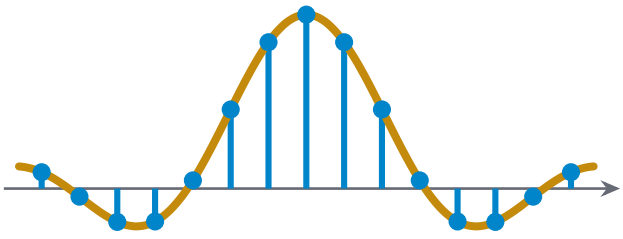
SIGNALS AND SYSTEMS

PROBLEM BOOK

SAMPLE PAGES

9 chapters · 162 problems · step-by-step solutions



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How to use this book

- Each chapter opens with a reminder box summarizing the definitions and formulas the problems rely on.
- Difficulty: ★☆☆ basic ★★☆ intermediate ★★★ advanced. Each chapter has a few multiple-choice concept questions; its last problem is an exam-style mixed problem combining several topics.
- The dotted area below each problem is for your own work. Solve first; if stuck, go back to the reminder box and the lecture notes, and look at the solution last.
- All solutions are at the end of the book. The Solution → link next to a problem takes you to its solution, and ← Problem takes you back. Short answers are in the **ANSWER** box.
- **Shared chapters:** Chapters 7–9 (discrete-time Fourier transform, sampling, z-transform) also appear in *Digital Signal Processing*, for schools whose second course begins with these topics.
- **Midterm and final:** The midterm usually covers signals, system properties, convolution, and Fourier series (Chapters 1–4); the final adds the continuous-time Fourier and Laplace transforms, the discrete-time Fourier transform, sampling, and the z-transform (Chapters 5–9). The order and scope of topics vary from school to school; choose the chapters to study according to your own course syllabus.

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Solutions 93

CHAPTER 3

CONVOLUTION

convolution sum	$y[n]=\sum_{k=-\infty}^{\infty} x[k]h[n-k]$	causality	$h[n]=0\ (n<0),\quad h(t)=0\ (t<0)$
convolution integral	$y(t)=\int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$	stability (BIBO)	$\sum_n \lvert h[n]\rvert<\infty,\quad \int \lvert h(t)\rvert\,dt<\infty$
convolution with δ	$x(t)*\delta(t-t_0)=x(t-t_0)$	discrete step response	$s[n]=\sum_{k=-\infty}^{n} h[k]$
series and parallel	$h_1*h_2,\qquad h_1+h_2$	continuous step response	$s(t)=\int_{-\infty}^{t} h(\tau)\,d\tau$
geometric sum	$\sum_{k=0}^{N-1} a^k=\frac{1-a^N}{1-a}$	inversion	$h[n]=s[n]-s[n-1],\quad h(t)=\frac{ds(t)}{dt}$
area and sum	$\sum_n y[n]=\sum_n x[n]\sum_n h[n]$		
doublet $\delta'(t)$	$\frac{du(t)}{dt}=\delta(t),\quad x(t)*\delta'(t)=\frac{dx(t)}{dt}$		

In this chapter, unless stated otherwise, systems are linear time-invariant (LTI) and are completely determined by their impulse response ($h[n]$ or $h(t)$); the output is the convolution of the input with the impulse response, $y=x*h$. Finite sequences are written inside $\{\dots\}$, and the index of the first element is stated separately; samples not shown in stem plots are zero. In the figures the input is drawn in blue and the impulse response in red. Systems given by difference and differential equations are initially at rest: the output is zero before the input is applied. $s[n]$ and $s(t)$ denote the step response. Stability is meant in the bounded-input bounded-output (BIBO) sense. When no unit is given, t is in seconds and n is the sample index.

LTI systems and the convolution sum

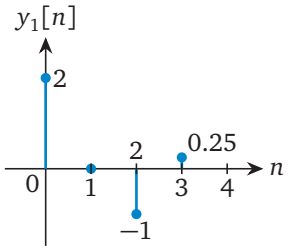
3.1 ★☆☆ Solution →

$x[n]$ is nonzero only for $2\le n\le 9$, and the impulse response $h[n]$ of an LTI system is nonzero only for $-2\le n\le 4$. What is the largest interval over which the output $y[n]=x[n]*h[n]$ can be nonzero?

A) $0\le n\le 13$ **B)** $0\le n\le 14$ **C)** $-2\le n\le 9$ **D)** $2\le n\le 4$ **E)** $4\le n\le 13$

3.3 ★★☆ Solution →

A wired communication channel is modeled as a causal LTI system with a finite impulse response. To identify the channel, the known training sequence $x_1[n] = 2\delta[n] - \delta[n-1]$ is applied to the input, and the output $y_1[n]$ shown in the figure is measured at the receiver ($y_1[n] = 0$ for $n \geq 4$; no noise).



