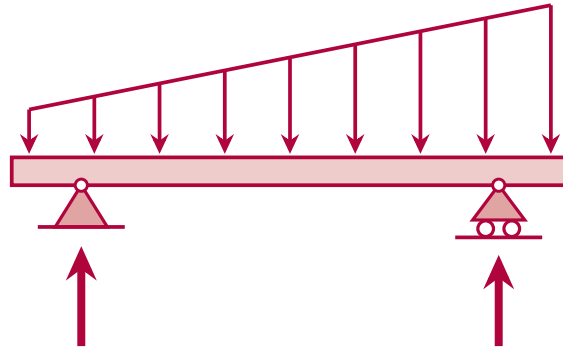


STATICS

WORKBOOK

11 chapters · 160 problems · step-by-step solutions



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CHAPTER 1

FORCE VECTORS

law of cosines

$$C^2 = A^2 + B^2 - 2AB \cos c$$

direction cosines

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

law of sines

$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

force along a line

$$\vec{F} = F \hat{u}_{AB} = F \frac{\vec{r}_{AB}}{r_{AB}}$$

coplanar resultant $R_x = \sum F_x$, $R_y = \sum F_y$, $\tan \theta = \frac{R_y}{R_x}$

dot product, projection

$$\vec{A} \cdot \vec{B} = AB \cos \theta, F_{\parallel} = \vec{F} \cdot \hat{u}$$

Unless stated otherwise, angles are measured counterclockwise from the $+x$ axis. In space the x, y, z axes form a right-handed system, and the z axis points upward. The forces act at a single point; moments are not used in this unit.

Scalars, vectors and the parallelogram law

1.1

☆☆☆ Solution → p. 109

Two forces of magnitude 5 kN and 9 kN act at the same point, and the angle between them can be set to any value. Which of the following *cannot* be the magnitude of the resultant force?

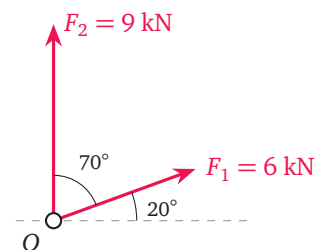
- A) 4.0 kN B) 3.5 kN C) 9.0 kN D) 12 kN E) 14 kN

1.2

☆☆☆ Solution → p. 109

Two forces act on a ring. The force $F_1 = 6$ kN points to the right at 20° above the horizontal, and the force $F_2 = 9$ kN acts vertically upward.

- Use the parallelogram law to determine the magnitude of the resultant force.
- Determine the angle that the resultant makes with the horizontal.

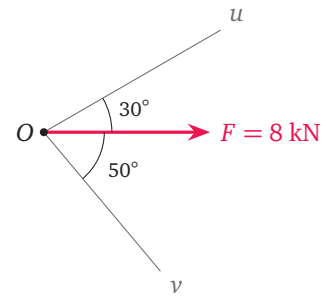


1.3

★★★ Solution → p. 109

The horizontal force $\vec{F}$ of 8 kN points to the right. The u axis is directed 30° above the horizontal, and the v axis 50° below it.

- Resolve $\vec{F}$ into components along the u and v directions (F_u and F_v).
- Determine the orthogonal projections of $\vec{F}$ onto the u and v axes. Why are they not the same as the components in part (a)? When would they be the same?

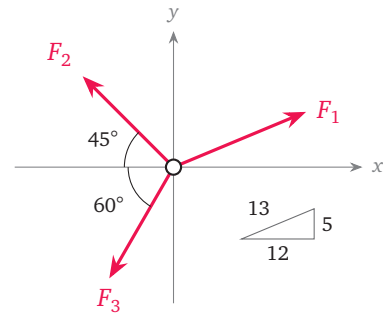


Addition of coplanar forces

1.4

☆☆☆ Solution → p. 110

The three forces shown act on an eyebolt: $F_1 = 520$ N (its slope is given by the 5-12-13 triangle), $F_2 = 350$ N and $F_3 = 300$ N. Determine the magnitude of the resultant force and its direction, measured counterclockwise from the $+x$ axis.

