uniform w	$\lvert y\rvert_{\max}=\dfrac{wL^4}{8EI},\ \lvert\theta\rvert_{\max}=\dfrac{wL^3}{6EI}$
singularity function	$\langle x-a\rangle^n=(x-a)^n\ \ (x\geq a)$	simple, midspan P	$\lvert y\rvert_{\max}=\dfrac{PL^3}{48EI},\ \lvert\theta\rvert_{\max}=\dfrac{PL^2}{16EI}$
integral	$\displaystyle\int\langle x-a\rangle^n dx=\frac{\langle x-a\rangle^{n+1}}{n+1}$	simple, uniform w	$\lvert y\rvert_{\max}=\dfrac{5wL^4}{384EI},\ \lvert\theta\rvert_{\max}=\dfrac{wL^3}{24EI}$
moment-area I	$\theta_D-\theta_C=\displaystyle\int_C^D\frac{M}{EI}\,dx$	simple, end M_0	$\lvert\theta\rvert_{\text{near}}=\dfrac{M_0L}{3EI},\ \lvert\theta\rvert_{\text{far}}=\dfrac{M_0L}{6EI}$
moment-area II	$t_{D/C}=\displaystyle\int_C^D\frac{M}{EI}(x_D-x)\,dx$		

x is measured along the beam axis, and y is positive upward (a downward deflection is negative). The slope $\theta=dy/dx$ is positive when counterclockwise. The bending moment is positive when it compresses the top fiber. A singularity function is zero for $x<a$. The tabulated values are magnitudes. $t_{D/C}$ is the vertical distance of point D from the tangent at C. Unless stated otherwise, the modulus of elasticity is taken as 200 for steel, 70 for aluminum and 12 GPa for wood; the self-weight of the beams is neglected. Deflections are drawn magnified in the figures.

Elastic Curve by Integration

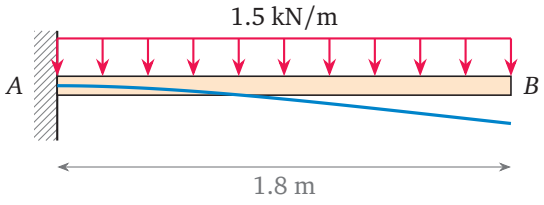
9.4 ★☆☆ Solution →

The aluminum support arm of a solar-panel array is a cantilever 1.8 m long and carries a uniformly distributed load of 1.5 kN/m from the panels and snow. To keep the panels from cracking, the tip deflection must not exceed $L/180$.

a. Using x measured from the fixed end, find the equation of the elastic curve by integration.

b. Find the minimum required moment of inertia.

c. What is the deflection of the midpoint of the arm for this section?



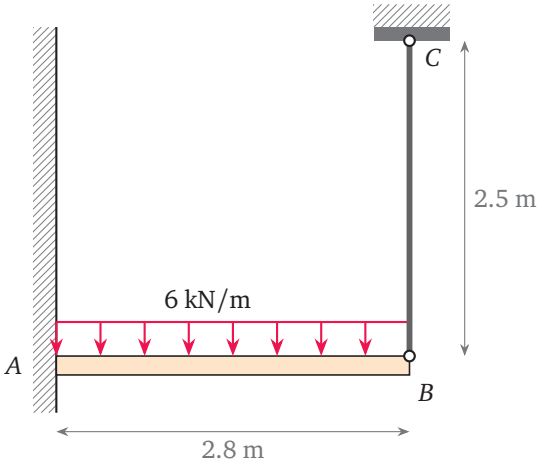
Method of Superposition

9.13 ★★★ Solution →

The steel beam AB of a canopy at a building entrance is fixed to the wall at A, is 2.8 m long and carries a distributed load of 6 kN/m; $I = 20 \times 10^6$ mm^4. End B of the beam is suspended from a bracket on the upper floor by a vertical steel hanger rod (BC), 2.5 m long and 16 mm in diameter. Before the load is applied, the rod is unstressed but has no slack.

a. Find the force and stress in the rod.

b. Find the deflection of B and the moment at A; what would M_A be if the rod were rigid, and if there were no rod at all?