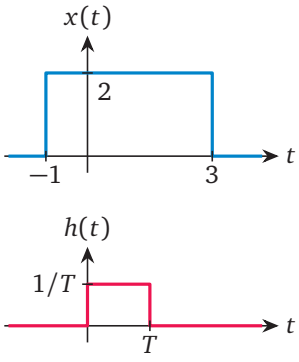
a. How many samples long is $h[n]$? Find $h[n]$.

b. The symbol sequence $x_2[n] = \delta[n] + \delta[n-1] - \delta[n-2]$ is sent through the same channel. What is measured at the receiver?

The convolution integral

3.7 ★★☆ Solution →

The pulse envelope $x(t)$ detected in a radio receiver (figure) is passed through a moving-average filter of T seconds: $h(t) = \frac{1}{T}[u(t) - u(t-T)]$.



a. For $T = 1.5$ s, find $y(t)$ for all t and sketch it.

b. The time during which the output stays equal to 2 (flat top) must be at least 3.2 s. What is the largest allowable T, in seconds?

c. If $T = 5$ s is chosen, what is the peak value of the output, and over what interval does it last?

CHAPTER 5

CONTINUOUS-TIME FOURIER TRANSFORM

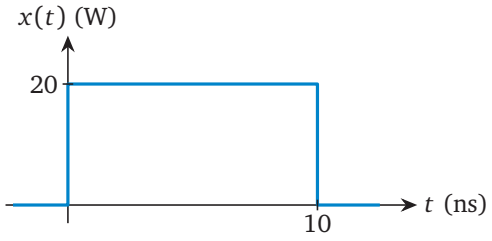
analysis	$X(j\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$	time shift	$x(t-t_0) \overset{\mathcal{F}}{\longleftrightarrow} e^{-j\omega t_0}X(j\omega)$
synthesis	$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$	time scaling	$x(at) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
exponential ($a>0$)	$e^{-at}u(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{a+j\omega}$	differentiation	$\frac{dx}{dt} \overset{\mathcal{F}}{\longleftrightarrow} j\omega X(j\omega)$
rectangular	$1\ (\lvert t\rvert<T_1) \overset{\mathcal{F}}{\longleftrightarrow} \frac{2\sin\omega T_1}{\omega}$	integration	$\int_{-\infty}^{t} x\,d\tau \overset{\mathcal{F}}{\longleftrightarrow} \frac{X(j\omega)}{j\omega}+\pi X(0)\delta(\omega)$
sinc	$\frac{\sin Wt}{\pi t} \overset{\mathcal{F}}{\longleftrightarrow} 1\ (\lvert\omega\rvert<W)$	convolution	$x*h \overset{\mathcal{F}}{\longleftrightarrow} X(j\omega)H(j\omega)$
impulse, const.	$\delta(t) \overset{\mathcal{F}}{\longleftrightarrow} 1,\quad 1 \overset{\mathcal{F}}{\longleftrightarrow} 2\pi\delta(\omega)$	modulation	$x\cos\omega_c t \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2}\sum_{\pm}X(j(\omega\mp\omega_c))$
cosine	$\cos\omega_0 t \overset{\mathcal{F}}{\longleftrightarrow} \pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	Parseval	$\int \lvert x(t)\rvert^2 dt=\frac{1}{2\pi}\int \lvert X(j\omega)\rvert^2 d\omega$
sine	$\sin\omega_0 t \overset{\mathcal{F}}{\longleftrightarrow} \frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$		

In this chapter ω is angular frequency (rad/s) and $f=\omega/2\pi$ (Hz). The output is $Y(j\omega)=X(j\omega)H(j\omega)$; an input $\cos(\omega_0 t+\phi)$ emerges with its amplitude multiplied by $|H(j\omega_0)|$ and its phase shifted by $\angle H(j\omega_0)$. Systems described by differential equations are initially at rest. The cutoff frequency is the frequency at which $|H|$ falls to $1/\sqrt{2}$ times (-3 dB) its passband value; for an ideal filter $|H|$ is 1 in the passband and 0 in the stopband. In the figures, signals are blue and systems red.