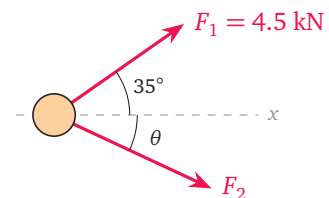


1.5

☆☆☆ Solution → p. 110

A tree stump is to be pulled out by means of two ropes. The force $F_1 = 4.5$ kN acts 35° above the $+x$ axis, and F_2 acts at an angle θ below it. The resultant is to be directed along the $+x$ axis.

- If $\theta = 25^\circ$, determine F_2 and the magnitude of the resultant.
- What should θ be so that F_2 is as small as possible? What are F_2 and the resultant, in kN, in this case?



1.4

← Problem p. 6

Resolve each force into components and add them. For F_1 the slope triangle gives $\cos = \frac{12}{13}$ and $\sin = \frac{5}{13}$.

$$\begin{aligned} R_x &= 520\left(\frac{12}{13}\right) - 350 \cos 45^\circ - 300 \cos 60^\circ \\ &= 480 - 247.5 - 150 = 82.5 \text{ N}, \\ R_y &= 520\left(\frac{5}{13}\right) + 350 \sin 45^\circ - 300 \sin 60^\circ \\ &= 200 + 247.5 - 259.8 = 187.7 \text{ N}. \end{aligned}$$

$$\begin{aligned} R &= \sqrt{82.5^2 + 187.7^2} = 205 \text{ N}, \\ \theta &= \arctan \frac{187.7}{82.5} = 66.3^\circ. \end{aligned}$$

Both R_x and R_y are positive, so the resultant lies in the first quadrant.

ANSWER $R = 205 \text{ N}$, $\theta = 66.3^\circ$

1.5

← Problem p. 6

a. Since the resultant acts along the x axis, $R_y = 0$:

$$\begin{aligned} \sum F_y : \quad & 4.5 \sin 35^\circ - F_2 \sin 25^\circ = 0 \\ \Rightarrow \quad & F_2 = \frac{2.581}{0.4226} = 6.11 \text{ kN}, \\ R = R_x &= 4.5 \cos 35^\circ + 6.11 \cos 25^\circ = 9.22 \text{ kN}. \end{aligned}$$

b. For a general θ , the equation in (a) gives

$$F_2 = \frac{4.5 \sin 35^\circ}{\sin \theta}.$$

The numerator is constant, so F_2 is smallest when $\sin \theta = 1$, that is, when $\theta = 90^\circ$.

$$\begin{aligned} F_{2,\min} &= 4.5 \sin 35^\circ = 2.58 \text{ kN}, \\ R &= 4.5 \cos 35^\circ = 3.69 \text{ kN}. \end{aligned}$$

The only job of F_2 is to cancel the vertical component of F_1 . When F_2 is vertical, all of its magnitude goes into this job; if it were inclined, part of it would go into a horizontal component. The price is a smaller resultant: only the horizontal component of F_1 is left to pull on the stump.

ANSWER a. $F_2 = 6.11 \text{ kN}$, $R = 9.22 \text{ kN}$
b. $\theta = 90^\circ$, $F_2 = 2.58 \text{ kN}$, $R = 3.69 \text{ kN}$

1.6

← Problem p. 7

First add the two fixed forces:

$$\begin{aligned} \vec{F}_1 + \vec{F}_2 &= (500 + 400 \cos 120^\circ, 400 \sin 120^\circ) \\ &= (300, 346.4) \text{ N}. \end{aligned}$$

This sum has a magnitude of 458.3 N and acts at 49.1° from $+x$. The resultant is

$$R_x = 300 + 500 \cos \theta, \quad R_y = 346.4 + 500 \sin \theta.$$

a. For a vertical resultant, $R_x = 0$, so $\cos \theta = -0.6$.