9.14 ★☆☆ Solution →

A simply supported beam of rectangular cross section carries a uniformly distributed load w. The span and the section depth are doubled; the section width, the material and the load w per unit length stay the same. By what factor does the midpoint deflection change from its initial value?

A) $\frac{1}{2}$ **B)** 1 **C)** 16 **D)** 2 **E)** 4

CHAPTER 10
COLUMNS

Euler critical load	$P_{cr} = \dfrac{\pi^2 EI}{L_e^2}$	fixed–pinned	$K = 0.7$
critical stress	$\sigma_{cr} = \dfrac{\pi^2 E}{(L_e/r)^2},\ r = \sqrt{\frac{I}{A}}$	fixed–fixed	$K = 0.5$
effective length	$L_e = KL$	allowable load	$P_{all} = \dfrac{P_{cr}}{\text{F.S.}}$
pinned–pinned	$K = 1$	secant formula	$\sigma_{max} = \dfrac{P}{A}\left[1 + \dfrac{ec}{r^2}\sec\theta\right]$
fixed–free	$K = 2$	maximum deflection	$y_{max} = e(\sec\theta - 1)$
Euler validity	$\sigma_{cr} \le \sigma_Y$		$\theta = \dfrac{\pi}{2}\sqrt{\dfrac{P}{P_{cr}}}$

Columns are initially perfectly straight, the load acts (unless stated otherwise) at the centroid of the cross section, and the material is linearly elastic. Column loads are given as compressive forces by magnitude. A column buckles in the plane in which the slenderness ratio (L_e/r) is largest; if the end conditions differ in the two planes, both are checked. In the figures a fixed lower end is shown by the hatched ground and a pinned lower end by the triangular support. A pinned upper end is restrained horizontally by a roller support bearing against the adjacent wall. A fixed upper end is a rigid cap guided on both sides. A free upper end has no support. Unless stated otherwise, take $E = 200$ GPa and $\sigma_Y = 250$ MPa for steel, and $E = 70$ GPa and $\sigma_Y = 240$ MPa for aluminum.

Stability and Euler's Formula

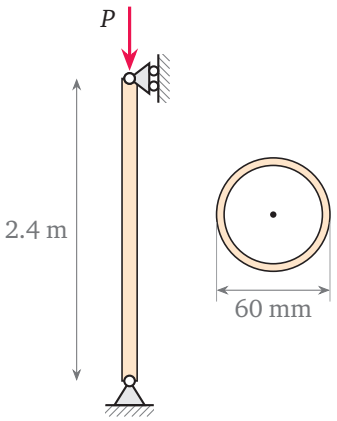
10.3 ★☆☆ Solution →

The compression member of a space-truss system is a steel pipe 2.4 m long with an outer diameter of 60 mm and an inner diameter of 52 mm; both ends are pinned.

a. Find the critical load and the critical stress. Is Euler's formula valid here?

b. If the factor of safety is 2.5, what is the largest load that can be applied to the member?

c. What is the shortest length of this same pipe for which Euler's formula is valid?