The Fourier transform and basic pairs

5.2 ★☆☆ Solution →

The laser diode of a lidar emits a rectangular light pulse $x(t)$ that starts at $t=0$, lasts 10 ns and has a peak power of 20 W (see the figure). Take time in ns and ω in rad/ns (1 rad/ns $=10^9$ rad/s).



a. Find $X(j\omega)$; write $|X(j\omega)|$ and $\angle X(j\omega)$.

b. What is $X(0)$? What is its physical meaning?

c. At which ω is the first zero of $|X(j\omega)|$? How many MHz does this correspond to?

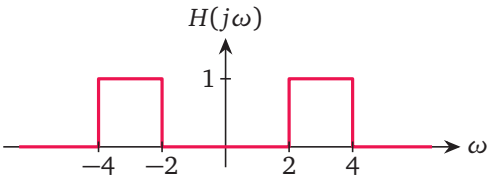
d. At 50 MHz, what is the ratio of $|X|$ to $X(0)$ (in dB)?

Filters, linear phase and delay

5.13

★★☆ Solution →

The signal $x(t) = \sin(6t)/(\pi t)$ is applied to an ideal bandpass filter (zero phase, see the figure) with passband $2 < |\omega| < 4$.



a. Sketch $X(j\omega)$ and $Y(j\omega)$.

b. Find $y(t)$; write it as the product of a cosine and a sine.

c. Find the energies of the input and the output. How much of the energy passes?

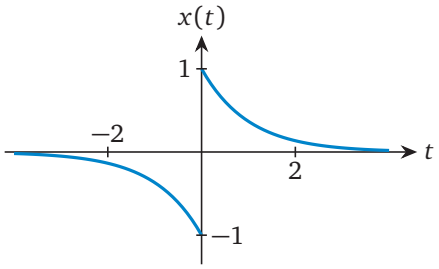
d. What is $y(0)$?

Mixed problems

5.18

★★★ Solution →

Consider the signal $x(t) = e^{-t}u(t) - e^{t}u(-t)$ shown in the figure.



a. Find $X(j\omega)$. Interpret the result using the conjugate symmetry property ($X(-j\omega) = X^*(j\omega)$ if x is real).

b. Find the transform $Y(j\omega)$ of $y(t) = x(t) * x(t)$ and write it in the form $A/(1+\omega^2) + B/(1+\omega^2)^2$.

c. Given that $e^{-|t|} \overset{\mathscr{F}}{\longleftrightarrow} 2/(1+\omega^2)$ and $(1+|t|)e^{-|t|} \overset{\mathscr{F}}{\longleftrightarrow} 4/(1+\omega^2)^2$, find $y(t)$.

d. Find $y(0)$ both from $y(t)$ and from $\frac{1}{2\pi}\int Y\,d\omega$.

CHAPTER 8

SAMPLING

impulse train

$$p(t)=\sum_n \delta(t-nT) \overset{\mathscr{F}}{\longleftrightarrow} \frac{2\pi}{T}\sum_k \delta(\Omega-k\Omega_s)$$

multiplication $\quad x(t)\,p(t) \overset{\mathscr{F}}{\longleftrightarrow} \frac{1}{2\pi}X(j\Omega)*P(j\Omega)$

sampled spectrum $\quad X_p(j\Omega)=\frac{1}{T}\sum_k X\big(j(\Omega-k\Omega_s)\big)$

discrete-time spectrum

$$X(e^{j\omega})=X_p(j\omega/T), \quad x[n]=x(nT)$$

sampling theorem $\quad \Omega_s > 2\Omega_M \;\; (f_s > 2f_M)$

ideal reconstruction

$$x_r(t)=\sum_n x[n]\,\frac{\sin\big(\pi(t-nT)/T\big)}{\pi(t-nT)/T}$$

zero-order hold

$$H_0(j\Omega)=Te^{-j\Omega T/2}\,\frac{\sin(\Omega T/2)}{\Omega T/2}$$

discrete-time processing (no aliasing)

$$H_{\text{eq}}(j\Omega)=H(e^{j\Omega T}),\ |\Omega|<\pi/T$$

downsampling $\quad x_d[n]=x[nM]$

$$X_d(e^{j\omega})=\frac{1}{M}\sum_{i=0}^{M-1}X\big(e^{j(\omega-2\pi i)/M}\big)$$