Two angles satisfy this:

- $\theta = 126.9^\circ$ ($\sin \theta = 0.8$): $R_y = 346.4 + 400 = 746 \text{ N}$, upward.
- $\theta = 233.1^\circ$ ($\sin \theta = -0.8$): $R_y = 346.4 - 400 = -53.6 \text{ N}$, that is, 53.6 N downward.

b. $\vec{R} = (\vec{F}_1 + \vec{F}_2) + \vec{F}_3$. The sum of two vectors is largest when they point in the same direction, so $\vec{F}_3$ must have the same direction as $(\vec{F}_1 + \vec{F}_2)$.

$$\theta = 49.1^\circ, \quad R_{\max} = 458.3 + 500 = 958 \text{ N}.$$

c. The smallest resultant occurs when $\vec{F}_3$ points the opposite way ($\theta = 229.1^\circ$): $R_{\min} = 500 - 458.3 = 41.7 \text{ N}$. The resultant can never be zero, because $F_3 \neq |\vec{F}_1 + \vec{F}_2|$.

ANSWER a. $\theta = 126.9^\circ$ ($R = 746 \text{ N} \uparrow$) or $\theta = 233.1^\circ$ ($R = 53.6 \text{ N} \downarrow$) b. $\theta = 49.1^\circ$, $R_{\max} = 958 \text{ N}$ c. $41.7 \text{ N} \leq R \leq 958 \text{ N}$

1.7

← Problem p. 8

The direction cosines are the components of a unit vector, so the sum of their squares is 1:

$$\cos^2 \gamma = 1 - \cos^2 60^\circ - \cos^2 135^\circ = 1 - \frac{1}{4} - \frac{1}{2} = \frac{1}{4}.$$

Hence $\cos \gamma = \pm \frac{1}{2}$, so $\gamma = 60^\circ$ or $\gamma = 120^\circ$. Only 120° appears among the choices.

- (A) and (E): $\cos^2 \gamma = \frac{3}{4}$; the sum of the squares would be 1.5.
- (B): $\cos^2 \gamma = \frac{1}{2}$; the sum would be 1.25.
- (C): $\cos^2 \gamma = 0$; the sum would be 0.75.

ANSWER D

1.8

← Problem p. 8

a. First resolve $\vec{F}$ into a vertical component and its projection F' on the xy plane; then resolve F' into x and y components:

$$F_z = 900 \sin 30^\circ = 450 \text{ N},$$

$$F' = 900 \cos 30^\circ = 779.4 \text{ N},$$

$$F_x = F' \cos 60^\circ = 389.7 \text{ N},$$

$$F_y = F' \sin 60^\circ = 675 \text{ N}.$$

$$\vec{F} = (390\hat{i} + 675\hat{j} + 450\hat{k}) \text{ N}.$$

b.

$$\alpha = \arccos \frac{389.7}{900} = 64.3^\circ,$$

$$\beta = \arccos \frac{675}{900} = 41.4^\circ,$$

$$\gamma = \arccos \frac{450}{900} = 60.0^\circ.$$

The result $\gamma = 90^\circ - 30^\circ$ is expected: if $\vec{F}$ makes 30° with the xy plane, it makes 60° with the z axis.