End Conditions and Effective Length

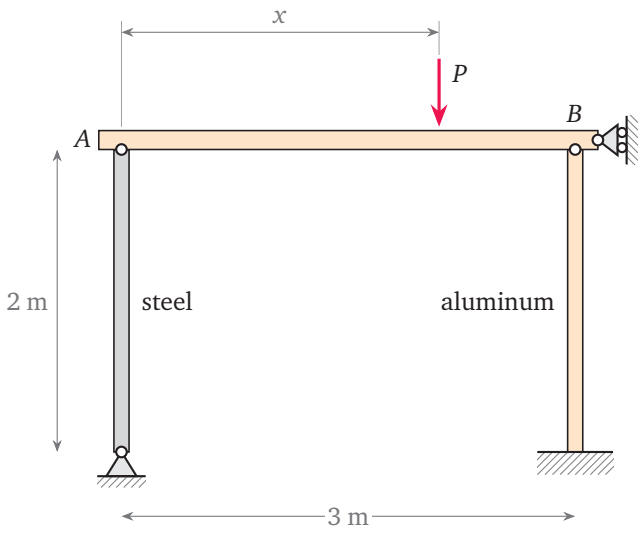
10.8 ★★★ Solution →

In the maintenance walkway of a storage tank, a rigid platform AB, 3 m long, rests on two posts, to which it is pinned; end B is held laterally by a roller bearing against the adjacent wall. The post at A is a solid steel bar 30 mm in diameter, pinned at both ends. The post at B is an aluminum tube with an outer diameter of 56 mm and an inner diameter of 46 mm; it is fixed at its base and pinned at its top end. Both posts are 2 m long. A vertical load P is placed on the platform at a distance x from A.

a. Find the critical load of each post.

b. Find the largest load P the platform can carry with respect to buckling, and the position x at which this load must be placed.

c. What is the largest P if the load is placed at the middle of the platform?



SOLUTIONS

7 TRANSFORMATIONS OF STRESS AND STRAIN

7.1

← Problem

When the transformation equations are added, the $\cos 2\theta$ and $\sin 2\theta$ terms cancel: the sum of the normal stresses on two perpendicular faces does not change with rotation.

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y = 70 - 30 = 40 \text{ MPa},$$

$$\sigma_{y'} = 40 - 50 = -10 \text{ MPa}.$$

B is correct. (A) is a sign error; (C) results if σ_y is assumed not to change with rotation; (D) is the difference $\sigma_x - \sigma_{x'}$. (E): Because the sum is invariant, the result can be found without knowing θ.

ANSWER B

7.6

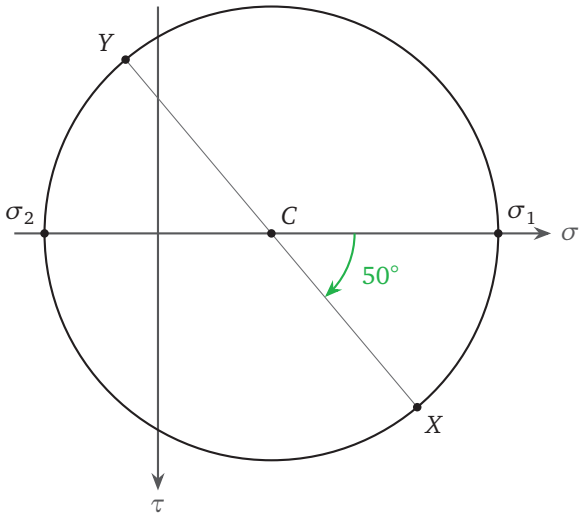
← Problem

Center and radius:

$$C = \frac{90 + (-30)}{2} = 30 \text{ MPa},$$

$$R = \frac{90 - (-30)}{2} = 60 \text{ MPa}.$$

The x axis is found by rotating 25° clockwise from the direction of σ_1. On the circle, too, one turns $2(25°) = 50°$ clockwise from the point σ_1; since τ is positive downward, X falls below the axis.



a. Coordinates of point X ($\cos 50° = 0.6428$, $\sin 50° = 0.7660$):

$$\sigma_x = 30 + 60(0.6428) = 68.6 \text{ MPa},$$

$$\sigma_y = 30 - 60(0.6428) = -8.57 \text{ MPa},$$

$$\tau_{xy} = 60(0.7660) = 46.0 \text{ MPa}.$$

b. Since point X is below the axis, τ_{xy} is positive. Check: $\tan 2\theta_p = 2(45.96)/(68.57 + 8.57) = 1.192$, $2\theta_p = 50°$.