upsampling

$$x_e[n]=x[n/L]\ (n=0,\pm L,\dots), \quad X_e(e^{j\omega})=X(e^{j\omega L})$$

frequencies $\quad \omega=\Omega T, \quad \Omega_s=\frac{2\pi}{T}=2\pi f_s$

In this chapter two frequency variables are used together: Ω is the continuous-time angular frequency (rad/s) and ω is the discrete-time frequency (rad/sample); they are related by $\omega=\Omega T$. Frequency in hertz is f, and the sampling frequency is $f_s=1/T$. Sampling is the multiplication of $x(t)$ by an impulse train; in the frequency domain this multiplication corresponds to the convolution of the spectra and produces replicas of the spectrum spaced Ω_s apart. If the replicas overlap, aliasing occurs. The ideal C/D (continuous-to-discrete-time) converter produces $x[n]=x(nT)$; the spectrum of the sequence is its discrete-time Fourier transform (DTFT), $X(e^{j\omega})$. The ideal D/C (discrete-to-continuous-time) converter performs reconstruction with an ideal lowpass filter of gain T and cutoff frequency $\Omega_s/2$. Unless stated otherwise, signals are real; spectrum plots are real and even, and in plots drawn only for $\Omega \geq 0$ the negative side is the mirror image. The Nyquist rate is $2f_M$ (f_M is the highest frequency in the signal).

Impulse-train sampling and the sampling theorem

8.1 ★☆☆ Solution →

The spectrum of the real signal $x(t)$ is zero for $|f|>2.5$ kHz. Which of the following signals has a Nyquist rate of 15 kHz?

A) $x(t)*x(t)$ (convolution)

B) $x^3(t)$

C) $x(t)\cos(2\pi\cdot 4000\,t)$

D) $x^2(2t)$

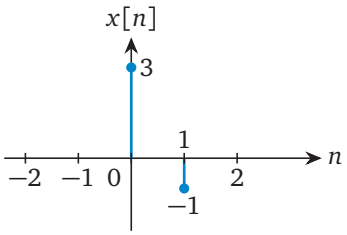
E) $x(t)\,\frac{dx(t)}{dt}$

Reconstruction and the anti-aliasing filter

8.6

★★☆ Solution →

The sequence shown in the figure is applied to an ideal D/C converter ($T = 0.5$ ms): $x[0] = 3$, $x[1] = -1$, and all other samples are zero.



a. Write the expression for the reconstructed $x_r(t)$.

b. Find the values $x_r(0.25\text{ ms})$ and $x_r(0.75\text{ ms})$.

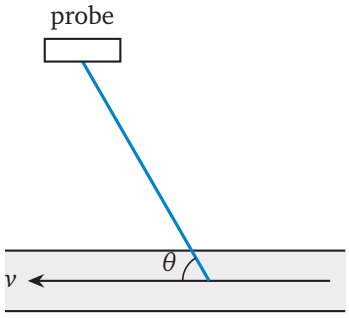
c. Find the spectrum $X_r(j\Omega)$ of $x_r(t)$. At which Ω is $|X_r(j\Omega)|$ largest?

Mixed problems

8.15

★★★ Solution →

A pulsed Doppler ultrasound device has probe frequency $f_0 = 5$ MHz; the speed of sound in tissue is $c = 1540$ m/s, and the beam–flow angle is $\theta = 60°$ (figure; blood flows toward the probe). The Doppler shift is $f_D = 2vf_0 \cos\theta / c$. The Doppler signal is sampled at the pulse repetition frequency (PRF): $f_s = \text{PRF} = 6$ kHz. The device takes complex samples; the apparent frequency is brought into $-f_s/2 \le f < f_s/2$, and its sign (negative or positive) gives the flow direction.



a. What is the largest velocity measurable without aliasing?

b. For blood with true velocity $v = 1.2$ m/s, find the apparent Doppler frequency and the apparent velocity.

c. The vessel is at depth $d = 12$ cm; a pulse cannot be sent before the echo returns: $\text{PRF} \le c/(2d)$. Is increasing the PRF enough? If not, what is the maximum f_0 in MHz?

SOLUTIONS

3 CONVOLUTION

3.1

← Problem

In the sum $y[n]=\sum_k x[k]h[n-k]$, a term is nonzero only if both $2\le k\le 9$ and $-2\le n-k\le 4$. The values of n for which the two conditions can hold together are found by adding the lower limits and adding the upper limits:

$$2+(-2)=0\le n\le 9+4=13.$$

The end samples consist of a single term: $y[0]=x[2]h[-2]\neq 0$ and $y[13]=x[9]h[4]\neq 0$. The number of samples is $8+7-1=14$.

- (B): The lengths were added to give $8+7=15$ samples; the correct count is $8+7-1=14$.
- (C): This is the union of the two intervals; in convolution the intervals are not merged, the limits are added.
- (D): This is the intersection of the two intervals; it is the interval where the product $(x[n]h[n])$ is nonzero.
- (E): The lower limit was found by subtraction, $2-(-2)=4$; the starting indices are added.