ANSWER a. $\vec{F} = (390\hat{i} + 675\hat{j} + 450\hat{k}) \text{ N}$

b. $\alpha = 64.3^\circ$, $\beta = 41.4^\circ$, $\gamma = 60.0^\circ$

1.9

← Problem p. 8

Find γ_1 from the direction cosines:

$$\cos^2 \gamma_1 = 1 - \cos^2 60^\circ - \cos^2 120^\circ = \frac{1}{2}.$$

Since $\gamma_1 > 90^\circ$, $\cos \gamma_1 = -0.7071$ and $\gamma_1 = 135^\circ$.

$$\begin{aligned} \vec{F}_1 &= 600(0.5, -0.5, -0.7071) \\ &= (300, -300, -424.3) \text{ N}. \end{aligned}$$

The required resultant is $\vec{R} = (0, 0, 1000) \text{ N}$. Since $\vec{F}_1 + \vec{F}_2 = \vec{R}$,

$$\vec{F}_2 = \vec{R} - \vec{F}_1 = (-300, 300, 1424.3) \text{ N},$$

$$F_2 = \sqrt{300^2 + 300^2 + 1424.3^2} = 1486 \text{ N}.$$

$$\alpha_2 = \arccos \frac{-300}{1486} = 101.6^\circ,$$

$$\beta_2 = \arccos \frac{300}{1486} = 78.4^\circ,$$

$$\gamma_2 = \arccos \frac{1424.3}{1486} = 16.6^\circ.$$

$\vec{F}_2$ must cancel the horizontal components of $\vec{F}_1$.

Vertically, it must both balance the 424.3 N downward

component of F_1 and supply 1000 N upward. That is why F_2 is larger than the resultant.

ANSWER $F_2 = 1.49 \text{ kN}$, $\alpha_2 = 101.6^\circ$, $\beta_2 = 78.4^\circ$, $\gamma_2 = 16.6^\circ$

1.10

← Problem p. 9

a. The position vector is found by subtracting the coordinates of its tail from those of its head:

$$\vec{r}_{AB} = (6-2)\hat{i} + (5-1)\hat{j} + (0-7)\hat{k} = (4\hat{i} + 4\hat{j} - 7\hat{k}) \text{ m}.$$

$$\text{b. } r_{AB} = \sqrt{4^2 + 4^2 + 7^2} = \sqrt{81} = 9.00 \text{ m}.$$

c.

$$\hat{u}_{AB} = \frac{\vec{r}_{AB}}{r_{AB}} = \frac{4}{9}\hat{i} + \frac{4}{9}\hat{j} - \frac{7}{9}\hat{k}.$$

$$\alpha = \beta = \arccos \frac{4}{9} = 63.6^\circ,$$

$$\gamma = \arccos \left(-\frac{7}{9} \right) = 141.1^\circ.$$

$\gamma > 90^\circ$: the pipe slopes downward as it goes from A to B.

ANSWER a. $\vec{r}_{AB} = (4\hat{i} + 4\hat{j} - 7\hat{k}) \text{ m}$ b. 9.00 m

c. $\hat{u}_{AB} = (\frac{4}{9}, \frac{4}{9}, -\frac{7}{9})$; $\alpha = \beta = 63.6^\circ$, $\gamma = 141.1^\circ$

1.11

← Problem p. 9

a. Each wire force is directed from A toward the other end of the wire (the wire pulls on the antenna).

$$\vec{r}_{AB} = (11, -2, -10) \text{ m}, \quad r_{AB} = 15 \text{ m},$$

$$\vec{F}_{AB} = 750 \frac{(11, -2, -10)}{15} = (550, -100, -500) \text{ N},$$

$$\vec{r}_{AC} = (-2, 11, -10) \text{ m}, \quad r_{AC} = 15 \text{ m},$$

$$\vec{F}_{AC} = 600 \frac{(-2, 11, -10)}{15} = (-80, 440, -400) \text{ N}.$$

b.

$$\vec{R} = (470, 340, -900) \text{ N},$$

$$R = \sqrt{470^2 + 340^2 + 900^2} = 1071 \text{ N}.$$

$$\alpha = \arccos \frac{470}{1071} = 64.0^\circ,$$

$$\beta = \arccos \frac{340}{1071} = 71.5^\circ,$$

$$\gamma = \arccos \frac{-900}{1071} = 147.2^\circ.$$

Most of the resultant acts downward and compresses the antenna mast; its horizontal component (580 N)