ANSWER a. $\sigma_x = 68.6$ MPa, $\sigma_y = -8.57$ MPa, $\tau_{xy} = 46.0$ MPa b. X below the axis: $\tau_{xy} > 0$

7.9

← Problem

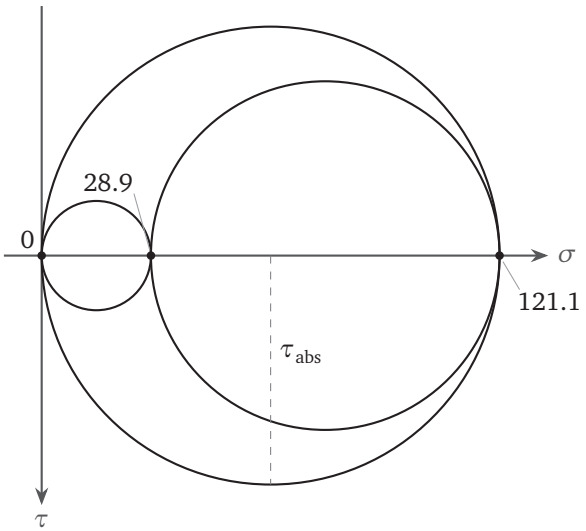
a. In-plane principal stresses:

$$\sigma_{\text{ave}} = 75, \quad R = \sqrt{35^2 + 30^2} = 46.10 \text{ MPa},$$

$$\sigma_a = 121.1 \approx 121 \text{ MPa}, \quad \sigma_b = 28.9 \text{ MPa},$$

$$\sigma_c = \sigma_z = 0.$$

The maximum in-plane shear is $R = 46.1$ MPa. Since the three principal stresses are 121.1, 28.9 and 0, three Mohr circles are drawn; the largest is the one between 0 and 121.1:



$$\tau_{\text{abs}} = \frac{121.1 - 0}{2} = 60.5 \text{ MPa}.$$

When both in-plane principal stresses are tensile, the maximum shear occurs out of plane (on planes that

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contain the σ_b direction and are inclined at 45° to the a and z axes) and is larger than the in-plane value.

b. For $\sigma_y = -40$:

$$\sigma_{\text{ave}} = 35, \quad R = \sqrt{75^2 + 30^2} = 80.78 \text{ MPa},$$

$$\sigma_a = 115.8 \text{ MPa}, \quad \sigma_b = -45.8 \text{ MPa}.$$

$\sigma_z = 0$ lies between these two; the absolute maximum shear equals the in-plane value:

$$\tau_{\text{abs}} = R = 80.8 \text{ MPa}.$$

If the principal stresses have opposite signs, the in-plane circle is the largest circle; if they have the same sign, the point $\sigma_z = 0$ must be taken into account.

ANSWER a. 121 and 28.9 MPa; in-plane 46.1, absolute 60.5 MPa b. 80.8 MPa (equal to the in-plane value)

9 DEFLECTION OF BEAMS

9.4

← Problem

a. From the load to the right of the section: $M = -w(L-x)^2/2$ (top fiber in tension).

$$EI\,y'' = -\frac{w(L-x)^2}{2},$$

$$EI\,y' = \frac{w(L-x)^3}{6} + C_1,$$

$$EI\,y = -\frac{w(L-x)^4}{24} + C_1 x + C_2.$$

$y'(0) = 0 \Rightarrow C_1 = -wL^3/6$; $y(0) = 0 \Rightarrow C_2 = wL^4/24$:

$$y = -\frac{w}{24EI}\left[(L-x)^4 + 4L^3 x - L^4\right].$$

b. At the tip ($x = L$):

$$y_B = -\frac{w}{24EI}(3L^4) = -\frac{wL^4}{8EI}.$$

Limit $1800/180 = 10$ mm:

$$I \geq \frac{wL^4}{8E(0.010)} = \frac{(1.5)(1.8)^4}{8(70\times10^6)(0.010)},$$

$$I_{\min} = 2.81\times10^{-6} \text{ m}^4 = 2.81\times10^6 \text{ mm}^4.$$

c. $x = L/2$:

$$y = -\frac{w}{24EI}\left(\frac{L^4}{16} + 2L^4 - L^4\right) = -\frac{17}{384}\frac{wL^4}{EI}.$$