ANSWER A

3.3

← Problem

a. For finite sequences the output length is $L_y=L_x+L_h-1$: $4=2+L_h-1$, so $L_h=3$ $(n=0,1,2)$. The convolution is written in terms of the input samples:

$$y_1[n]=2h[n]-h[n-1].$$

Since the channel is causal, $h[-1]=0$, and the equation is solved step by step starting from $n=0$:

$$h[0]=\frac{y_1[0]}{2}=1,\qquad h[1]=\frac{y_1[1]+h[0]}{2}=0.5,$$

$$h[2]=\frac{y_1[2]+h[1]}{2}=\frac{-1+0.5}{2}=-0.25.$$

Check: The last measurement contains only $h[2]$: $y_1[3]=-h[2]=0.25$, the same as the measured value.

b. $y_2[n]=h[n]+h[n-1]-h[n-2]$; the copies are added:

n	0	1	2	3	4
$h[n]$	1	0.5	−0.25		
$h[n-1]$		1	0.5	−0.25	
$-h[n-2]$			−1	−0.5	0.25
$y_2[n]$	1	1.5	−0.75	−0.75	0.25

The signs of the first three samples are the same as those of the transmitted symbols $(+,+,-)$, but their amplitudes have changed and the sequence has become two samples longer: the channel spreads each symbol onto its neighbors (intersymbol interference).

ANSWER a. 3 samples; $h[n]=\{1,0.5,-0.25\}$, $n=0,1,2$ b. $y_2[n]=\{1,1.5,-0.75,-0.75,0.25\}$, $n=0,\dots,4$

3.7

← Problem

a. $h(t-\tau)$ equals $1/T$ for $t-T\le\tau\le t$; the output is the average of the input over the last T seconds:

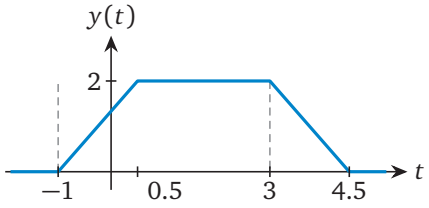
$$y(t)=\frac{1}{T}\int_{t-T}^{t}x(\tau)\,d\tau.$$

For $T=1.5$ the window $[t-1.5,\,t]$ slides over the pulse ($[-1,3]$):

- $t<-1$ or $t-1.5>3$ $(t>4.5)$: no overlap, $y=0$.
- $-1\le t\le 0.5$: the overlap is $[-1,t]$, $y=\frac{2}{1.5}(t+1)=\frac{4}{3}(t+1)$.
- $0.5\le t\le 3$: the window is inside the pulse, $y=2$.
- $3\le t\le 4.5$: the overlap is $[t-1.5,\,3]$, $y=\frac{4}{3}(4.5-t)$.

The expressions agree at the boundaries: $\frac{4}{3}(0.5+1)=2$. The output is a trapezoid whose rise and fall times are $T=1.5$ s:

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The dashed lines are the edges of the input: the moving average spreads sharp edges over a time T.

b. For $T \le 4$ the flat top is the interval $-1+T \le t \le 3$, in which the window lies entirely inside the pulse; its duration is $4-T$:

$$4-T \ge 3.2 \;\Rightarrow\; T \le 0.8 \text{ s}.$$

c. For $T = 5 > 4$ the window can contain the whole pulse; this happens when $t-5 \le -1$ and $t \ge 3$, that is, for $3 \le t \le 4$. There the output is the area of the pulse $(2 \cdot 4 = 8)$ divided by T:

$$y = \frac{8}{5} = 1.6 \qquad (3 \le t \le 4).$$

When the window is longer than the pulse, the peak value drops: the pulse fills only part of the window.

ANSWER a. $y = 0$ $(t < -1,\ t > 4.5)$; $(4/3)(t+1)$ $(-1 \le t \le 0.5)$; 2 $(0.5 \le t \le 3)$; $(4/3)(4.5-t)$ $(3 \le t \le 4.5)$ b. $T \le 0.8$ s c. 1.6; $3 \le t \le 4$

5 CONTINUOUS-TIME FOURIER TRANSFORM

5.2

← Problem

a. The pulse equals 20 only for $0 \le t \le 10$:

$$X(j\omega) = \int_0^{10} 20\, e^{-j\omega t}\, dt = \frac{20\,(1-e^{-j10\omega})}{j\omega}.$$

With $1-e^{-j10\omega} = e^{-j5\omega} \cdot 2j \sin 5\omega$:

$$X(j\omega) = 200\,\frac{\sin 5\omega}{5\omega}\, e^{-j5\omega} \text{ W ns},$$

$$|X| = 200\left|\frac{\sin 5\omega}{5\omega}\right|, \qquad \angle X = -5\omega.$$

(Where $\sin 5\omega$ becomes negative, π is added to the phase.) The linear phase comes from the shift of the pulse center to $t = 5$ ns.

b. $X(0) = \int x\, dt = 20(10) = 200$ W ns $= 200$ nJ: this is the light energy in the pulse.

c. $\sin 5\omega = 0$, $\omega \ne 0$: the first zero is at $5\omega = \pi$:

$$\omega = \frac{\pi}{5} = 0.6283 \text{ rad/ns},$$

$$f = \frac{\omega}{2\pi} = 0.1 \text{ GHz} = 100 \text{ MHz}.$$

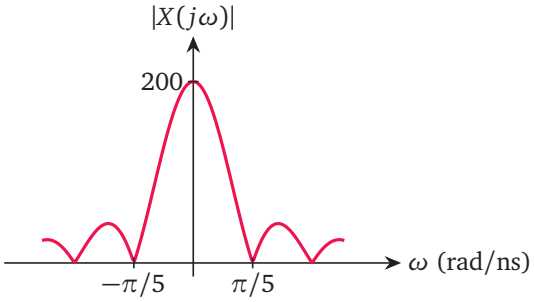
To three significant figures, $\omega = 0.628$ rad/ns. The frequency of the first zero is $f = 1/(\text{pulse width})$; the shorter the pulse, the wider the spectrum.

d. 50 MHz: $\omega = 2\pi(0.05) = \pi/10$ rad/ns, $5\omega = \pi/2$:

$$\frac{|X|}{X(0)} = \frac{\sin(\pi/2)}{\pi/2} = \frac{2}{\pi} = 0.6366,$$

$$20 \log_{10} 0.6366 = -3.92 \text{ dB}.$$

To three significant figures, the ratio is 0.637.

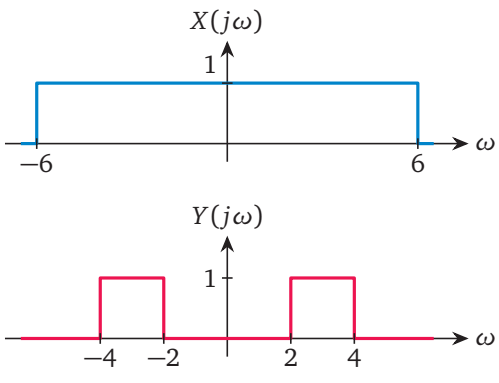


ANSWER
a. $X(j\omega) = 200\,[\sin 5\omega/(5\omega)]\, e^{-j5\omega}$ W ns; $|X| = 200|\sin 5\omega/(5\omega)|$, $\angle X = -5\omega$ ($\pm\pi$ added on intervals where $\sin 5\omega < 0$) b. 200 nJ (pulse energy) c. $\omega = \pi/5 = 0.628$ rad/ns; 100 MHz d. $2/\pi = 0.637$; −3.92 dB

5.13

← Problem

a. $\sin Wt/(\pi t) \overset{\mathcal{F}}{\longleftrightarrow} 1$ $(|\omega| < W)$: $X(j\omega) = 1$ $(|\omega| < 6)$. $Y = XH = 1$ $(2 < |\omega| < 4)$:



b. Y is the difference of two ideal lowpass spectra

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($W=4$ and $W=2$):

$$y(t)=\frac{\sin 4t}{\pi t}-\frac{\sin 2t}{\pi t}=\frac{2}{\pi t}\cos 3t\,\sin t,$$

($\sin 4t-\sin 2t=2\cos 3t\sin t$). y is the cosine at the band center (3 rad/s) multiplied by the envelope $\sin t/(\pi t)$.

c. Parseval:

$$E_x=\frac{1}{2\pi}\int_{-6}^{6}1\,d\omega=\frac{12}{2\pi}=\frac{6}{\pi}=1.910,$$

$$E_y=\frac{1}{2\pi}\cdot 2(4-2)=\frac{2}{\pi}=0.6366.$$

To three significant figures, $E_x=1.91$, $E_y=0.637$. 1/3 of the energy passes (the band is one third of the flat part of the spectrum).

d. $\sin at/(\pi t)\to a/\pi$ ($t\to 0$): $y(0)=(4-2)/\pi=0.6366$. **Check:** $y(0)=\frac{1}{2\pi}\int Y\,d\omega=4/2\pi$.