This is $\frac{17/384}{1/8} = \frac{17}{48}$ times the tip deflection: $10(17/48) = 3.54$ mm. The midpoint deflects slightly more than one third of the tip deflection; most of the curve develops in the region near the tip.

ANSWER
a. $y = -w\left[(L-x)^4 + 4L^3x - L^4\right]/(24EI)$
b. 2.81×10^6 mm^4 c. 3.54 mm (downward)

9.13

← Problem

The unknown rod force is F (tension); the beam is statically indeterminate to the first degree. Units are kN, m:

$$EI = (200\times10^6)(20\times10^{-6}) = 4000 \text{ kN}\cdot\text{m}^2.$$

The cross-sectional area of the rod is $A = \pi(8)^2 = 201.1$ mm^2.

Force–deformation. By superposition, the downward deflection of B:

$$\delta_B = \frac{wL^4}{8EI} - \frac{FL^3}{3EI} = 11.52 - 1.829F \text{ (mm)},$$

($wL^4/8EI = 6(2.8)^4/32\,000 = 11.52$ mm, $L^3/3EI = 1.829$ mm/kN). Elongation of the rod:

$$\delta_r = \frac{FL_r}{EA} = \frac{F(2500)}{(200)(201.1)} = 0.06216F \text{ (mm)}.$$

Compatibility. The rod is attached to B; it elongates as much as B descends:

$$11.52 - 1.829F = 0.06216F,$$

a. Rod force and stress:

$$F = \frac{11.52}{1.829 + 0.06216} = 6.092 \approx 6.09 \text{ kN},$$

$$\sigma = \frac{6092}{201.1} = 30.3 \text{ MPa}.$$

b. $\delta_B = 0.06216(6.092) = 0.379$ mm (downward). Equilibrium:

$$M_A = \frac{wL^2}{2} - FL = 23.52 - 6.092(2.8) = 6.46 \text{ kN}\cdot\text{m}.$$

For a rigid rod, $F = 3wL/8 = 6.30$ kN and $M_A = 23.52 - 17.64 = 5.88$ kN·m; without the rod, $M_A = 23.5$ kN·m. Even a slender hanger rod reduces the fixed-end moment to less than one third; the

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flexibility of the rod makes the result differ from the rigid-support case by only 10%.

ANSWER a. $F = 6.09$ kN, $\sigma = 30.3$ MPa
b. $\delta_B = 0.379$ mm, $M_A = 6.46$ kN · m; rigid: 5.88, no rod: 23.5 kN · m

9.14

← Problem

$|y_C| = 5wL^4/(384EI)$ and $I = bh^3/12$; therefore $|y_C| \propto L^4/h^3$:

$$\frac{y_2}{y_1} = \frac{2^4}{2^3} = 2.$$

D is correct. (C) is the effect of the span alone; (B) would result if the total load wL were kept constant (L^3/h^3); (E) results if $I \propto h^2$ is assumed, and (A) if the deflection is assumed proportional to L^3 and I to h^4.