ANSWER a. $X=1$ ($|\omega|<6$); $Y=1$ ($2<|\omega|<4$) b. $y=(\sin 4t-\sin 2t)/(\pi t)=(2/\pi t)\cos 3t\sin t$ c. $E_x=6/\pi=1.91$, $E_y=2/\pi=0.637$; 1/3 d. $2/\pi=0.637$

5.18

← Problem

a. $e^{-t}u(t)\overset{\mathcal{F}}{\longleftrightarrow}1/(1+j\omega)$; by time reversal, $e^{t}u(-t)\overset{\mathcal{F}}{\longleftrightarrow}1/(1-j\omega)$:

$$X(j\omega)=\frac{1}{1+j\omega}-\frac{1}{1-j\omega}=\frac{-2j\omega}{1+\omega^2}.$$

x is real and odd; its transform is purely imaginary and odd ($X(-j\omega)=-X(j\omega)=X^*(j\omega)$).

b. Convolution property:

$$Y(j\omega)=X^2=\frac{-4\omega^2}{(1+\omega^2)^2}.$$

Write $\omega^2=(1+\omega^2)-1$:

$$Y(j\omega)=\frac{-4}{1+\omega^2}+\frac{4}{(1+\omega^2)^2},\qquad A=-4,\ B=4.$$

c. With the given pairs, the first term is $-2e^{-|t|}$ and the second is $(1+|t|)e^{-|t|}$:

$$y(t)=-2e^{-|t|}+(1+|t|)e^{-|t|}=(|t|-1)\,e^{-|t|}.$$

y is even (the convolution of two odd signals is even) and real; Y is also real and even.

d. $y(0)=(0-1)e^{0}=-1$. From the frequency domain ($\int_{-\infty}^{\infty}\frac{\omega^2}{(1+\omega^2)^2}d\omega=\frac{\pi}{2}$):

$$y(0)=\frac{1}{2\pi}\int Y\,d\omega=\frac{-4}{2\pi}\cdot\frac{\pi}{2}=-1.$$

Also, $y(0)=\int x(\tau)x(-\tau)\,d\tau=-\int x^2\,d\tau=-1$; the energy of x is 1.

ANSWER a. $X=-2j\omega/(1+\omega^2)$ (purely imaginary, odd) b. $Y=-4/(1+\omega^2)+4/(1+\omega^2)^2$ c. $y(t)=(|t|-1)e^{-|t|}$ d. $y(0)=-1$

8 SAMPLING

8.1

← Problem

The Nyquist rate is twice the highest frequency in the signal. The effect of each operation on the bandwidth:

- Multiplication in time is convolution in frequency: the bandwidth of the product of two signals is the sum of their bandwidths. The bandwidth of x^3 is $3(2.5)=7.5$ kHz; the Nyquist rate is $2(7.5)=15$ kHz. (B) is correct.
- (A): Convolution in time is multiplication in frequency: $X^2(j\Omega)$; the bandwidth is unchanged (2.5 kHz), and the Nyquist rate is 5 kHz.
- (C): Multiplication by the cosine shifts the spectrum to ± 4 kHz; the highest frequency is $4+2.5=6.5$ kHz, and the Nyquist rate is 13 kHz.
- (D): The bandwidth of x^2 is 5 kHz; compression by a factor of 2 in time multiplies the bandwidth by 2: 10 kHz, Nyquist rate 20 kHz.
- (E): Differentiation does not change the bandwidth ($j\Omega X(j\Omega)$); the bandwidth of the product of two 2.5 kHz signals is 5 kHz, and the Nyquist rate is 10 kHz.

ANSWER B

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8.6

← Problem

a. In ideal reconstruction each sample multiplies the interpolating function $\phi(v)=\sin(\pi v)/(\pi v)$ centered at nT ($v=t/T-n$); only two terms remain:

$$x_r(t)=3\,\phi\Big(\frac{t}{T}\Big)-\phi\Big(\frac{t}{T}-1\Big),$$

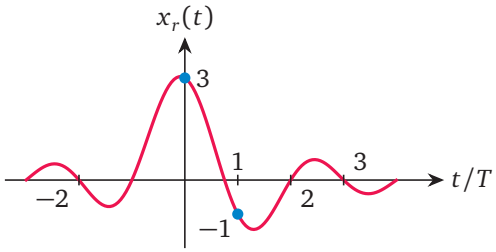
$$x_r(t)=3\,\frac{\sin(\pi t/T)}{\pi t/T}-\frac{\sin\big(\pi(t-T)/T\big)}{\pi(t-T)/T}.$$

b. $t=0.25\text{ ms}=T/2$: the two arguments are $\pi/2$ and $-\pi/2$; $\sin(\pi/2)/(\pi/2)=2/\pi=0.6366$:

$$x_r(T/2)=(3-1)\frac{2}{\pi}=\frac{4}{\pi}=1.273.$$

$t=0.75\text{ ms}=1.5T$: the arguments are 1.5π and 0.5π; $\sin(1.5\pi)/(1.5\pi)=-2/(3\pi)=-0.2122$:

$$x_r(1.5T)=3\Big(-\frac{2}{3\pi}\Big)-\frac{2}{\pi}=-\frac{4}{\pi}=-1.273.$$



At the sampling instants ($t=nT$) the curve passes through the sample values (blue dots), and it is zero at the other integer values of t/T; it oscillates between and outside the samples.

c. The DTFT of $x[n]$ is $X(e^{j\omega})=3-e^{-j\omega}$. The ideal D/C passes $X(e^{j\Omega T})$ with gain T in the band $|\Omega|<\pi/T$:

$$X_r(j\Omega)=T\big(3-e^{-j\Omega T}\big)$$

for $|\Omega|<\pi/T=2000\pi$, and zero outside. $|3-e^{-j\Omega T}|^2=10-6\cos(\Omega T)$ is largest at $\Omega T=\pm\pi$: as $|\Omega|\to 2000\pi$ rad/s (1 kHz), $|X_r|\to 4T=2\times10^{-3}$. Because successive samples alternate in sign, the energy is concentrated at the upper end of the band.

Check: At $t=T$ the first term is $3\sin(\pi)/\pi=0$ and the second is -1: $x_r(T)=x[1]=-1$.

ANSWER a. $x_r(t)=3\phi(t/T)-\phi(t/T-1)$, $\phi(v)=\sin(\pi v)/(\pi v)$ b. $4/\pi=1.273$; $-4/\pi=-1.273$ c. $X_r=T(3-e^{-j\Omega T})$ ($|\Omega|<2000\pi$); largest as $|\Omega|\to 2000\pi$, $4T=2\times10^{-3}$

8.15

← Problem

a. The alias-free range is $|f_D|<f_s/2=3$ kHz. With $\cos 60^\circ=0.5$, $f_D=2v(5\times10^6)(0.5)/1540=3247v$ (Hz, with v in m/s):

$$v_{\max}=\frac{(f_s/2)\,c}{2f_0\cos\theta}=\frac{3000\cdot1540}{2(5\times10^6)(0.5)}=0.924\text{ m/s}.$$

b. True shift:

$$f_D=\frac{2(1.2)(5\times10^6)(0.5)}{1540}=3896\text{ Hz}.$$

$3896>3000$: it aliases. Subtracting f_s brings it into the range: $3896-6000=-2104$ Hz. Apparent velocity:

$$v_a=\frac{-2104\cdot1540}{2(5\times10^6)(0.5)}=-0.648\text{ m/s}.$$

The device shows the flow in the opposite direction, at 0.648 m/s. On the spectral display the peak is cut off at the top edge and re-enters from the bottom.

c. $\text{PRF}_{\max}=1540/(2\cdot0.12)=6417$ Hz; the alias-free limit is $6417/2=3208$ Hz <3896 Hz. Increasing the PRF is not enough (the required value is $2(3896)=7792$ Hz). f_D is proportional to f_0; for $f_D\le3208$ Hz:

$$f_0\le5\text{ MHz}\cdot\frac{3208}{3896}=4.117\text{ MHz}.$$

To stay within the limit, round down: $f_0\le4.11$ MHz. A lower probe frequency reduces the shift; this is why low-frequency probes are chosen for deep vessels. (Increasing the angle also reduces $\cos\theta$, but it increases the angle-correction error.)

Check: With $f_0=4.117$ MHz,

$$f_D=3896\cdot\frac{4.117}{5}=3208\text{ Hz};$$

the shift is exactly at the limit (with 4.12 MHz it would be 3210 Hz, exceeding the limit).

ANSWER a. 0.924 m/s b. $f_D=3896$ Hz; apparent −2104 Hz, −0.648 m/s (reversed direction) c. not enough ($\text{PRF}_{\max}=6417$ Hz, limit 3208 Hz); $f_0\le4.11$ MHz

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