ANSWER D

10 COLUMNS

10.3

← Problem

Section properties:

$$I = \tfrac{\pi}{64}\left(60^4 - 52^4\right) = 277\,264 \text{ mm}^4,$$

$$A = \tfrac{\pi}{4}\left(60^2 - 52^2\right) = 703.7 \text{ mm}^2,$$

$$r = \sqrt{I/A} = 19.85 \text{ mm}.$$

a. Both ends pinned: $L_e = L = 2400$ mm.

$$P_{\text{cr}} = \frac{\pi^2(200\times10^3)(277\,264)}{(2400)^2} = 95.02\times10^3 \text{ N},$$

$$P_{\text{cr}} = 95.0 \text{ kN},$$

$$\sigma_{\text{cr}} = \frac{95\,020}{703.7} = 135 \text{ MPa}.$$

135 MPa $< \sigma_Y = 250$ MPa: the column buckles before yielding, so Euler's formula is valid.

b. Allowable load:

$$P_{\text{all}} = \frac{95.02}{2.5} = 38.0 \text{ kN}.$$

c. The limit is the slenderness at which $\sigma_{\text{cr}} = \sigma_Y$:

$$\frac{L}{r} = \pi\sqrt{\frac{E}{\sigma_Y}} = \pi\sqrt{\frac{200\,000}{250}} = 88.86,$$

$$L = (88.86)(19.85) = 1764 \text{ mm} = 1.76 \text{ m}.$$

The slenderness ratio of the pipe is $2400/19.85 = 120.9$; it is above the limit. In a pipe shorter than 1.76 m the Euler load exceeds the yield load; such a column yields before it buckles.

ANSWER a. $P_{\text{cr}} = 95.0$ kN, $\sigma_{\text{cr}} = 135$ MPa $< \sigma_Y$, valid b. 38.0 kN c. 1.76 m

10.8

← Problem

a. Post A: pinned–pinned, $L_e = 2000$ mm.

$$I_A = \tfrac{\pi}{64}(30)^4 = 39\,761 \text{ mm}^4,$$

$$P_{\text{cr},A} = \frac{\pi^2(200\times10^3)(39\,761)}{(2000)^2} = 19.62 \approx 19.6 \text{ kN}.$$

Post B: fixed–pinned, $L_e = 0.7(2000) = 1400$ mm.

$$I_B = \tfrac{\pi}{64}\left(56^4 - 46^4\right) = 262\,963 \text{ mm}^4,$$

$$P_{\text{cr},B} = \frac{\pi^2(70\times10^3)(262\,963)}{(1400)^2} = 92.69 \approx 92.7 \text{ kN}.$$

Check: $\sigma_A = 19\,620/706.9 = 27.8$ MPa, $\sigma_B = 92\,690/801.1 = 115.7$ MPa; both are below their own yield strengths (steel 250, aluminum 240 MPa), so Euler is valid.

b. The platform is rigid and statically determinate: $R_A = P(3-x)/3$, $R_B = Px/3$. The load is largest when the two posts reach their critical loads *at the same time*:

$$P_{\max} = P_{\text{cr},A} + P_{\text{cr},B} = 19.62 + 92.69 = 112.3 \approx 112 \text{ kN}.$$

Moment about A: $R_B = Px/3 = P_{\text{cr},B}$

$$x = 3\,\frac{P_{\text{cr},B}}{P_{\max}} = 3\,\frac{92.69}{112.3} = 2.48 \text{ m}.$$

If the load is placed anywhere other than x, one of the posts buckles first and P remains smaller.

c. At the middle $R_A = R_B = P/2$; the weaker steel post buckles first:

$$P = 2P_{\text{cr},A} = 2(19.62) = 39.2 \text{ kN}.$$

If the position of the load is chosen correctly, the capacity increases about threefold.

ANSWER a. $P_{\text{cr},A} = 19.6$ kN, $P_{\text{cr},B} = 92.7$ kN
b. 112 kN, $x = 2.48$ m c. 39.2 kN

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IN THE FULL BOOK

- 7 chapters and 112 problems, all with step-by-step solutions.
- A reminder box at the start of each chapter, summarizing the relations the problems rely on.
- Basic, intermediate and advanced problems, each with a dotted area for your own work.
- One-click links from each problem to its solution and back.

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